SOUND

A PHYSICAL TEXT-BOOK

BY

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PREFACE TO FIRST EDITION

This branch of physics has received renewed attention from research workers during the past decade, stimulated no doubt in part by the European War and by the development of broadcasting. The need for a text-book on sound, which will give the results of recent experimental research, has often been expressed. This book is an attempt to fill that gap, and while it is hoped that sufficient mathematical theory has been given to enable the physical conceptions to be followed, it does not pretend to take the place of the mathematical treatises of Rayleigh and Lamb, to which the reader is referred for a more rigorous and complete analysis when necessary. A knowledge of physics and mathematics to Intermediate Science standard, and of elementary calculus is assumed. The book covers all that a candidate for the pass and honours examinations of British and American Universities should require, and at the same time the needs of the research worker and technician have been met by describing the important work which has been done in the subject up to the present, with copious references to original papers where further details may be discovered. It has been the aim to include references to all complete papers which have appeared in the generally circulating scientific periodicals during the last twenty years, though certain have been left out as lacking physical interest, or on account of unconscious plagiarism. Prior to 1917 references are given only to the authors cited in the text, as the presence of a copy of Winkelmann's monumental *Handbuch der Physik* in most university libraries renders a bibliography of papers previous to 1907 unnecessary. Dimensions of certain pieces of apparatus have been given, where it was thought that they would make useful laboratory experiments, or would be of general use to the worker in sound.

I have tried to meet the criticism sometimes levelled against physical treatises on this subject, that they ignore or run counter to musical practice ; sections on all the important types of musical instruments are included. Nor should any apology be needed for

the inadequate digression at the beginning of Chapter VI on the important theories of Prandtl concerning viscous fluids; work which has not yet found its way into the text-books of physics, despite the test of twenty years' successful application to aeronautics.

It has been my endeavour, though I have not been able to carry it out in its entirety, to allot to each concept a distinct symbol. For this reason, and for the sake of consistency, certain quantities have had to lose their conventional symbols.

I wish to express my great indebtedness to Mr. D. Orson Wood, M.Sc., who suggested the compilation of this book, and without whose valuable suggestions and assistance in reading the manuscript, the book could scarcely have seen the light in a presentable form. To Prof. A. W. Porter, D.Sc., F.R.S., Mr. R. C. Richards, B.A., M.Sc., and Mr. R. S. Maxwell, B.A., B.Sc., also I am indebted for helpful suggestions and criticism. I owe my best thanks to Mr. H. J. Smith for the drawings, and to my father, Mr. E. G. Richardson, M.A., B.Sc., for assistance in the preparation of the manuscript.

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PREFACE TO THIRD EDITION

In the second edition (1935), new chapters on impedance, supersonics and sound reproduction were added, and the rest of the book fairly thoroughly revised. In this, the third edition, these new chapters have been further extended and brought up to date, since it is in these departments that most of the research work of the past four years has taken place. The remainder of the book has required only detailed amplification. The position in sound now is very different from that in 1927; there are now a number of works on different aspects of the new developments as well as three periodicals dealing solely with the subject, while in industry the penetration of applied acoustics has been far-reaching, so much so that it has been impossible to adhere to the original intention and print all references. The reader may add material from year to year from the annual reports on the progress of physics which the Physical Society publishes. I am indebted to many teachers, students and research workers for their continued support of the book, particularly to those who have written me proffering suggestions, most of which I have incorporated in the new edition. Certain paragraphs are printed in small type. This is to indicate that they deal with more advanced aspects of sound and may be omitted by the reader who wishes to gain a general knowledge of the subject. There is no intention to suggest that the matter therein is any less important than that in the larger type.

I wish to thank Dr. H. L. Penman for carefully correcting the manuscript of the new matter.

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CHAPTER ONE

THE VELOCITY OF SOUND

First Principles. In seeking the origin of the phenomena which we classify under "sound" we have to deal with a sudden compression at a point in a medium, and the *raison d'être* of the phenomena we are about to study lies in the elasticity of the medium. If the latter were perfectly compressible, our interest in the matter would cease at its inception, but the particles which are pressed together tend to resume their original positions, and, in doing so, react on the surrounding particles, which are in turn compressed; so that the original sudden change in the state of the medium travels out from the point at which it arose in the form of a wave.

Again owing to inertia, the return of the disturbed portions of the medium is never "dead beat"; inertia causes the particles of the medium to overshoot their position of rest, so that in the track of the compression there follows a rarefaction, and another (but less intense) compression, as the oscillations of the particles about their former position are more or less rapidly damped. A single wave of compression is thus only an ideal conception, but is approximately realized by a short, sharp noise called a "pulse." The state of things at our point of origin may be made to pass through a recurring series of changes by artificial means, resulting in the emission of a train of compressions and rarefactions at regular intervals: this we term a musical note. The type of wave-motion (transverse, longitudinal, torsional) may be changed during propagation as it passes from one medium to another, but finally becomes apparent to us by an excitement of the auditory nerves; or, if a registering apparatus is used instead of the ear, of the optical nerves, since a recording apparatus usually functions by moving a ray of light. The velocity with which the disturbance travels will obviously depend on the medium for the

type of deformation involved, and on the closeness of packing of the material of the medium (its density).

The study of this subject as a branch of physics has had a peculiar history. It lay at first entirely in the hands of musicians who accumulated a great deal of more or less empirical data. It was known to the ancient Greeks that rapid vibration produced a tone of "high pitch"; and slow vibration, one of "low pitch." The pioneer physicists in modern times who endeavoured to set the subject on a scientific basis were: Mersenne,[1] Newton,[2] Young,[3] Chladni,[4] the brothers Weber,[5] and Savart.[6]

Velocity of Longitudinal Waves in Gases, especially Air. The sounds which affect a normal ear come through the atmosphere, so that the propagation of sound in the air is of prime importance. When the air follows a simple pendular motion under the maintaining action of the source of sound, the ear hears a single tone, of which the number of complete oscillations per second is called the "frequency" of the source; and if an instantaneous picture of the medium be imagined, the length of the unit of the "pattern," or the distance between successive compressions, is known as the "wave-length."

The fractional decrease in volume, or the decrease in volume per unit of original volume, which a compressed body of gas may undergo, is known as the "condensation" s; in symbols, $s = -\frac{\delta v}{v}$. The compressibility may be defined as the fractional decrease of volume produced by unit change of pressure. The elasticity E is the inverse of this; the more compressible the gas the less elastic it is; in symbols

$$E = -\delta p \frac{v}{\delta v} \qquad \ldots \ldots \ldots \quad (1)$$

(the negative sign shows that increase of pressure produces decrease of volume).

Consider a tube of gas (Fig. 1) of unit cross section, and two plane sections A, B, of this tube, whose co-ordinates measured along the tube are, before the passage of the wave, x and $x + \delta x$ respectively; so that the initial volume of the slice so cut off is δx.

[1] *Harmonicarum*, 1636. [2] *Principia*, 1687.

[3] *Principal Phenomena of Sounds*, 1784.

[4] *Theorie des Klanges*, 1787.

[5] *Wellenlehre*, 1825. [6] *Ann. de Chemie*, 1820, etc.

At an instant of time δt later, let the arrival of the disturbance have displaced the end A of our slice by an amount ξ to A', so that its co-ordinate is now $x + \xi$. Other sections of the slice will have a different displacement, for the displacement will vary with x at a rate $\frac{\partial \xi}{\partial x}$, so that, e.g., the displacement of the end B will be $\xi + \frac{\partial \xi}{\partial x}\delta x$, and this plane will now occupy a position B' given by $x + \delta x + \xi + \frac{\partial \xi}{\partial x}\delta x$, and the new volume of the slice will be $\delta x + \frac{\partial \xi}{\partial x}\delta x$, representing an increase of volume of $\frac{\partial \xi}{\partial x}\delta x$; this, divided by the original δx, is therefore, by definition, the condensation $s = -\frac{\partial \xi}{\partial x}$. Now the pressure on a section differs from the normal by an amount which varies with its position; so that, if that on the plane through A' is δp, that on B' is :—

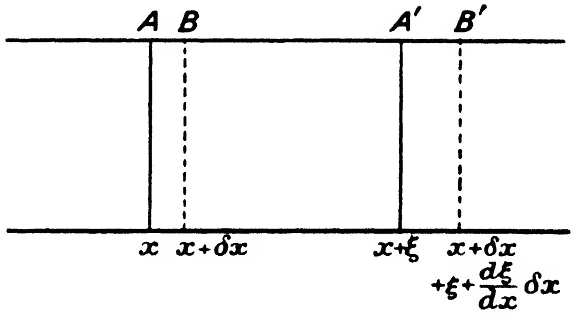


FIG. 1.—Velocity of Longitudinal Waves.

$$\delta p + \frac{\partial(\delta p)}{\partial x}.\delta x = Es + E\frac{\partial s}{\partial x}.\delta x, \text{ in virtue of (1).}$$

The total force on the slice = the difference of pressure on the two ends :—

$$-E\frac{\partial s}{\partial x}.\delta x = E\frac{\partial^2 \xi}{\partial x^2}.\delta x.$$

Equating this to the mass × acceleration of the slice, the mass being $\rho\delta x$ where ρ is the density of the gas, we have :—

$$E\frac{\partial^2 \xi}{\partial x^2}.\delta x = \rho\delta x\frac{\partial^2 \xi}{\partial t^2};$$

or

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{E}{\rho}\cdot\frac{\partial^2 \xi}{\partial x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Putting

$$\frac{E}{\rho} = c^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

we find that $\sqrt{\frac{E}{\rho}}$, i.e., c, has the dimensions of a velocity. In the succeeding chapters it will be shown that the solution of (2) represents longitudinal waves travelling with this velocity.

This formula is only true for plane waves, and in the case where the original disturbance is of infinitesimal amplitude, but is approximately fulfilled for sounds of ordinary strength. Having regard to the correct value of the elasticity E for the type of wave in question, this formula also applies to longitudinal waves in liquids and solids, and to torsional waves in solids, and to the transverse vibrations of strings (but not, without modification, to the transverse vibrations of bars and plates).

Value of E in Free Air—Newton's Formula and Laplace's Correction. From the formula (2) we can calculate the velocity (c) of sound in terms of the pressure and density, if we assume some relation between the pressure and volume (or density) of a gas. Newton [1] obtained, by other reasoning, a similar formula, and adopted the isothermal relation (pv = a constant) implying that the heat produced in the compression was rapidly conducted away from the layer of air affected, so as not to raise its temperature appreciably. Differentiating,

$$\frac{\delta p}{p} + \frac{\delta v}{v} = 0,$$

$$E = -\,\delta p \frac{v}{\delta v} = p$$

From (3) $c = \sqrt{\frac{p}{\rho}}$ . . . . (4)

At 0° C., $p = 76 \times 13{\cdot}6 \times 981 = 10^6$ dynes per sq. cm. (approx.).

$\rho = {\cdot}0013$ gms. per c.c.

$\therefore c = 280$ metres per second.

Now the earliest experimental determinations had shown a value considerably in excess of this. Newton ascribed this discrepancy to the fact that he had neglected to allow for the space occupied by the molecules in the actual atmosphere. This would imply that if one used a formula taking account of this space, there would be agreement between the experimental and theoretical results. Newton endeavoured to make some such correction by assuming that the molecules of air were incompressible, so that

[1] *Principia.*

the disturbance passed instantaneously from one side of a molecule to the other, but he could not by this assumption raise his value to meet the experimental results.

It was left to Laplace[1] some 120 years later to provide the explanation which has borne the test of 100 years' subsequent experiment. He held that the compressions and rarefactions succeed each other so rapidly that the adiabatic relation is pertinent. The heat produced in compressing each stratum of the gas has not time to be dissipated in the body of the gas, but is expended in warming the compressed particles themselves. There are, therefore, no exchanges of heat between neighbouring strata, and the appropriate equation is

$$pv^{\gamma} = \text{const.}$$

$$\log p + \gamma \log v = \log \text{const.}$$

$$\frac{\delta p}{p} + \gamma \frac{\delta v}{v} = 0$$

$$E = -v\frac{\delta p}{\delta v} = \gamma p.$$

$$c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma p}{\rho}} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which gives a value for c of 331·5 m. per sec. at 0° C.

Signal Method. What may be termed the large-scale method of finding the velocity of sound in air has remained the same in principle, though improved in technique, until the present day. A gun is fired and at a distant point observations are made of the instants (1) at which the gun is fired to produce the sound, and (2) at which the sound arrives at the observation post.

In the first experiments of Mersenne,[2] and of Gassendi,[3] the first instant was noted by looking for the flash accompanying the report, light being propagated over a few miles instantaneously in comparison with sound. The instant of arrival was determined by ear. The time for the sound to travel from the source to the observer was taken as the time on a clock between the incidence of the flash and the report. By employing distances greater than 3,000 metres, and allowing errors of starting and stopping the clock not exceeding ½ sec., an accuracy of 5 per cent. was possible. On the appearance of Laplace's Memoir, the Bureau des

[1] *Ann. d. Physik*, 57, 234, 1817. [2] *Harmon*·· *rum*, 12, 1636.
[3] *Opera omnia*, 3, 1658.

Longitudes[1] organized extensive researches between five stations round Paris, with eminent observers, over distances of about 18,000 metres, cannon being fired consecutively from alternate ends of each line in an endeavour to eliminate the effect of the wind. The mean result gave 333·2 m./sec., when reduced to 0° C. The fault of their method lay in the "personal equations" of the observers, whereby the recorded times depended on the quickness of the perception and response of the experimenter. Stone,[2] in an endeavour to estimate this "psychological time (t)," fired small guns near the observers, so that sound of the same intensity was heard by them, as from the large guns far away.

Assuming an approximate value c_1 for the velocity of sound over this small distance l, the time "clocked" by an observer

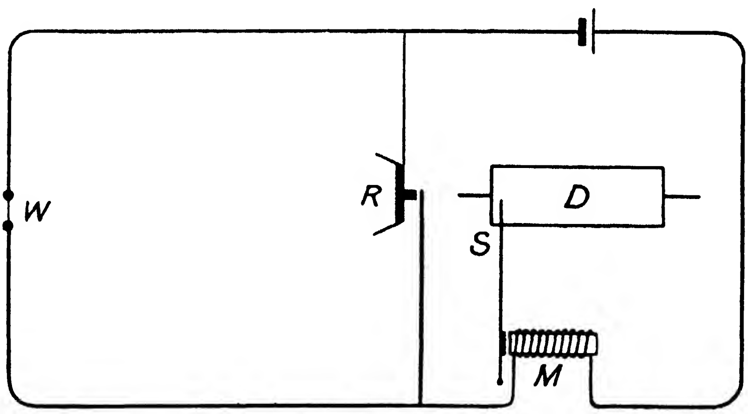


FIG. 2.—Regnault's Apparatus.

for the small gun was $\dfrac{l}{c_1} + t$, from which t was found and substituted in the formula of the actual velocity c over the larger distance L, $c = \dfrac{L}{T - t}$, T being the observed time over this range.

Regnault[3] first endeavoured to get rid of the human element, by using electrical registration apparatus. A drum D (Fig. 2) was revolved by clockwork at constant speed, on which a style S, normally attracted by an electromagnet M, marks a straight line. At the sending station, a wire W in the circuit is broken by the firing of the gun, releasing S which now makes a mark parallel to the former but a little to the left. The sound travels out till it reaches the receiving station, and pushes in the membrane R, so that it momentarily re-establishes the circuit, and

[1] *Mém. de l'Académie*, 1738. [2] *Phil. Mag.*, 43, 153, 1872.
[3] *Comptes Rendus*, 66, 209, 1868.

pulls the style suddenly to the right. The time of travel of the sound is thus found by measuring the distance between two kinks in the mark on the drum, the speed of rotation of the drum being known. Regnault found however that the time lag in the instrument was, at least, equal to that of a trained experimenter.

Another method which, although subjective, does not involve human error in the estimation of time, is that of Bosscha,[1] which depends on the observation of coincidences. Two small electromagnetic hammers are arranged so that periodic interruption of the current which excites them causes them to produce simultaneously a loud tap at equal intervals of time. The periodic "make and break" usually consists of a vibrating reed having a short wire attached to its free end, alternately entering and leaving a cup of mercury placed in the primary circuit of an induction coil, of which the secondary circuit contains the tappers. The observer takes one of these tappers with him, and walks away from the other stationary one. Owing to the time taken by the sound to cross the intervening space, the sound from the stationary tapper lags behind that from the one with the observer. If he goes farther away the ticks will again be in step; but this time the tick produced nearby will coincide with the previous tick of the distant instrument. If n ticks are being made per second, and l is the distance of separation, this time is obviously $\frac{1}{n} = \frac{l}{c}$, whence c is determined.

If a reflecting wall is used only one ticker is necessary; the experimenter moves away from the wall until the tap and the echo from the preceding one are in phase. This reduces the distance required, but some intensity is lost in reflection. Another modification in the coincidence method consists in fixing the distance between the tappers, and varying the number of ticks per second, until coincidence is obtained. Obviously, the greater the distance, the greater the accuracy of the results, but then the intensity of the sound from the distant source is overpowered by that from the one close at hand. Probably, if the relative intensity of emission were made adjustable so as to produce equal intensity at the observer's ears, the technique of the method would be improved.

An objective method on "coincidence" lines is that of Hebb.[2]

[1] *Ann. d. Physik*, **92**, 485, 1854.

[2] *Phys. Rev.*, **20**, 89, 1905 and **14**, 74, 1919.

He placed a telephone transmitter T_1 in the focus of a paraboloid of plaster of Paris, with a corresponding receiver T_2 in another paraboloid (Fig. 3). These were set facing each other, so that plane waves were sent out from one mirror, and were brought by the other to a focus on the receiver. Both transmitter and receiver were connected in two primary circuits wound on the same secondary coil, I. The receiving apparatus could be moved away from the transmitter until the two primary currents round the coil were "dead out of phase" and so produced no effect in the secondary; a telephone R connected across the latter would then remain silent. Knowing the frequency of the note maintained in the transmitter, the velocity of sound was calculated precisely as in Bosscha's experiment.

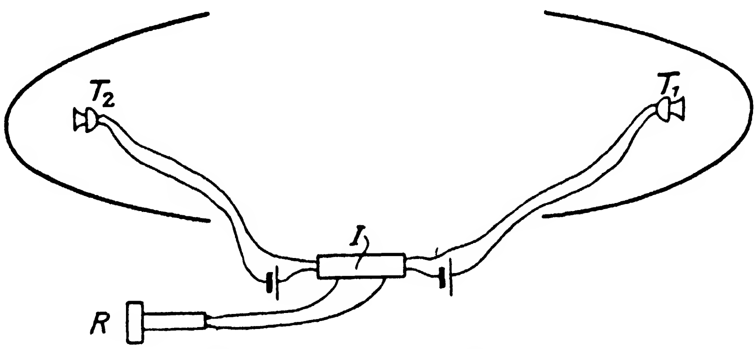


FIG. 3.—Hebb's Apparatus.

The latest series of observations reverts to the objective method of Regnault, and was made on the one hand by Angerer and Ladenburg[1] in the Bavarian Mountains, during the European War. Instead of a style and a revolving drum, they used an Einthoven string galvanometer, whose fluctuations were recorded by the movement of a spot of light on sensitized paper moving with constant speed. As before, the explosion (in this case of powder alone) broke a wire in the primary of a circuit, the galvanometer being in the secondary circuit. The arrival of the sound-pulse at a distant microphone caused oscillations in another primary on the same coil. A helium tube controlled by a tuning-fork gave time intervals, and in this way they claimed to measure lapses of time as short as two-thousandths of a second. A number of stations 6 or 10 km. apart, over mountainous country round a rough triangle, were chosen, and their positions carefully surveyed. The anomalies observed will be mentioned in a later

[1] *Ann. d. Physik*, **66**, **293**, **1921**; Kukkamäki, *ibid.*, **31**, **398**, **1938**

section, but the reduced result is 330·8 ± 0·1 m. per sec., comprising many observations.

During the course of researches on gunfire, Esclangon [1] obtained 330·9 m. per sec. (reduced to 0° C.) as the mean of a large number of experiments by a similar method over a fixed range of 14 km. in all kinds of weather. He found that, over this distance, the time of propagation was independent of the calibre of the gun, i.e., of the amplitude of the initial pulse.

Atmospheric Influences—Temperature. It appears from the formula (3) that any cause that will alter the density of the medium, supposed gaseous, will alter c. The most frequent source of change in the velocity will be the temperature. According to the well-known law of Gay-Lussac, the density of a gas varies inversely as the absolute temperature. Equation (3) then shows that the velocity of sound will vary directly as the square root of the absolute temperature, or if one wishes to reduce the velocity c—observed at $\theta°$ Cent.—to the velocity c_0 at 0° Cent., one can use the approximate formula

$$c_0 = \frac{c_\theta}{\sqrt{1 + \alpha\theta}} = c_\theta(1 - \tfrac{1}{2}\alpha\theta) \quad . \quad . \quad . \quad . \quad (6)$$

where α is the coefficient of expansion on the Centigrade scale. Data to test this formula have rested mainly on small-scale researches by the methods described on p. 27; allowing for rather uncertain corrections, the agreement with theory is good. Open-air observations by Greeley [2] during a Polar expedition add confirmation down to − 45° C.

Influence of Pressure. According to Boyle's Law, the relation between pressure and density is constant, so that change of pressure, *per se*, can have no influence on the velocity. There is no reason to believe that this is not the case, though the secular range of pressure in the atmosphere is hardly sufficient to test the deduction.

Influence of Humidity, and Fog. Like the temperature-effect, the effect of humidity is mainly an alteration of density, but admixture of the foreign vapour requires the use of a new value of γ for the ratio of the specific heats. Fog itself—as distinct from humidity—has no influence on the propagation of a pulse,

[1] *Comptes Rendus*, **168**, 165, 1919.

[2] *Phil. Mag.*, **30**, 507, 1890. Humphreys, *Frank. Inst. Journ.*, **197**, 821, 1924.

but the accompanying temperature-changes in the air strata may cause anomalous propagation.

Influence of Wind. When a steady air stream is imposed on the motion of the sound waves, its velocity is to be added to or subtracted from c, if it blows directly in the line of observation; in this case the arithmetical mean of the times of travel of sound in the two opposite directions will give the true value of c with sufficient accuracy. If, however, there is a side wind, the state of affairs is quite complex, and not then merely the resolved sum; while, with a wind at right angles to the direction of propagation of the sound, the observed velocity in both directions is the same, i.e., the wind has no effect. The actual conditions in the atmosphere involve, however, so many different effects, that it is generally impossible to pick out those due to the wind alone, nor is there any advantage in choosing a still day for velocity determinations. We must assume that the mean of a large number of observations under all possible atmospheric conditions will give the best results.

Fall of Intensity in Propagation. It is a matter of common observation, that under normal conditions, the intensity of the sound rapidly diminishes as one recedes from the source. This is to be explained (quite naturally) if one remembers that energy must be conserved. It can be shown that, following the principle of energy conservation, the intensity falls off as the square of the distance from the source. For if O be the source (Fig. 4) which has just sent out a sound impulse, then the wave-front of the emitted sound, travelling out as an expanding sphere, has now covered a sphere of radius r_1, of which $A_1B_1C_1$ is a section. A receiver, such as the ear placed at A_1 on the surface of the sphere, gains energy I_1 from the unit area of the surface, so that the total energy over the surface $= I_1 4\pi r_1^2$. An instant later the wave-front covers a larger sphere of radius r_2, and the receiver at A_2 collects energy I_2 from unit area of the new wave-front,

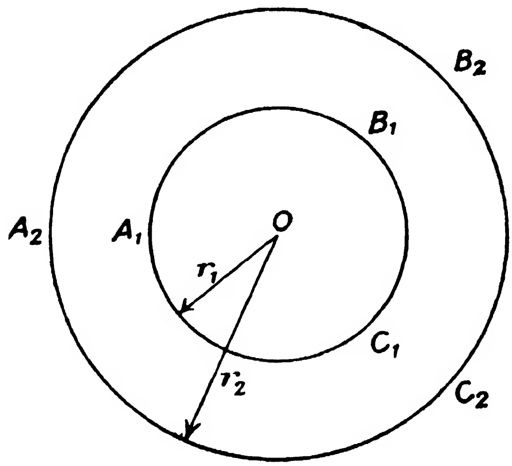


Fig. 4.—Inverse Square Law.

on which the total energy crossing is $I_2 4\pi r_2^2$. Then

$$I_1 4\pi r_1^2 = I_2 4\pi r_2^2$$

and
$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

There are two remarks which should be made with regard to the application of equation (7). Firstly, we have assumed a point-origin of sound. In practice, the source of the vibrations is of considerable extent, and the equation will not be true for distances of the same order as the linear dimensions of the vibrating body; at large distances the conditions of the equation will be satisfactorily realized. Secondly, the energy has been assumed to travel out from the origin in straight lines. Lastly, no account has been taken of the loss of energy dissipated in friction, and so converted into heat in the medium.

On account of these diverse influences in the actual atmosphere, the fall of intensity is much more rapid than that given by the inverse square law, especially of sound waves passing over the surface of the earth.[1]

There is another factor which affects the propagation of musical tones, as distinct from pulses, in the atmosphere. This factor is absorption, and its effects vary with frequency. Generally the tones of shortest wave-length are most affected, the mechanism of the absorption being similar to that produced by the scattering of light waves in a turbid fluid. Neklepajev [2] showed that sounds having wave-length of the order of 1 mm., were strongly absorbed by the air. Continuing these observations, Altberg and Holtzmann,[3] by producing very high-pitched tones, found that the absorption by an artificial fog increased five or six times, when the wave-length was reduced from 6·5 to 1·5 cm. In fact we may say that this type of absorption becomes very conspicuous as the wave-length approaches the average size of the particles forming the medium.

Under disturbed atmospheric conditions, one hears the lower components in the very complex note emitted by an aeroplane more plainly than the higher tones; whereas on a still night, when the dissipative forces are less active, the higher tones are prominent in the sound heard.[4]

[1] Rayleigh, *Advis. Comm. Aero.*, 1916; *see also* Drzewiecki, *Comptes Rendus*, **189**, 122, 1929; Rocard, *J. de Physique*, **1**, 426, 1930 and **4**, 118 1933.

[2] *Ann. d. Physik*, **35**, 175, 1911.

[3] *Phys. Zeits.*, **26**, 149, 1925. *See also* Mallock, *Roy. Soc. Proc.*, **84**, 39, 1910; Hart, *Roy. Soc. Proc.*, **165**, 80, 1924.

[4] Stewart, *Phys. Rev.*, **14**, 376, 1919. *See also* Milne, *Phil. Mag.*, **42**, 96, 1921.

Reflection and Refraction of Sound. The phenomena of reflection and refraction of sound which occur when the sound reaches the boundary of a medium, can be demonstrated as in the parallel case of light; except that, as sound is propagated (except in bodies of limited width) by longitudinal waves, no question of polarization can arise. The apparent differences between light and sound in reflection are merely questions of scale. The average wave-length of sound being 100,000 times the average wave-length of light-radiation, it requires rugosities of the surface 100,000 times the corresponding ones in light to produce diffuse reflection or scattering. A mirror or lens to produce concentration of sound must be enormous compared with the mirrors and lenses used in optical work. The same remark applies to gratings for producing diffraction. This difficulty of scale troubled Hertz

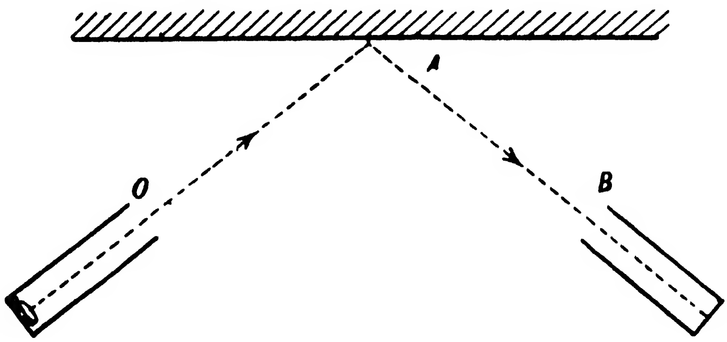


Fig. 5.—Reflection from Plane Surface.

in his work on the analogous electro-magnetic radiation which bears his name.

In place of the optical "screen," any of the sound detectors described later, microphones, sensitive flames, resonators, etc., may be used.

The law of reflection, that the angle which the incident ray makes with the normal to the surface is equal to that made by the reflected ray, can be proved by taking a directional source of sound, such as a watch at the end of a cardboard tube which is pointed at a large plane wall. A tubular resonator or a detector at the end of a tube must be placed so that the tube points to the same spot on the wall as the sending tube, and makes an equal angle with the normal (Fig. 5) in order that the detector may respond to the source. With curved surfaces the existence of foci can be demonstrated in a similar manner. In particular a small source of sound at the focus of a mirror of parabolic section

made of iron or cardboard sends out a train of plane waves, which can be brought to a focus at another similar paraboloid, and there detected by a microphone or a sensitive flame, as in Hebb's experiment (p. 8).

If the source gives a pulse instead of a continuous note, and if its distance from the wall is considerable, at a point such as B (Fig. 5) sound coming direct from O may be heard considerably in advance of the reflected sound travelling along a path such as OAB. When there is more than one-fifth of a second (corresponding to a distance of 70 m.) between the two times, an observer at B gets the impression of a distinct "echo."

Harmonic Echo. Reflection plays an important part in Building Acoustics, which, because of its technical importance, will be dealt with in the last chapter. There are several interesting cases, however, which are best dealt with now. The echo of a complex note returned by a reflecting surface may not necessarily be a faithful reproduction of the original. If the reflector has the property of reflecting certain wave-lengths better than others, then these components, if present in the source, will predominate over the others in the echo. Cases of this kind, in which the upper components of a complex note predominated in the echo, were noticed by Rayleigh,[1] who gave them the name of "harmonic echoes," as the pitch of the returned note was apparently raised; the edge of a wood, for example, seemed to return a note as its octave, owing to the predominance of the octave in the reflected sound.

Musical Echo. The conversion of a single pulse into a succession of reflected pulses (or a musical note) is accomplished by wooden fences, the palings of which overlap in echelon formation. In consequence of this, the portion of the wave-front striking each paling and being there reflected, has to travel a little farther, and consequently arrives a little later at the listener's ear, than that portion which struck the preceding paling in the row. A clap of the hand is thus reflected as a succession of pulses, at regular intervals depending on the spacing of the palings, and is perceived as a short musical note.

Whispering Gallery. Another interesting case of reflection is that which may be observed in the Whispering Gallery of St. Paul's Cathedral in London. There the slightest sound, made at one point near

[1] *Nature*, **8**, 319, 1873. *See also* Eve, *Frank. Inst. Journ.*, **202**, 627, 1926.

the wall beneath the dome, may be heard at the opposite point, while it remains audible at intervening points round the gallery. This phenomenon, according to Rayleigh,[1] is not due to reflection, but due to the waves hugging the walls of the dome as they travel outwards, and being gradually guided round to the conjugate point of the hemisphere.

Raman and Sutherland [2] have conducted experiments in St. Paul's and found Rayleigh's theory so far confirmed in that the effect was most marked when the source of sound was directed tangentially along the surface of the gallery wall, instead of directly at the opposite side. They found, however, certain fluctuations of intensity both radially and tangentially to the gallery, which the theory does not explain. Sabine [3] ascribes a great deal of the efficacy of this and other Whispering Galleries to the inward slope of the wall, which keeps a good deal of the sound down to the level of the gallery, which would otherwise find its way up to the roof of the building, and never reach the listener.

Refraction may be shown to follow the same laws as light, i.e., if i is the incident angle in a medium where the velocity of sound is c, and r is the refracted angle in the second medium where the velocity is c', then $\frac{\sin i}{\sin r} = \frac{c}{c'} =$ a constant. Sondhauss [4] contructed an acoustic bi-convex lens of a large balloon containing carbon dioxide. Lenses made of pitch, and rubber vessels containing water may also be used. Reflection and refraction may take place at any surface where there is a change of density, *inter alia* at surfaces between layers of gas at different temperatures (cf. pp. 21–24).

Should the longitudinal waves be incident at a large angle—nearly "grazing" incidence—on the surface of a medium in which c' is greater than c of the first medium, it may suffer "total internal reflection." Under such circumstances none of the energy penetrates the second medium; all the sound is reflected by the surface. This will arise if $\sin r$ is greater than 1, i.e., if r is imaginary, the critical angle being given by $\sin r = \frac{c'}{c} \sin i = 1$. At an air-water surface, $c = 340$ and $c' = 1440$, therefore the critical value of $i = 13{\cdot}5°$. This explains why the voices of bathers whose heads are naturally close to the surface of the water, can be heard so

[1] *Phil. Mag.*, **20**, 1001, 1910, and **27**, 100, 1914. [2] *Roy. Soc. Proc.*, **100**, 424, 1922; also Bate, *Phys. Soc. Proc.*, **50**, 293, 1938.

[3] *Coll. Papers*, *p.* 255. *See also* Raman, *Proc. Ind. Assoc.*, **7**, 159, 1922.

[4] *Ann. d. Physik*, **85**, 378, 1852.

plainly on the shore. The phenomenon is also of technical importance (cf. p. 324).

Diffraction of Sound. The elementary phenomena of light and sound find their readiest explanation in the assumption that light and sound travel in straight lines. There are some phenomena in light, and more particularly in sound, which will not bear this assumption. Every one knows that sound can be heard round corners. In order to explain such facts, we avail ourselves of the fruitful Principle of Huygens,[1] according to which every point P_1 P_2 P_3 (Fig. 6*a*) on a wave-front, as it vibrates, becomes the origin of secondary waves, which diverge in spheres; so that the wave-front at a succeeding instant is the envelope E of these secondary waves. Why the disturbance is not, *ipso facto*, propagated back to the origin O by an enveloping wave such as E',

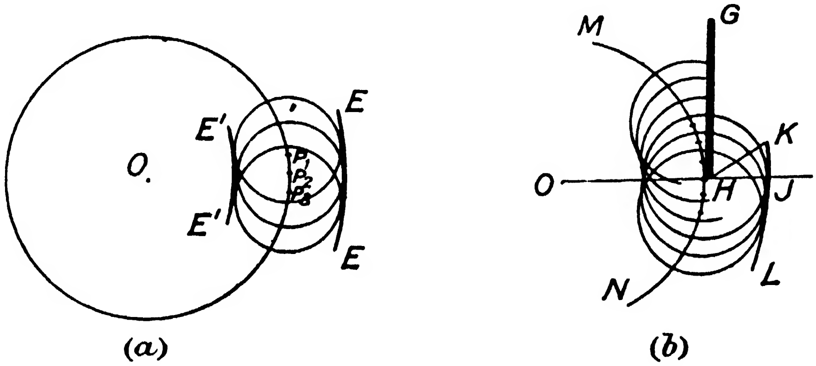


FIG. 6.—Huygens' Construction.

required the Principle of Interference for its explanation. This was added by Fresnel [2] and Kirchhoff,[3] and from the calculation of the latter it appears that the effects of the secondary waves mutually destroy each other round the envelope E', so that the wave is propagated only in the direction away from the origin. When now an obstacle (GH, Fig. 6*b*) which will not allow the sound to pass, is placed across the path of the waves, secondaries arising from the wave-front HN teach us to expect, not only a consecutive half wave-front JL, but also a spreading of the waves into the geometrical shadow at JK; only, since half of the wave-front MH is now cut off from this side, the intensity of the sound received at a point in the shadow is less than it would have been, if the obstacle had been absent. Here again the question of wave-length comes in, and Rayleigh [4] has shown that, e.g. in the case

[1] *Traité de la Lumière*, 1690. [2] *Mém. de l'Académie*, **5**, 339, 1826.
[3] *Ann. d. Physik*, **18**, 663, 1883. [4] *Sound*, **2**, 254, 1896.

of a spherical obstacle whose circumference is twice the wave-length, the secondary waves conspire in the centre of the shadow, and make a concentration of sound along the axis through the origin and the centre of the obstacle.

Acoustic Shadow of a Sphere. The human head is a type of "spherical obstacle" in relation to the propagation of the sounds of the human voice to points behind the head. This case has been idealized for theoretical discussion by Rayleigh [1] in the form of a sphere having a source of sound very close to its surface. By mathematical analysis too complex for reproduction here, he, and later, Stewart [2] traced the "acoustic shadow" of such a sphere, by determining the relative intensities produced by diffraction at points behind the sphere. The theory has been checked by Stewart and Stiles [3] using for the sphere a large ball on the end of a narrow stem, which served as a conduit for the sound from a tuning fork in a box below, the sound finally debouching upon the atmosphere at a point on the surface of the sphere. Naturally in testing the effect of such an obstacle on the sound received in its shadow, reflections from neighbouring surfaces are to be excluded. Stewart placed his sphere so that it projected from the edge of a roof, covering the latter with felt to obviate reflection of the sound.

Other local concentrations of sound produced by diffraction are formed by acoustic gratings in the same way as in optics. Altberg [4] constructed a grating of glass rods, about 1 cm. apart, using the powerful spark discharge of a condenser as origin of sound. The sound waves were made plane by a wooden concave mirror and impinged on the grating; after being diffracted by this on to another concave mirror, they were finally detected at the focus of the latter. By moving the grating on an axis, one can trace out a sound spectrum just as with a spectrometer. Sparks have the property of originating very short waves a few millimetres only in length, and this reduces the size of the apparatus to workable proportions.[5]

Spark Photography. A method which has been of such use in studying the progression of sound waves as to deserve detailed

[1] *Sound*, **2**, 255, 1877. [2] *Phys. Rev.*, **33**, 467, 1911, and **7**, 442, 1916.

[3] *Phys. Rev.*, **1**, 309, 1913.

[4] *Ann. d. Physik*, **23**, 267, 1907; Abello, *Nat. Acad. Sci. Proc.*, **13**, 699, 1927; Humby, *Phys. Soc. Proc.*, **39**, 435, 1927; Lindsay, *Phys. Rev.*, **32**, 515, 1928; Sivian and O'Neil, *J. Acoust. Soc.*, **3**, 483, 1932; Meyer and Thienhaus, *Zeits. tech. Phys.*, **15**, 630, 1934.

[5] *See also* Lebedew, *Ann. d. Physik.*, **35**, 171, 1911. Stewart and Stiles, *Phys. Rev.*, **3**, 256, 1914; Palaiologus, *Zeits. f. Physik.*, **12**, 375, 1923.

treatment, was developed by the Toeplers (father[1] and son[2]) and called by them the Schlieren method. The sound is made by the discharge of a large condenser across a spark gap; this produces a very intense pulse of sound, sufficient to cause considerable changes of density in its passage through the air. If the field over which the wave travels is illuminated instantaneously the position of the wave will be visible as a slight shadow on the background, owing to the somewhat different optical properties of the compressed air. In the modern method an instantaneous photograph of the wave is usually made. It may readily be understood, that if waves of considerable curvature are required, as they must be to demonstrate the reflection, etc., of sound waves, the time that elapses between the emission of the sound by the spark and the moment at which the wave or waves are photographed must be a very small fraction of a second. The photography is performed by the light from a discharge across an auxiliary spark gap.

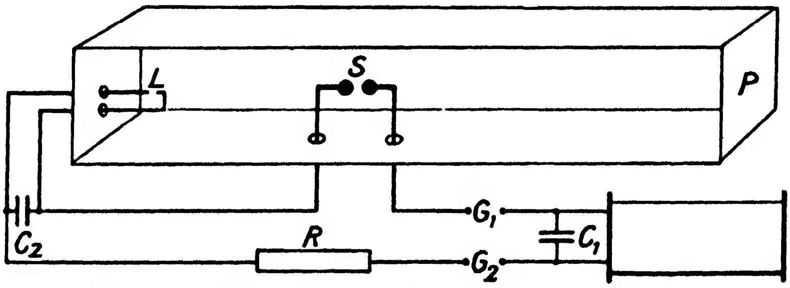


FIG. 7.—Photography of Sound Wave by Spark.

The condenser C_1 (Fig. 7) is charged to a high potential by a large influence machine or induction coil, high enough to allow it to jump the gaps S and L, when the larger gaps G_1 and G_2 are short-circuited. The two spark gaps, S the sound producer, and L the light producer, are placed in parallel across the terminals of the condenser. As soon as G_1 and G_2 are closed, a discharge takes place almost simultaneously across the gaps L and S. Under such conditions no sound wave would be registered on the screen, as it would not have had time to leave the shadow of the knobs of the impulse gap. In order to make the illuminating spark occur a fraction of a second later, when the wave shall have attained

[1] *Ann. d. Physik*, **131**, 180, 1867.

[2] *Ann. d. Physik*, **27**, 1043, 1908. *See also* Petropoulos, *J. de Physique*, **2**, 317, 1921; Huguenard, *Comptes Rendus*, **177**, 744, 1923; Okuba and Matuyama, *Phys. Rev.*, **34**, 1474, 1929.

a radius of several centimetres, a condenser C_2 is added to the branch of the circuit containing the illuminating spark. Then, if a discharge is made to pass across the gaps, it jumps the impulse gap a little before the other; the condenser causes the necessary lag in the passage of the illuminating spark, and the extent of the lag depends on the capacity of the interposed condenser. The light spark may take place along the surface of a piece of wet chalk, or between magnesium wires, in order to produce a brilliant light casting on the photographic plate P a shadow of the impulse, which left S earlier.

In the arrangement used at the National Physical Laboratory, C_1 is a battery of 10 Leyden jars and C_2 of 5 jars; a high liquid resistance R of about 10,000 ohms steadies the action. The sparks are set off by lowering glass plates into the "trigger" gaps G_1 and G_2; L consists of two pieces of No. 24 S.W.G. magnesium wire, with tips 3 cm. apart, enclosed in a capillary tube pointing in the direction SP; S has platinum points 1·5 cm. apart. The total length LP is about 7 ft., and P is a "whole plate." [1]

It may be remarked, in parenthesis, that there is nothing essentially different between the two sparks, although S is intended to cause compression, and L is primarily light-producing. Both discharges are both sound and light-producing, but the light from the first, S, casts a shadow of the knobs alone on the plate, whereas L casts an additional shadow of the sound wave which left S earlier; again, the sound wave which L starts is never subsequently registered by light on the plate. Thus, as far as the photograph is concerned, S produces a sound wave and L makes it visible.

When it is desired to study reflection problems, a piece of bent wood or card is placed below S and parallel to the plane of the screen. For refraction, a lens or small tank is similarly placed. By enclosing S with a model section of a building, the acoustical properties of it can be studied [2] (see Chap. XIII).

The radius of the unreflected wave in these photographs represents the distance travelled by the sound in the time which has elapsed between the impulse and the illuminating spark; if this time is known we have a means of determining the velocity of sound. To this end Foley [3] found the value of this short interval of time,

[1] Davis and Fleming, *Journ. Sci. Instruments*, **3**, 393, 1926.

[2] Sabine, *Ann. Arch.*, **104**, 257, 1913; Davis and Fleming, *loc. cit.*

[3] *Phys. Rev.*, **16**, 449, 1920. *See also* Foley, *Phys. Rev.*, **14**, 143, 1919; **20**, 505, 1922; Foley and Sonder, *Phys. Rev.*, **35**, 373, 1912.

by making the light from each spark cast a shadow of a revolving cog-wheel on another sensitized plate. The speed of revolution being known, the required time could be calculated from the angular separation of the two shadows.

Ripple Photography. Most of the problems that can be studied by the Schlieren method can be examined by following the motion of small ripples on shallow water or mercury. This method was first used by Vincent[1] for demonstrating interference of waves (see Chap. II), but, by putting obstacles in the tank in which ripples are formed to represent mirrors or diffracting edges, the method may be extended to problems in the propagation of sound; it has been shown mathematically by Lamb[2] that, although the motion of such "capillary waves" is in two dimensions only, it is analogous to the three-dimensional motion of actual sound waves, if we picture a plane section of the latter; and such a section is, of course, what we get in our spark-pulse photographs. To start a pulse it is sufficient to direct a puff of air on to, or to dip a rod smartly into the water; the pulse is, however, always followed by a number of lesser waves, more so in the water than in the mercury. To imitate a musical note, a prong is placed on the end of a vibrating reed or tuning fork, so that the prong dips into the water periodically. In order to produce an acoustically denser medium, part of the tank is made more shallow than the rest; over this portion the waves travel with slower speed, and so on reaching it, are bent towards the normal of the line of separation. When the pulse is formed the motion is slow enough to be followed by the eye; and when a series of waves is being sent out, it is usually arranged that the vibrator periodically lights a helium discharge tube, which illuminates the tank from below (water) or by reflected light (mercury), and so renders the motion stationary to the eye. The ripple tank is less irksome to set up than the spark-pulse apparatus; on the other hand its indications are not so well defined.[3]

Anomalous Propagation of Explosive Sounds. The study of the velocity of sounds from explosions has been facilitated in recent years by the need of disposing of the large dumps of ex-

[1] *Phil. Mag.*, **43**, 17, 1897. [2] *Hydrodynamics*, Arts. 189 and 172.

[3] *See* Schultze, *Zeits. für Instk.*, **26**, 151, 1907; Waetzmann, *Phys. Zeits.*, **12**, 866, 1911; Palmer, *Phys. Rev.*, **33**, 528, 1911; Watson and Shewhart, *Phys. Rev.*, **7**, 226, 1916; Davis, *Phys. Soc. Proc.*, **38**, 234, 1926; **40**, 90, 1928; Bekesy, *Zeits. f. Phys.*, **79**, 668, 1932.

plosives left over from the European War. The earlier experiments of Mach and his pupils [1] were confined to the propagation of the explosion itself in vessels and tubes of explosive gas. As this is rather a problem in combustion for the chemist, it will not be considered further in this treatise. It is sufficient to point out that these experimenters recognized that the abnormal velocity of the waves of compression so generated was due to the intensity of the explosion, which brought it outside the pale of the small amplitudes, to which the theory leading to the velocity-equation applies. The anomalies to be noticed in the propagation of sounds from large explosions capable of penetrating many miles from the source, are: (1) abnormally large velocities in the neighbourhood of the explosion, with considerable mechanical movement of the air; (2) a second zone of normal velocity; (3) a third zone of complete silence; (4) a fourth zone where the sound reappears with unusual intensity but taking an exceptionally long time to arrive.

Of these, the occurrence of a region where the sound is inaudible, in spite of the fact that it is distinctly audible at a greater distance from the source, is the most striking. This was noticed early in the present century, and two rival theories were put forward to account for it. Both agree in stating that the sound heard in the first and second zones travels directly over the surface of the earth. Apart from the diminution of intensity accounted for by equation (6), there is a much more powerful source of damping in the friction which the waves experience in passing over the broken surface of the earth, the tops of forests, etc., so that after an average course of 60 miles, the sound becomes inaudible to a listener. They also accede that the sound heard with renewed intensity beyond the silent zone has reached its objective by travelling up into the higher parts of the atmosphere, and being gradually bent down again to reach the earth's surface by a devious route; which accounts for the abnormal lapse of time between the explosion and the arrival of the sound in the fourth zone. They differ, however, in assigning the cause of this curved trajectory. We have noted that a bending of the direction of the sound will occur when the waves pass into a medium of different density, and wherein, therefore, their velocity is different. Such a change may occur in the atmosphere at regions where the temperature is different, or where the composition of the gas is different, and may be a sudden or a gradual change.

[1] *Akad. Wiss. Wien. Ber.*, **92**, 225, 1885.

At a sudden discontinuity in the medium reflection may be produced. Bending of the wave-front can also be produced by a gradient of wind speed, resulting in parts of the wave-front gaining on the rest. All three of these possibilities have been invoked to explain the curved trajectory of waves leaving the origin at an inclination to the earth's surface. The meteorological theory postulates the existence of regions of decreasing temperature as one ascends from the earth, whereby these sounds pursue a path which is concave upwards. At a height of about 10 miles, however, there is believed to be a temperature-inversion layer, wherein warmer regions are rapidly entered. This layer causes, either reflection of the waves, or refraction in the opposite sense (the path convex upwards) or indeed both, until the sound ultimately reaches the ground again, its renewed intensity being accounted for by the concurrence of waves having traversed slightly different paths. The physical theory put forward by von den Borne,[1] allows the upward bending in the lower regions, but ascribes the bending back to the fall of velocity in the rarefied upper regions of the atmosphere (which Borne describes as mainly helium and hydrogen), occasioned by the low density there. This theory requires that the sound should penetrate into more rarefied regions, where the sound would have difficulty in progressing. One is reminded of the well-known experiment in which an electric bell is placed under a bell-jar, which is then exhausted; with a good vacuum the sound fails to penetrate the few inches between the bell and the open air.

It is then a matter of putting forward hypotheses for the velocity of sound above 17 km. and testing these against the known time of propagation of the sound through this region into the abnormal zone of the earth, and such hypotheses must make the summit-velocity high, otherwise these high-penetrating rays would not get back to earth within the prescribed surface distance (180 to 300 km.). The simplest hypothesis seems to be to suppose that from 17 km. the velocity rises again at such a rate that at 30 to 40 km. the surface velocity is equalled and exceeded. This fits the observed time of passage of sound to the abnormal zone, giving the rays which penetrate to the outer edge of the abnormal zone an original inclination of about 40° at the source, and a "summit level" of

[1] *Phys. Zeits.*, **11**, 483, 1910. *See also* Nölke, *Phys. Zeits.*, **17**, 31 and 283, 1916; 18, 501, 1917; Schmidt, *Phys. Zeits.*, **17**, 333, 1916; Kommerell, *Phys. Zeits.*, **17**, 172, 1916.

70 km. (The assumed paths of such sound waves are shown in Fig. 8.)

Measurements of the propagation of sound from these explosions enable us to get an approximate estimate of the velocity of sound in the stratosphere, and from this we can obtain valuable information on the physics of this upper atmosphere, a subject on which little information is available. Pressure *per se* can have no effect on the velocity, but any other factor which alters the density will *ipso facto* alter the velocity of sound. The increase in the velocity in passing up through the stratosphere is therefore ascribed to a reduction in density. Granted that the density of the air progressively decreases in the stratosphere, there are two main factors which may produce this effect ; either (1) the constitution of the stratosphere becomes richer in the lighter gases

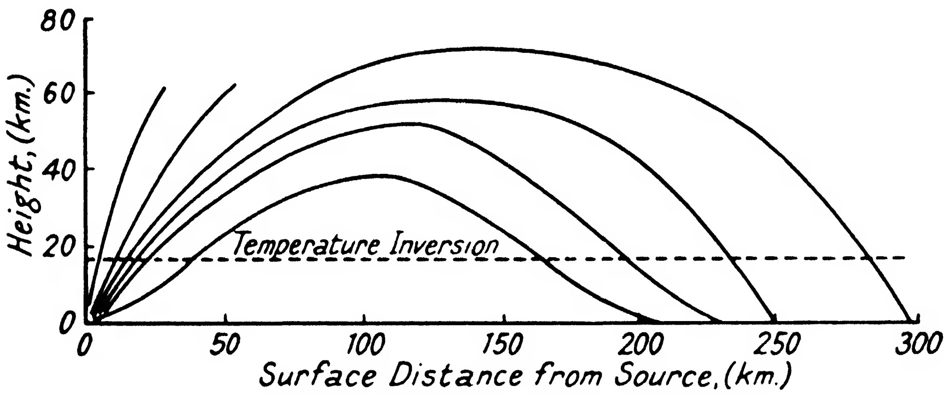


FIG. 8.—Sound Rays from Explosive Source.

—this would also affect γ—or (2) the temperature progressively increases upwards.

The first hypothesis is the older and was put forward by von den Borne. This theory considers that the temperature does not increase in the stratosphere, but that the necessary decrease in density is to be credited to a decrease in molecular weight due to the presence of lighter gases—e.g. helium and hydrogen—in these regions. Such an equilibrium would not arise if diffusion could freely take place, and it is difficult to account for the congregation of these gases in the upper region, except by postulating a vertical streaming which would upset diffusion equilibrium. It has also been pointed out that the hydrogen molecule would be more or less dissociated at such heights, and must have been dispersed in space long ago. A second objection lies in the fact that, so far, examination of the spectrum of the *aurora borealis* (a light phe-

nomenon having its origin at great heights in the atmosphere) has failed to show any lines which can be ascribed to either of the elements named.

We are therefore led to adopt the temperature theory, which leads to the surprising result that, from the temperature inversion at 17 km., the temperature rises with height, reaches the surface temperature again at 35 km., and at 60 km. has passed 70° C! This conclusion is completely against the older ideas on the atmosphere but is supported by certain observations on meteors. Attempts which are made from time to time to penetrate the stratosphere with sealed balloons carrying observers as well as instruments may bring evidence to support the acoustical data.

Data from Explosions. The first statistical records of the audibility of an explosion were collected by Davison [1] after the accidental blowing-up of a munition factory at Silvertown, London, in 1917. In order that more accurate data might be obtained by persons prepared for the explosion, a larger quantity of explosives has been blown up on two previously announced occasions, one at Oldebroek in Holland, 1923, and the other at La Courtine,[2] France, in 1924. In the first there was a definite silent zone extending from 60 to 100 miles from the source. There was no trustworthy return from this region recording that the sound had been heard. It is curious that in the inaudible zone of the Silvertown explosion, windows were shaken, and pheasants, which seem to be sensitive to low frequency sounds, became restive; all of which indicated a pulse of very low pitch, not audible to the human ear (cf. p. 276). Van Everdingen,[3] who has studied results of several explosions, favours the meteorological theory, as Borne's physical theory would require the zones to be symmetrically distributed with respect to the origin; whereas the confines of the zones are irregular, and there are in addition isolated patches of audibility in the silent zone, and *vice versa*. These would be accounted for by irregular conditions of wind and temperature in parts of the atmosphere. On the other hand, the outer limit of the silent zone where the sound reappears from the upper path, seems to be quite a definite line, unchanged by considerable irregularities in wind and surface temperature.

[1] *Quart. Rev.*, **452**, 51, 1917.

[2] Bigourdan, *Comptes Rendus*, **178**, 25, 1924; Ritter, *Zeits. tech. Phys.*, **7**, 152, 1926; Perot and Baldet, *J. de Physique*, **6**, 79, 1925.

[3] *K. Akad. Amsterdam Proc.*, **18**, 933, 1916; Deslandres, *Comptes Rendus*, **178**, 1741 and 1869, 1924; Dufour, *Comptes Rendus*, **179**, 759, 1924.

Some meteorologists, indeed, have gone so far as to postulate large seasonal movements of the upper atmosphere, which would account for the fact that, in Europe, the return of the sound to the earth is more marked on the west side of the source in summer, on the east side in winter. The direction in which the tails of meteors in this region point is said to favour the supposition. From the time and *direction* of return of the sound attempts have been made to calculate the velocity in the higher strata, but these must of necessity be merely qualitative.[1]

The results of the La Courtine explosion in France are shown on Fig. 9 (after Maurain [2]). In the dotted regions, the sound was registered at normal velocity; in the hachured regions its tardy arrival indicated a long trajectory. The large areas of silence are obvious. Esclangon [3] calculates that they correspond roughly to reflection at about 17 km. in the inversion layer, according to the meteorological theory. There seems to be some evidence also for reflection by cloud masses,[4] but wind velocity-gradients near the ground seem to play quite a secondary part.[5] That the surface winds have little influence on the velocity higher up is proved by observations at Paris on the audibility of gunfire from the Western Front. It was noticed on some days that the

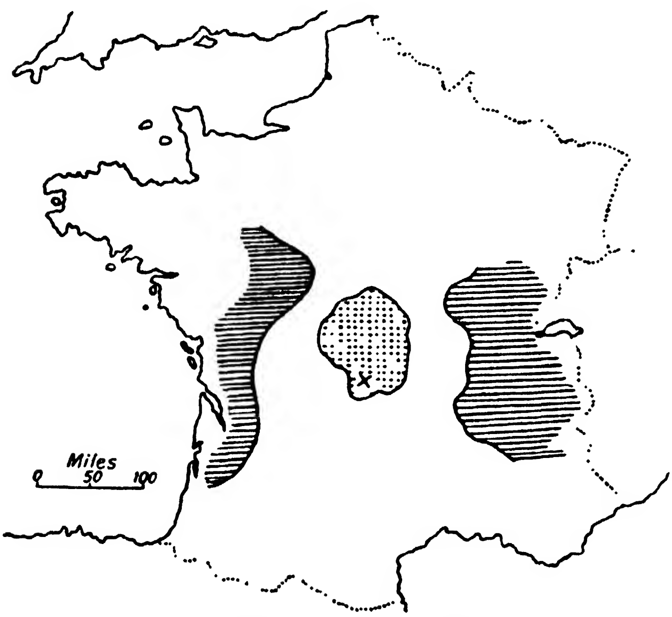


Fig. 9.—Explosion at La Courtine.

[1] Meteorological refs. In *Meteor. Zeits.*, Kölzer, **42**, 457, 1925; Wiechert, **43**, 8, 1926; Witkiewitsch, **43**, 91, 1926. In *Zeits. f. Geophysik*, Wegener, **1**, 297, 1925; Augenheister, **1**, 314, 1925; Gütenberg, **2**, 101, 1926. *See also Phys. Zeits.*, **27**, 84, 1926; Galbrun, *Comptes Rendus*, **183**, 652 and 1019, 1926.

[2] *Comptes Rendus*, **179**, 284, 1924.

[3] *L'acoustique des Canons*, 369, 1925. *See also Comptes Rendus*, **172**, 160 and **173**, 167, 1916; **178**, 1892, 1924; **180**, 1412, 1924.

[4] Gazaud, *Comptes Rendus*, **179**, 284, 1916.

[5] Perot, *Comptes Rendus*, **163**, 272, 1916.

gunfire was better heard when the wind blew in a contrary direction to the sound, than when the wind was blowing from the front to Paris.[1]

We have not yet discussed the region in the immediate neighbourhood of a large explosion. Angerer and Ladenburg noticed that the velocity of the sound as measured by their instruments was abnormally high at first, but reached its normal value after about 200 m. This is evidently due to the abnormal intensity in this region. Riemann [2] has proposed a formula for such a case—

$$c' = \frac{dr}{dt} = c\sqrt{1 + \frac{b^2}{r^2}}$$

giving the change of velocity with distance r from the source ; b is a quantity dependent on the width of the layer of air compressed at the instant of explosion. After some distance has been traversed the formula evidently gives the normal velocity c. A pulse from such an explosion is very strongly damped, meaning that it is followed perhaps by only two or three waves of very low frequency. According to Villard,[3] the explosion at La Courtine produced in Paris (230 miles away), a train of waves of 1 sec. period, lasting for 3 secs. only. The most striking effect was an actual forward motion of the air, and a compression causing serious physiological derangements to those persons close to and unprotected from the explosion. On another occasion the same writer noticed an air movement of about 1 cm., at a point 1 mile away from an explosion. The mere noise is quite insignificant in comparison with these very slow pulsations, whose period apparently grows with the quantity of material exploded.[4]

Sounds from Bodies Travelling Faster than Sound. Some very interesting facts come to light when one examines the propagation of sound from moving bodies whose speed exceeds the normal value of c. In earlier times quite normal values had been

[1] Perot, *Comptes Rendus*, **163**, 272, 1916.

[2] *Gött. Nach.*, **19**, 1859. [3] *Comptes Rendus*, **179**, 617, 1924.

[4] *Other refs.* Dörr, *Wien. Ber.*, **122**, 1683, 1913; Bigourdan, *Comptes Rendus*, **162**, 928 and 965, 1916; *Comptes Rendus*, **163**, 78, 1916; Houssay, *Comptes Rendus*, **163**, 350, 1916; Collignan, *Comptes Rendus*, **167**, 333, 1918; *Comptes Rendus*, **172**, 213, 1921; Nolke, *Phys. Zeits.*, **28**, 302, 1927; Whipple, *Roy. Astron. Soc. Proc.*, **2**, 89, 1928 and *Terr. Mag.*, **38**, 13, 1933; Sandmann, *Beit. z. Geophys.*, **28**, 241, 1930; Gutenberg, *ibid.*, **27**, 217, 1930; Meisser, *Phys. Zeits.*, **30**, 170, 1929; Benndorf, *ibid.*, **30**, 97, 1929; Gowan, *Nature*, **124**, 452, 1929; Whipple, *Met. Soc. J.*, **61**, 285, 1935.

obtained from gunfire, but latterly it has been observed that the sound, produced when bullets were shot from powerful guns, travelled faster than the normal. It was at first thought that this was an effect of abnormal intensity, but attempts to collate the intensity and the velocity were unproductive. Researches of Journée [1] led him to think that as long as the bullet travelled faster than sound, the sound travelled with it; as the bullet lost speed the sound forged ahead with the normal velocity. It will readily appear that, knowing the muzzle velocity of the projectile and its deceleration, the time for the sound to travel in the direction of the line of fire is readily calculable. At points to one side, however, the calculation becomes very complicated. There is also, in practice, the complication introduced by the curved path of the projectile. Actually a double sound may be heard at a point to one side, one direct from the muzzle, and the other which has

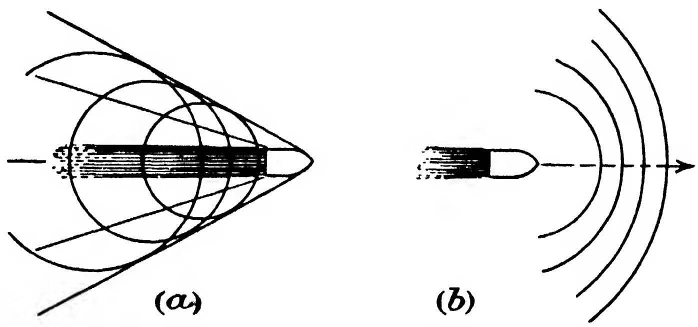


Fig. 10.—Sound Waves from Projectile.

travelled part of the way with the projectile; but, more often, a continuous roll of sound is heard, showing that the bullet during its course is a continuous centre of emitted sound waves, which reach the hearer by paths of continuously increasing or decreasing length. Mach, on the other hand, ascribed the "roll" to reflection from the earth, buildings, etc.—the cause of the rolling of thunder, too—as at such high speeds the bullet is incapable of producing a continuous series of compressions. He and Salcher [2] obtained photographs of these waves by spark photography. The missile is made to break a wire in its flight, which sets off a spark and projects a shadow of the missile and its accompanying waves on to a sensitized plate. When the missile is travelling faster than sound, the waves sent out in its path are enveloped by a conical wave having the bullet at its vertex (Fig. 10*a*).

[1] *Comptes Rendus*, **106**, **244**, **1888**.

[2] *Akad. Wiss. Wien. Ber.*, **95**, **764**, **1887**.

When the bullet's velocity is below that of sound, the sound waves gain upon it, as in Fig. 10*b*. There is always a surface of discontinuity spreading out in cone fashion from the rear of the projectile. The conical waves appear as two pairs of straight lines on the photographic plate (10*a*) because along the surface of this cone we have a number of wave-fronts originating from different points in the missile's path, combining to produce a sufficient compression of the air to be manifest on the plate. In the absence of reflecting surfaces a listener in the neighbourhood of the trajectory of a missile travelling faster than sound, hears first this head-wave, or *onde de choc* and after an interval the muzzle-wave, which has travelled from the gun at the speed of sound throughout, accompanied by a continuous whistling due to the turbulence behind the bullet, often apparent in the photographs. We shall revert to these two sounds in connection with sound ranging.

Velocity of Sound in Tubes. A number of direct investigations were made by Regnault [1] on the velocity of sound in air in pipes up to 3 miles in length. These pipes were intended for a water system in Paris. The method employed was to fire a pistol at one end, and to receive the sound on a membrane at the other end, in conjunction with the electro-magnetic apparatus described above (p. 6). By allowing the sound to be reflected at each end, so that it passed a number of times up and down the tube, the precision of the method was improved. The now well-established fact that the velocity decreases with the width of the tube, emerged from the results. When the tube was 1 foot wide the same value was found as in the open air. The method has been repeated without considerable modification,[2] but has been superseded by the methods described below, which are workable on quite a small scale with short lengths of tube. The tubes we are concerned with here are all at least an inch wide. Very narrow tubes, where friction is paramount, will be considered in Chap. VII.

Kundt's Dust Figures. The method of all small-scale methods which has been most fertile in development was devised by Kundt in 1866,[3] and has provided, in addition to measurements of the

[1] *Comptes Rendus*, **66**, 209, 1868.

[2] Dixon and Greenwood, *Roy. Soc. Proc.*, **105**, 199, 1924, and **109**, 56, 1925; Vautier, *Ann. de Physique*, 1926; Thovert, *Comptes Rendus*, **184**, 517, 1927; Lichte, *E.N.T.*, **4**, 304, 1927; Vautier, Boulaye, Balmé, *Ann. de Physique*, **14**, 263, 1930, and **16**, 311, 1931; *Comptes Rendus*, **184**, 76, 1927, and **193**, 317, 1931; Canac, *Rev. d'Acoust.*, **1**, 52, 1932.

[3] *Ann. d. Physik*, **127**, 497, 1866.

velocity of sound in gases under all conditions, valuable information on the molecular aggregation of their constituents. The air is contained in a glass tube closed at one end B by an adjustable stopper (Fig. 11a). The tube is about 150 cm. long and 3 cm. wide. Projecting into the other end is a metal or glass rod, terminating in a cork or wooden stopper at A, which just clears the tube inside. The rod is usually clamped at the centre, and is stroked with a resined cloth (if of wood) or a damp cloth (if of glass or metal) in order to produce longitudinal vibrations in the rod. This tone will have a definite frequency n and therefore a definite wave-length λ in the rod, determined by the fundamental relation $V = n\lambda$, where V is the velocity of sound in the rod. In addition, if the column of air AB is of the correct length, it will be able to vibrate with the same frequency n, and with a wave-length λ_0, where $c = n\lambda_0$.

One possible mode of vibration of the air column is that corresponding to the half-length of the vibrating bar, that is with the stopped end B fixed, and with maximum amplitude of vibration at the open end D. When the adjustment is made so that both metal and air-column are of such length (dependent on λ and λ_0) that they can both vibrate in this fashion to the same frequency, on exciting the tone in the rod the air column will also vibrate in sympathy with the rod. This is, in fact, an example of the universal principle of Resonance. As the velocities of these longitudinal sound waves will not be the same in the metal as in the air, the wave-lengths and hence the lengths of the vibrating portions are not the same. It will be seen in the next chapter that these lengths are directly proportional to the respective wave-lengths. From the ratio of these wave-lengths the ratio $\frac{V}{c}$ can be found, since $\frac{V}{c} = \frac{n\lambda}{n\lambda_0} = \frac{\lambda}{\lambda_0}$; and the value of c found if that of V is known.

Now equation (3) applies to all longitudinal vibrations, in solid, liquid, or gas, so that if we know the value of the appropriate elasticity for the rod (and this is readily found by experiment) V is simply determined. We have only then to adjust the stopper B, and measure a few distances in air and metal to be able to calculate the velocity in the air in the tube. It is in the detection of the resounding air-column and its measurement that the speciality of Kundt's method lies; another method will be described in the

next section. Instead of having the air in the tube vibrating in the manner described above, Kundt extended the tube until, in order to respond to the tone in the rod, the air-column was obliged to break up into segments, having at E' maximum vibration, at B' (Fig. 11*b*) no vibration (corresponding to the distance EB under former conditions), and then beyond this point a series of segments extending with alternate positions of maximum and no vibration down to the end of the tube B, which, by reason of the unyielding stopper, remains a point where longitudinal motion of the air is impossible.

In order to make these points visible, lycopodium or cork or pith [1] dust is strewn lightly along the tube. At points like B', it remains undisturbed even when the air is responding to the rod, but at points such as E, and to a less extent at points in between, it is thrown into violent motion, ultimately coming to rest in peculiar bands across the tube—the significance of these

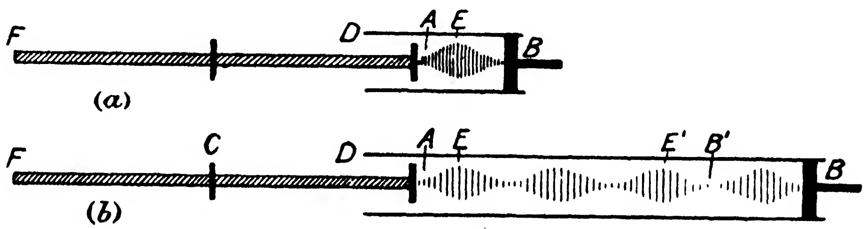


FIG. 11.—Kundt's Tube.

bands will appear later (p. 220). The lengths AC in the rod and EB in the air are vibrating under corresponding conditions, and their ratio represents the ratio of the wave-lengths of the respective media. As Regnault's work showed, the velocity by this method will in general be less than that in free air. In order to reduce these velocities to "absolute" values, Helmholtz [2] employed an empirical formula, which fitted Kundt's and Regnault's results. It is

$$c' = c\left(1 - \frac{k}{r}\right),$$

where r = radius of tube, and k is a constant as long as the gas, tone, and material of the tube are unchanged. If we obtain a second value from another tube of different radius, $c'' = c\left(1 - \frac{k}{r_1}\right)$ we can eliminate k, for

$$r_1\left(1 - \frac{c''}{c}\right) = k = r\left(1 - \frac{c'}{c}\right) \quad \therefore c = \frac{r_1c'' - rc'}{r_1 - r}$$

[1] Cook, *Nature*, **118**, 157, 1926.

[2] *Wiss. Abhand.*, **1**, 338, 1882.

giving a value for the velocity in the open air. Another way of detecting when the tube responds to the exciter is to have a narrow side-tube leading off near A; the stopper is then adjusted until the sound transmitted down this tube to the ear by the vibrating air is a maximum. Quincke [1] and others used this method, with either a telephone transmitter or tuning-fork as exciter in place of the rod. A tuning fork was used by Quincke, who applied the listening-tube as a search-tube to find the actual positions of the maximum and minimum of vibration of the air in the tube.

Velocity in Other Gases.[2] By introducing other gases into the tube it is possible to obtain values of the velocity in these gases. An objection to doing this is that the free passage which must be left near A for the rod to vibrate in, will permit the outside air to diffuse into the tube. Another arrangement due to Kundt [3] is to place a second tube at F containing the gas, and some dust. When the note in the rod is excited, both tubes are adjusted to resonance, and the ratio of the wave-lengths in the air and in the gas, measured from the dust figures, gives the ratio of the speeds of sound. The latter ratio gives important information on the molecular aggregation in the gas, as the ratio of the specific heat of the gas at constant pressure to that at constant volume is a definite function of the number of atoms which go to make up the molecule.

In order to enable a chemically purified gas to be kept from contamination by the outer air, Behn and Geiger [4] introduced the ingenious improvement of enclosing the gas itself in the rod which is rubbed to produce the note. To this end the rod is made hollow and of glass, and contains dust. The gas is introduced at the mid-point which is clamped. The column of *gas* in the rod is of invariable length, and may therefore not resound to the note in its *glass* container, but the effective length of the rod is increased by screwing on metal washers to a threaded metal extension-piece at each end of the rod, until the dust inside shows that resonance has been reached. The adjustment is thus the reverse of what it was in the original apparatus, since here the rod is adjusted to resonance with the gas. It is usual to retain the air-tube, and, after the first

[1] *Ann. d. Physik.*, **128**, 190, 1866. [2] *See also* Ch. xi.

[3] *Ann. d. Physik*, **135**, 337 and 527, 1868. *See also* Röbitzch, *Ann. d. Physik*, **38**, 1027, 1912.

[4] *Ber. deut. phys. Ges.*, **5**, 657, 1907. *See also* Schöler, *Ann. d. Physik*, **45**, 913, 1914.

adjustment, to adjust this for resonance. Wave-lengths and velocities in the two gases can then be compared. The complete arrangement and method of measuring the figures is shown in Fig. 12. Corresponding points in the pattern near where the disturbance of the dust is first apparent, like A, A', A'', and B, B', B'', are chosen and distances measured across them. The Behn and Geiger apparatus has been much used by other chemists and physicists, notably by Partington.[1]

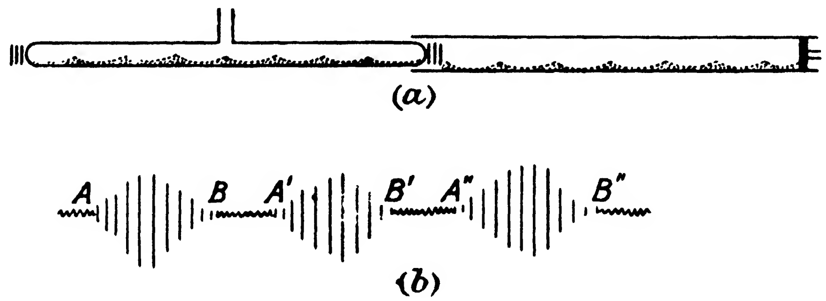


FIG. 12.—Modified Kundt's Tube (Behn & Geiger).

The gas tube should be about 2·5 cm. in diameter, and from 40 to 80 cm. long. The discs were cut from lead, 1 mm. thick and of diameter a trifle less than 2·5 cm., and stuck on by wax. The air tube should have a slightly larger internal diameter. In place of the discs, caps of metal foil hammered to fit the rounded ends of the gas tube may be used.[2] The gas and dust must be scrupulously dried before being admitted into the tube.

Velocity in Gases at Different Temperatures. Kundt's method provides the possibility of measuring the velocity of sound in gases at different temperatures, if the tube be surrounded by a thermostat. At low temperatures it would be inconvenient to surround a long tube by liquid air, but Himstedt and Wedder[3] have been able to measure velocities at low temperatures, by using a very high-pitched source of sound (a Galton whistle, see p. 277) in place of the glass rod, requiring only a short length of the gas-

[1] *Phys. Zeits.*, **15**, 601, 1914; Partington and Shilling, *Faraday Soc. Trans.*, **18**, 386, 1923 and *Phil. Mag.*, **45**, 416, 1923; Partington and Cant, *Phil. Mag.*, **43**, 369, 1922; Shilling, *Phil. Mag.*, **3**, 177, 1927. *See also* in the *Ann. d. Physik*, Kalähne, **20**, 398, 1906; Thiesen, **25**, 506, 1908; Furstenau, **27**, 735, 1908; Wenz, **33**, 951, 1910; Küpper, **43**, 905, 1914; Schweikert, **48**, 593, 1915, and **49**, 433, 1915; Schultze and Rathjen, **49**, 457, 1916; Grüneisen and Merkel, **66**, 344, 1921; Grüneisen and Goens, **72**, 193, 1923; Shilling, *Phil. Mag.*, **3**, 273, 1927.

[2] Partington, *loc. cit.*

[3] *Zeits. f. Physik*, **4**, 355, 1921.

tube accommodated in a Dewar vacuum vessel. In this case resonance in the column of gas was detected by a microphone at the bottom of the gas-tube, which was of invariable length. The frequency of the note given by the whistle, when the gas was thrown into maximum vibration, was determined by a subsidiary Kundt's tube containing air.[1]

Velocity of Sound in Liquids. Obviously, water is the only liquid with which large-scale methods are practicable. Colladon and Sturm [2] measured the velocity of sound in the Lake of Geneva in 1827, a bell being struck under water at one side of the lake, and the sound received through the water at the other. A flash of powder made by the hammer which struck the bell, took the place of the muzzle flash in the overland gunfire experiments. This classic work gave 1,435 metres per second at 8° C., against 1,441, calculated from (2), where the inverse of the compressibility of water was substituted for E.

The great strides which the science of submarine signalling has made in recent years have drawn the attention of physicists to the velocity of sound in the sea. A series of experiments in the roadstead of Cherbourg at a depth of 13 m. was made by Marti,[3] during the European War. Small submarine charges were exploded, and the propagation of the pulse registered by a number of submarine microphones 900 m. apart. The explosion and the response of each microphone were registered on the same chronograph. The average result was high in comparison with those of other investigators (1,503 m. per sec. at 14·5° C., density 1·0245).

Stephenson [4] made careful measurements on the American coast, in which a half-kilogram bomb of T.N.T. was detonated at a depth of 10 m., and simultaneously a radio signal, instead of a light signal, was sent out. The sound signals were received on five microphones at an average distance of 10 miles from the source.

[1] *See also* Dixon, Campbell and Parker, *Roy. Soc. Proc.*, **100**, 1, 1921 ; Irons, *Phil. Mag.*, **3**, 1274, 1927 ; Keesom and Itterbeck, *Roy. Soc. Amsterdam Proc.*, **34**, 204 and 988 and 996, 1931 and **33**, 440, 1930 ; *Rev. d'Acoust.*, **2**, 81, 1933 ; Cornish and Eastman, *Amer. Chem. Soc. J.*, **50**, 627, 1928 ; King, Shilling, Partington, *Phil. Mag.*, **5**, 1920, 1928 and **9**, 1020, 1930 ; Sheratt and Awbery, *Phys. Soc. Proc.*, **43**, 242, 1931.

[2] *Ann. de Chim. et Phys.*, **36**, 113 and 225, 1827.

[3] *Comptes Rendus*, **169**, 281, 1919.

[4] *Phys. Rev.*, **21**, 181, 1923. *See also* Eckhardt, *Phys. Rev.*, **24**, 452, 1924 ; Service, *Frank. Inst. J.*, **206**, 779, 1928 ; Pooler, *Phys. Rev.*, **35**, 832, 1930 ; Dorsay, *Acoust. Soc. J.*, **3**, 428, 1932.

The response of these microphones and the receipt of the radio signal· were registered by a six-string Einthoven galvanometer recording on a moving sensitized film. The average result was 1453·5 m. per sec., at − 0·3 deg. C.; salinity 3·35 per cent.; estimated error less than 0·1 per cent.

The estimated velocity in the neighbourhood of the source rises with the quantity of explosive.[1] This perhaps explains Marti's high value. The changes produced by the possible variations in salinity S and temperature θ are reckoned to range over one or two metres per second only.[2] The following formula contains the results of the experiments of Wood and Browne [3] off St. Margaret's, Kent:

$$V = 4{,}756 + 13{\cdot}8\,\theta - {\cdot}12\theta^2 + 3{\cdot}73S$$

(where V is in ft./sec.; θ in degs. C.; S in parts per thousand).

Kundt's tube has been applied to the determination of the velocity in water, but it is extremely difficult to get wet dust to form the figures. Using oils, one achieves a greater measure of success, but the movements of the dust are still strongly damped. Using a resined wheel continually rubbing against a glass rod to maintain the note, Dorsing [4] was able to get values of the velocity in alcohol and in ether from a Kundt's tube, containing carefully dried powdered pumice-stone. He found that the vibrating liquid tended to force the tube containing it into sympathetic vibration, so that it is advantageous to get rod, liquid column and tube all vibrating to the same note.

Among other "tube methods" we may mention Wertheim's [5] organ pipes, which were not only blown by, but filled with water, enabling V to be estimated from the frequency of the note emitted. Recently two Roumanian physicists [6] have measured the speed of impulses transmitted along water-pipes several metres in length, in the manner of Regnault's researches on air (p. 27). It was found that even after applying a "tube correction," the velocity depended greatly on the type of shock transmitted.

[1] Threlfall and Adair, *Roy. Soc. Proc.*, **45**, 450, 1889.

[2] Lichte, *Phys. Zeits.*, **20**, 385, 1919.

[3] *Phys. Soc. Proc.*, **35**, 183, 1923.

[4] *Ann. d. Physik*, **25**, 227, 1908; *see also* Busse, *Ann. d. Physik*, **75**, 657, 1924; Lechner, *Akad. Wiss. Wien. Ber.*, **118**, 1,035, 1909.

[5] *Ann. d. Physik*, **77**, 427, 1849.

[6] Jonescu, *J. de Physique*, **5**, 377, 1924; Cisman, *J. de Physique*, **7**, 344 1926.

CHAPTER TWO

VIBRATING SYSTEMS

Simple Harmonic Motion in a Straight Line. In most natural phenomena which we meet, the motion must be treated as two or three dimensional. For simplicity of mathematical treatment we shall consider first the periodic motion of a particle moving along a straight line between the points A and A' (Fig. 13a). One dimension will then suffice to describe the position of the particle at any instant. The motion is periodic if, after an interval T, the motion repeats itself; T is then called the

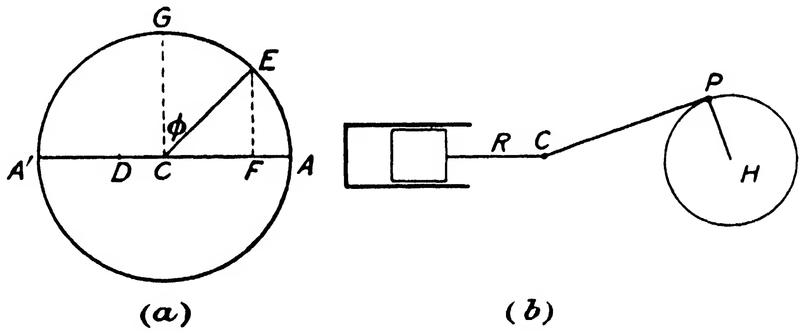


FIG. 13.—Simple Harmonic Motion.

" period " of the motion. Moreover, the movements of the particle are identical from period to period, so that if, after a certain fraction of the period, or at a certain " phase," the particle has moved from A to D, and we wait for its return to A, we shall find in the next period and after the same fraction of the period has elapsed, the particle will be at D again. In sound we deal almost entirely with vibrations symmetrical with regard to the mid-point of the motion; the particle goes through evolutions on the right-hand side of C similar to those on the left.

Mathematically the simplest type of vibration that we can imagine happens to be that obtained by projecting circular motion on to a straight line. We imagine a circle drawn on AA' as diameter, and a point to move round it with angular velocity ω,

and suppose the particle to lie always on the projection of the point E on AA', i.e., at F (Fig. 13a). At the instant pictured let the radius vector CE make an angle ϕ with the normal, and put $CF = y$, the particle's displacement from its central position. Then, if t is the time that has elapsed since E was at G and the particle was at C, $\phi = \omega t$.

Also
$$y = a \sin \phi = a \sin \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$
where a = the radius of the circle = the "amplitude" CA of the vibration. Also the "period" T of the motion $= \frac{2\pi}{\omega}$, and the "frequency" or number of revolutions n per second $= \frac{\omega}{2\pi}$.

Equation (8) can also be written :—
$$y = a \sin \frac{2\pi t}{T} = a \sin 2\pi n t.$$

The velocity of the particle at any instant :—
$$\frac{dy}{dt} = 2\pi n a \cos 2\pi n t = \omega a \cos \omega t \quad . \quad . \quad . \quad . \quad (9)$$

When $\phi = 0$, the particle is at C, $\cos \omega t = 1$, and the velocity is
$$\omega a = 2\pi n a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$
This being the maximum velocity in the motion of the particle, the velocity decreases on each side of C until, when the particle is at A or A', the displacement is a maximum a, and the velocity is 0, because $\phi = \pm 90°$, $\cos \omega t = 0$.

Finally the acceleration $= \frac{d^2y}{dt^2}$
$$\begin{aligned} &= -\,4\pi^2 n^2 a \sin 2\pi n t \\ &= -\,\omega^2 a \sin \omega t \\ &= -\,\omega^2 y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11) \end{aligned}$$

Since ω^2 is a constant, we have demonstrated the very important property of this so-called Simple Harmonic Motion (S.H.M.), that the acceleration is always proportional to the displacement. The minus sign indicates that the acceleration opposes the displacement, the particle being accelerated when its displacement (from C) is decreasing, and decelerated when this distance is increasing. At C itself the acceleration is nothing. If we multiply both sides of equation (11) by m the mass of the particle, the left-hand side (mass $\times$ acceleration) represents the restoring force acting

on the particle, tending to bring it back to C. We may state then, alternatively, that the restoring force is proportional to the displacement.

It should be noticed that the character of the motion, as borne out by the above differential analysis, is unchanged by adding a constant under the trigonometric functions, thus:—

$$y = a \sin \left[\frac{2\pi}{T}(t + t_0)\right]$$

describes the same motion as (8) but reckons the time from another zero point. Another mode of expressing this is to say that this equation represents the motion of another particle on the same line AA' following the same motion as the one delineated by (8) but always lagging behind or leading the latter by a time t_0 according as t_0 is negative or positive. If this second particle be supposed to follow the projection of another point E' moving round the circle (Fig. 13a) at the same speed as E, but at a constant angle $ECE' = \delta$ behind or in front of E, then we may describe the motion of the second particle by the equation:—

$$y = a \sin \left[\frac{2\pi t}{T} + \delta\right] \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This is the most common way of representing the "phase lag" or "phase lead" of one S.H.M. in reference to another of the same period, i.e., by the angle δ.

Demonstration of S.H.M. Owing to that physical property known as elasticity, systems which, when displaced from their position of rest, experience a force tending to restore them to that position and proportional to the displacement, are quite common in nature; hence the motion discussed above is of universal importance, and forms the basis of those motions which produce sound. The sound-producing motions being too rapid to be followed by the unassisted eye, slower systems have to be called upon for demonstration purposes. The simplest of these is the conical pendulum. A weight hung on a string is made to rotate in a circle corresponding to the circular motion of Fig. 13a. If the observer stands at a distance from the pendulum, with his eye on a level with the weight, so as to view the motion edgewise, the circular motion appears projected on to an imaginary horizontal straight line producing simple harmonic motion, as it were along AA'. The pendulum bob itself can be made to reproduce this S.H.M., if pulled slightly aside and let go, so that it

is constrained to move in a vertical plane and on a small arc of a circle. An approximate S.H.M. may also be seen on all types of reciprocating engines (Fig. 13*b*). In such, the circular motion of the crank-pin P round the axle H is converted into the to and fro motion of the piston and piston rod R. If any point on this or on the cross-head C be watched, it will be seen to follow S.H.M. approximately. The maximum velocity of the piston occurs when the crank and connecting rod are at right angles to each other as shown. Note that the motion is symmetrical only when the connecting rod is of infinite length; but with a long rod and a short crank the motion is to all appearance S.H.M.

If, while the particle is vibrating along AA' (Fig. 13*a*), it be given an additional movement at constant speed in the direction at right angles to AA', it will trace out a sinuous curve which will be, from the nature of the motion, a sine curve representing displacement in the AA' direction, and time in the direction of the imposed motion. This provides a common way of representing vibratory motion graphically; displacements are plotted vertically as ordinates, against the time plotted horizontally.

Superposition of two Simple Harmonic Motions of equal Periods in the same Straight Line. Let the position of the particle under one periodic force be given by

$$y_1 = a_1 \sin(2\pi nt + \delta_1)$$

and, under another periodic force, by

$$y_2 = a_2 \sin(2\pi nt + \delta_2).$$

Under the action of the two forces acting simultaneously, the particle will execute a vibration such that its position at any instant will be equal to the sum of those which it would have under either force acting alone. This is the Principle of Superposition, and as we shall see, it is, in practice, applicable only when the amplitudes a_1 and a_2 are small. With this proviso, the resultant displacement:—

$$y = y_1 + y_2 = a_1 \sin(2\pi nt + \delta_1) + a_2 \sin(2\pi nt + \delta_2)$$

can be written for example:—

$$y = A \sin(2\pi nt + \Delta),$$

provided the coefficients of $\sin 2\pi nt$ and $\cos 2\pi nt$ in each expression are the same. This will be so if:—

$$a_1 \cos \delta_1 + a_2 \cos \delta_2 = A \cos \Delta,$$

and $$a_1 \sin \delta_1 + a_2 \sin \delta_2 = A \sin \Delta.$$

Squaring and adding the last two equations, we find the amplitude of the resultant is given by :—

$$A^2 = a_1{}^2 \cos {}^2\delta_1 + a_2{}^2 \cos {}^2\delta_2 + 2a_1a_2 \cos \delta_1 \cos \delta_2$$
$$+ a_1{}^2 \sin {}^2\delta_1 + a_2{}^2 \sin {}^2\delta_2 + 2a_1a_2 \sin \delta_1 \sin \delta_2$$
$$= a_1{}^2 + a_2{}^2 + 2a_1a_2 \cos (\delta_1 - \delta_2),$$

and the phase Δ of the resultant is given by :—

$$\tan \Delta = \frac{a_1 \sin \delta_1 + a_2 \sin \delta_2}{a_1 \cos \delta_1 + a_2 \cos \delta_2}.$$

The resultant amplitude is less than the sum of the component amplitudes, except when $\delta_1 = \delta_2 +$ a multiple of 2π.

An interesting case occurs when $a_1 = a_2$, and $(\delta_1 - \delta_2) = \pi$, or any *odd* multiple of π, so that $\cos (\delta_1 - \delta_2) = -1$, and therefore $A = 0$, and also $y = y_1 + y_2 = 0$. In fact, the particle is being given displacements simultaneously in two opposite directions and so remains at rest. This "wiping-out" of one vibration by a superposed vibration of opposite phase is of wide occurrence in all forms of wave motion, and is known as "interference." This term is often applied to the superposition of vibrations in general, but should strictly be limited to the mutual cancellation of motions.

Incidentally, our analysis shows that two S.H.M.'s of equal period acting in the same direction, add up to produce a vibration of equal period, but of different phase and amplitude. Analysis similar to the above can be applied to any number of vibrations superposed in the same straight line on the same particle.

Fourier Analysis. A very important theorem, or rather the physical interpretation thereof, enables us to resolve any periodic vibration into simple components. The theorem turns on the possibility (first recognized by Bernoulli[1] for the problem of the vibrating string) of representing a complex periodic function as a series of sines and cosines of continually increasing order, thus :—

$$f(x) = \tfrac{1}{2}a_0 + a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + b_1 \cos x$$
$$+ b_2 \cos 2x + b_3 \cos 3x, \text{ etc.} \quad . \quad . \quad . \quad (13)$$

Each of these terms will be recognized as a Simple Harmonic Motion in itself if ωt is put in place of x, so that the theorem states the possibility of representing any motion of a point along a line (as long as the motion is periodic) as the superposition of a series of S.H.M.'s whose frequencies gradually increase in the

[1] *Hist. de l'Acad. des Sciences*, **147, 1753.**

ratio of the natural numbers. It may be noted in passing that such a series is called a "harmonic series," and the separate motions $a_1 \sin \omega t$, $a_2 \sin 2\omega t$, $a_3 \sin 3\omega t$, etc., are called "harmonics" of which the first or "prime" or "fundamental" motion is represented by $a_1 \sin \omega t$. The test of the feasibility of such a separation into component vibrations lies in the possibility of determining the values of the amplitudes a_1, b_1, etc., of the components making up a given periodic function. This was accomplished by Fourier, who thus gave us a series of far-reaching importance in physics. For Fourier's[1] method of calculating the coefficients mathematical textbooks must be consulted[2]; suffice it to say that the general coefficients are given by other series :—

$$a_m = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x) \sin mx dx. \qquad a_0 = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x) dx.$$

$$b_m = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x) \cos mx dx.$$

It appears that the coefficients a_m and b_m are not calculable if the functions $f(x) \sin mx$ and $f(x) \cos mx$ are not directly integrable. With many of the functions occurring in physics this is not possible, but it usually is practicable to determine a_m and b_m by approximate integration.

The Fourier series can also be written as a series of sines alone, but representing component S.H.M.'s in different phases; indeed, the presence of the cosines in the above form (13) is merely a method of indicating these phase differences. Thus if in (13) we put $c_1 = \sqrt{a_1^2 + b_1^2}$; $\tan \delta_1 = \frac{b_1}{a_1}$, etc., we get the series in the form :—

$$f(x) = \tfrac{1}{2}a_0 + c_1 \sin (x + \delta_1) + c_2 \sin (2x + \delta_2) + \text{etc.}$$

representing a series of harmonics, all having different phase-angles of lead.

To obviate the tedious labour of calculating the coefficients required to complete the Fourier analysis of a given curve, a number of instruments have been devised to perform this operation

[1] *Mém. Acad. Paris*, **4**, 185, 1819.

[2] *See* Lamb, p. 89; Rayleigh, *Phil. Mag.*, **24**, 864, 1912. *See also* Russell, *Phys. Soc. Proc.*, **27**, 149, 1914.

in respect of the first few (and usually the most important) coefficients.[1]

Progressive Undulation. If we imagine our particle which has been executing S.H.M. to be connected by elastic springs to a series of similar particles, then the movement of the original particle will set in motion each succeeding particle but with a progressive phase-lag (owing to inertia) of each behind the one previously set in motion. The disturbance originally confined to one particle will be seen to progress along the row from end to end in the form of a wave, known as a progressive wave. If the particles and springs lie along a continuation of AA' (Fig. 13), so that the springs are compressed or extended by the motion, the wave is called longitudinal; this is the type of wave dealt with in the first chapter, but in one dimension instead of three. If the line of particles is at right angles to AA' and to the direc-

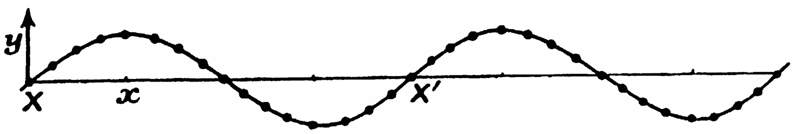


FIG. 14.—Progressive Undulation.

tion of motion of the particles, the vibration is called transverse Finally, if instead of moving along AA', the first particle is given a rotation about its axis, causing a twist of the springs and subsequent rotation of the other particles, the vibration is torsional.

In accordance with the convention previously mentioned, the displacement is plotted vertically, in whatever direction it actually lies. If the distances along the row of particles be represented horizontally, Fig. 14 represents the relative displacements of the particles at any instant, though it is only for the one case of transverse vibration that Fig. 14 represents the actual relative positions of the particles. The distance XX' represents the wavelength λ, and the motion is given by:—

$$y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right),$$

or, remembering that $n\lambda = V$, and $n = \frac{1}{T}$,

$$y = a \sin 2\pi n\left(t - \frac{x}{V}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

[1] *See* Miller, *Frank. Inst. Journ.*, **181**, 51 and **182**, 285, 1916; Kranz, *Frank. Inst. Journ.*, **204**, 245, 1927.

for the particle at x goes through the same evolutions as that at X, but at a time $\frac{x}{V}$ later. This lag progressively increases with x until at X' where $x = \lambda$ it has become $\frac{1}{n} = T$, so that the particles at X and at X' are in step. They are in step again at $x = 2\lambda$, etc. Equation (14) accordingly represents a wave progressing in the positive direction of x, the second term under the sine representing the gradual taking up of the motion by subsequent particles.

It is to be noted that we have taken no account of possible loss of amplitude in transmission. The velocity of a particle at any instant $= \frac{dy}{dt} = 2\pi an \cos 2\pi n\left(t - \frac{x}{V}\right)$, and therefore lags a quarter-period behind the displacement.

Stationary Waves. In this type of motion the successive particles, over a section at least, of the vibrating series all have the same phase but with amplitude continuously diminishing from a maximum at one place (called a loop or antinode) down to zero at the end of the section (called a node). In the simplest type of stationary vibration the decrease of amplitude with distance x from antinode to node follows a sine-law, thus :—

$$y = a_0 \sin \frac{\pi}{2l} x \sin 2\pi nt \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where a_0 = amplitude at antinode, and l = distance from node to antinode. This formula gives :—

$$y = a_0 \sin 2\pi nt$$

for the vibration of a particle at the antinode, falling to $y = 0$ at the node, the motion at all intervening points being in the same phase.

Stationary waves may also be produced in longitudinal, transverse or torsional form. The rod used in the Kundt's experiment (Chap. I) executed stationary longitudinal vibration, having a node at the clamped centre, and an antinode at each free end. A spiral spring clamped at the top and suspended with axis vertical will exhibit this motion if the lower end be slightly extended and let go. Stationary transverse vibration may be produced in the cord of a bow by twanging it at the centre.

A body may vibrate in a number of segments containing alternate nodes or antinodes by suitably arranging the fixed points

in it (cf. succeeding chapter). The distance between two successive antinodes represents the unit-pattern of the vibration, and may be termed, in accordance with a preceding definition, the wave-length of the stationary motion, but as in fact two succes-

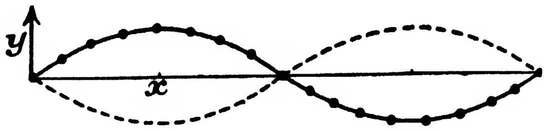


FIG. 15.—Stationary Vibration.

sive loops have opposite phases (see Fig. 15), it is more usual to call the length of *two* loops the wave-length, and this falls into line with the concept of stationary motion demonstrated in the next section.

Reflection of Progressive Waves. By means of our formula (14) we can show that a stationary wave can be regarded as the superposition of two progressive waves of equal amplitudes, frequencies and velocities, but travelling in opposite directions. Thus, as

$$y = a \sin 2\pi n\left(t - \frac{x}{V}\right) \quad . \quad . \quad . \quad . \quad . \quad (14)$$

represents a wave travelling in the positive direction of x from left to right,

$$y = a \sin 2\pi n\left(t + \frac{x}{V}\right)$$

represents a wave travelling from right to left. The resulting motion is obtained by adding these two equations :—

$$y = a \sin 2\pi n\left(t + \frac{x}{V}\right) + a \sin 2\pi n\left(t - \frac{x}{V}\right)$$

$$= 2a \sin 2\pi nt \cos \frac{2\pi x}{\lambda}.$$

If this be compared with the formula (15) it will be seen to represent stationary vibration in which the amplitude of particles at the antinodes $\left(\text{at } x = \text{a multiple of } \frac{\lambda}{2}\right)$ is $2a$, or twice the amplitude of either progressive wave. Also the distance l from node to antinode is a quarter of the wave length of the progressive undulation. By similar reasoning a contrary transformation may be demonstrated, i.e., that two superposed stationary vibrations, one advanced a half period in front of the other, make up a pro-

gressive wave advancing in the direction of the stationary vibration with the leading phase, and having amplitude equal to the antinodal displacement of either of the stationary waves.

Both the propositions in this section may be proved by graphical means, by drawing the two sets of waves after the style of Fig. 14, but at a number of succeeding instants in the period; a process more tedious than the analytical demonstration given here.

Phase Change on Reflection. If we regard stationary vibration as made up of progressive waves, it is of interest to examine the conditions at the end of the series of particles, or of the medium in vibration. When this end is free the maximum vibration is possible, and we find at such an end an antinode of stationary vibration; when the end is fixed, or, in the case of a fluid column is limited by a rigid wall, no motion at any instant is possible, and so this point becomes a node. At the antinode, the motion being given by $2a \sin 2\pi nt$, the actual displacement at any instant can be regarded as made up of two superposed displacements, each equal to $a \sin 2\pi nt$ and of the same phase. To get this displacement out of the two progressive waves we may imagine the incident wave to come up to the wall, and to recede without change of phase; this will make the total displacement equal to the arithmetical sum of the two separate displacements at every instant. In order to get no motion at the nodal end at any instant we must postulate a complete reversal of phase (π) on reflection, in order that the separate amplitudes may mutually destroy each other, thus:—

$$y = a \sin 2\pi nt + a \sin (2\pi nt + \pi) = 0.$$

The case of imperfect reflection is worthy of consideration. Suppose that owing to transmission of part of the vibration at an "end" into the neighbouring medium, there is a loss of amplitude in the reflected wave; then the superposed motions of the direct and retrograde waves will be represented by:—

$$y = a \sin 2\pi\left(nt - \frac{x}{\lambda}\right) + b \sin 2\pi\left(nt + \frac{x}{\lambda}\right)$$

where b is less than a. Expanding:—

$$y = (a + b) \sin 2\pi nt \cos 2\pi\frac{x}{\lambda} - (a - b) \cos 2\pi nt \sin 2\pi\frac{x}{\lambda}$$

This represents a set of imperfect stationary vibrations with pseudo-antinodes where the vibration is $(a + b) \sin 2\pi nt$, and pseudo-nodes where $y = (a - b) \cos 2\pi nt$. The motion in the former points is less

than before, while there are no real nodes, their place being taken by points of minimum vibration.

This type of motion will be met with in Melde's experiment (Chap. IV) and in the measurements of absorption coefficients (Chap. XIII).

Superposition of Vibrations in Directions at Right Angles. This type of superposition is purely artificial, that is to say, no natural sounds involve this type of vibration, but it leads to an important precision method for comparing the relative frequencies and phases of the vibrations executed by two bodies. Let a particle be acted on by a periodic force tending to displace it in the x direction according to the law :—

$$x = a \sin 2\pi nt \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and at the same time by another force which, acting alone would make it execute vibrations in the y direction at right angles to the former ;

$$y = b \sin (2\pi nt + \delta) \quad . \quad . \quad . \quad . \quad (17)$$

Then, under the combined forces, the particle will trace out a two dimensional figure in the plane of xy. The *form* of this figure, but not the rate at which it is traced, will be obtained by eliminating t from (16) and (17). Expanding (17) and substituting from (16) :—

$$\frac{y}{b} = \frac{x}{a} \cos \delta + \sqrt{1 - \frac{x^2}{a^2}} \sin \delta.$$

Squaring :
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos \delta = \sin^2 \delta \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This represents an ellipse, contained within a rectangle of sides a and b, but of an eccentricity and at an inclination depending on the phase difference, and the individual amplitudes.

A number of special cases will be noticed :—

(1) If $\delta = 0$ or π, or any multiple of π, the equation gives one of two straight lines, diagonals of the rectangle, $y = \pm \frac{b}{a} x$

(2) $\delta = \frac{\pi}{2}$ or any odd multiple of $\frac{\pi}{2}$ gives the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

symmetrically situated in the rectangle, and traced out in one or other direction. If in addition $a = b$, the ellipse becomes a circle.

When the components are vibrations of different frequency, the method of elimination of t follows the same lines, but the resulting curves are too complex to be treated mathematically in this treatise. The curve is in general bent on itself, and shows "double points," e.g., when the ratio is 1 to 2, one double point; when 1 to 3, two double points; in fact, the number of these points is a function of the period ratio. In these cases, though the phase for each vibration remains fixed, yet the relative phase (if one can speak of relative phase when the periods are different) is constantly changing, owing to dissimilarity of periods. For example, confining one's attention to successive passages of the faster vibration past its null point, the first time the slow vibration may have zero displacement; the second time the fast vibration reaches the null phase, it may have a small positive displacement; the third time, a larger positive displacement, etc., until they will reach the null-displacement point once more together, provided the periods are commensurate.

For pure demonstration purposes, one or other of the "harmonographs" based on the principle of Blackburne's Pendulum may be used, by which a pencil or ink-marker is given perpendicular motions, an adjustment being provided for varying the periods of the two pendulums by which this motion is caused. In sound, however, superposition of rectangular vibrations is of importance because it serves as a test for the equality of the periods of two vibrating bodies, and was first so used by Lissajous. The figures in Fig. 16 are usually known as Lissajous' Curves.[1]

The method was incorporated by Lissajous into a "vibroscope" consisting of a microscope, the object-glass of which was detached from the rest of the system and vibrated to and fro on one of the prongs of a tuning fork. The vibroscope is placed with its axis vertical, and the vibration which it is desired to examine takes place in a horizontal plane, but at right angles to the motion of the tuning fork; a polished speck on the vibrating body may serve as object. If the tuning fork is suited to the nominal frequency of the vibration, or has a frequency a small multiple or sub-multiple of it, one or other of the figures in Fig. 16 will be traced, and will be visible (owing to optical fatigue) as a whole in the microscope. Any slight change in the ratio of the frequencies will be at once detectable by the "wandering" of the form of the vibration from its regular evolution, so that it no longer follows

[1] *Comptes Rendus*, **44**, 727, 1857.

out a recurring sequence. When both vibrators are large bodies (e.g., two tuning forks) and can be made to carry a light mirror, the figures can be projected on a screen, by allowing light from a source to be successively reflected from the two mirrors before falling on the screen.[1]

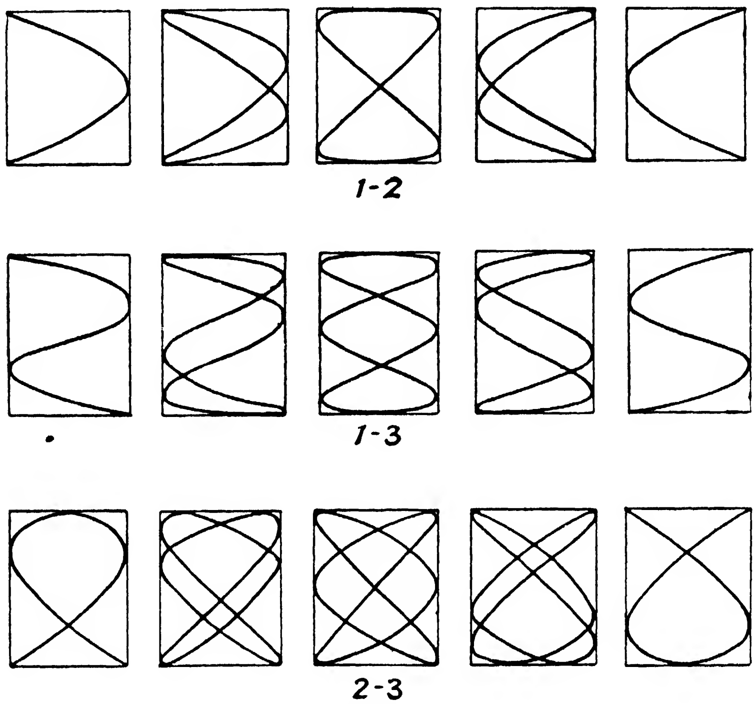


FIG. 16.—Lissajous' Curves.

Undamped Motion. In the simple cases of vibration which we have dealt with up to the present no account has been taken of the loss of energy by friction. When a particle has been given the amplitude a it has been tacitly assumed that it continues to vibrate with this amplitude undiminished, or else that energy has been supplied to maintain it so. Let us glance back over the ideal case, in which no degradation of energy takes place. The kinetic energy of the particle is expressed in the product of half its mass and the square of its velocity; by such a case where the elastic force is proportional to the displacement, the potential energy in its turn is proportional to the square of the displacement, because rate of change of potential energy in the direction of the displacement must be equivalent to the force, or alternatively, because the potential energy is the force integrated over the displacement.

If m is the mass of the particle (inertia of the system) and k

[1] *See* Carrière, *J. de Physique*, **3**, 355, 1932.

represents the elastic force per unit displacement, the principle of conservation of energy gives :—

$$\frac{1}{2}m\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}ky^2 = \text{a constant} \quad . \quad . \quad . \quad . \quad (19)$$

By differentiation with respect to time :—

$$m\frac{d^2y}{dt^2} = -ky \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

an expression which we have already obtained (with ω^2 in place of $\frac{k}{m}$) and whose solution we know to be :—

$$y = a \sin \left(\sqrt{\frac{k}{m}}t - \delta\right).$$

The frequency $n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ depends only on the ratio of the elastic to the inertia force, but it must be remembered that this is true only for small displacements. This being assured, we observe that increase of the elastic force or decrease of the mass, both raise the pitch of the emitted sound. We may note in passing that the kinetic energy in the motion is proportional to $\left(\frac{dy}{dt}\right)^2$ and therefore to $a^2\omega^2$; this latter quantity is often taken to represent the intensity of the sound.

Damped Oscillations. Under actual conditions, some of the kinetic energy is degraded and appears as heat, either in the system itself or in the surrounding medium. In any case the dissipative force introduced by friction is proportional to the velocity, or rather, to the relative velocities of the rubbing substances, provided these are not so great as to produce vortices in the surrounding medium. Adding such a frictional term to (20) we get :—

$$m\frac{d^2y}{dt^2} = -ky - \mu\frac{dy}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The solution of this equation is :—

$$y = ae^{-\alpha t} \sin [\omega t \pm \delta'],$$

and may be verified by differentiation. When this is done we find :—

$$\alpha = \frac{\mu}{2m}$$

and

$$\omega^2 = \frac{k}{m} - \frac{\mu^2}{4m^2}.$$

In acoustical phenomena, the rate of decay in amplitude represented by α is comparatively slow, so that the case in which, owing to $\frac{\mu}{2m}$ being greater than $\sqrt{k/m}$, the quantity ω is imaginary, may be dismissed. Ignoring this case, the effect of friction is twofold :—

(1) Loss of amplitude.
(2) Fall of frequency.

The frequency is $n' = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{\mu^2}{4m^2}}$

instead of $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$,

a change from n to $\sqrt{n^2 - \frac{\mu^2}{16m^2\pi^2}}$.

The effect of viscosity on the frequency is then a quantity of the second order, and in ordinary circumstances we can say that friction has no influence on the period, but only on the amplitude. When the vibrating body is surrounded by a medium which exerts a considerable dragging effect on the motion (e.g., bars and strings vibrating in liquids) we shall see that this approximate statement is insufficient (cf. p. 122).

Free and Forced Vibration. If the system now, instead of being given an impulse and let go, has a sustained periodic force acting upon it, the system reproduces what are called "forced" vibrations. The type described in this chapter up to the present is denoted "free" vibration. For a forced vibration, we must modify (21) by adding a term representing this external force. The simplest type of periodic force is the simple harmonic, so we write our equation :—

$$m\frac{d^2y}{dt^2} = -ky - \mu\frac{dy}{dt} + F\sin pt, \quad . \quad . \quad . \quad (22).$$

so that F is the amplitude of the force, and $\frac{2\pi}{p}$ the period of its alternation. The particular solution of this equation which denotes the forced vibration is :—

$$y = A\sin(pt - \delta).$$

Substituting in (22) and equating the coefficients of $\sin pt$ and $\cos pt$ we find :—

$$-mAp^2\cos\delta = -kA\cos\delta - \mu Ap\sin\delta + F \quad . \quad (23)$$

$$mAp^2\sin\delta = +kA\sin\delta - \mu Ap\cos\delta \quad . \quad . \quad . \quad (24)$$

From (24), $$\tan\delta = \frac{\mu p}{k - mp^2}.$$

Dividing (23) by $\cos\delta$, and substituting $\tan\delta$ for this expression and

$$\frac{k - mp^2}{\sqrt{\mu^2p^2 + (k - mp^2)^2}}$$

for $\cos\delta$, we get :—

$$A = \frac{F}{\sqrt{\mu^2p^2 + (k - mp^2)^2}}$$

whence $$y = \frac{F}{\sqrt{\mu^2p^2 + (k - mp^2)^2}}\sin(pt - \delta) \quad . \quad . \quad . \quad (25)$$

Another solution is obtained when $F = 0$; this corresponds to the free vibration, and we have already obtained it as a solution of (21). The general solution involves both particular solutions :—

$$y = ae^{-\alpha t}\sin(\omega t - \delta) + \frac{F}{\sqrt{\mu^2p^2 + (k - mp^2)^2}}\sin(pt - \delta) \quad . \quad (26)$$

The general solution includes two periodic functions, one the natural vibration of the driven system, and the other the vibration imposed by the driver. Usually, owing to the diminishing amplitude of the former, the frequency of the driver predominates over the other; but the particular case where the two periods are nearly, if not quite equal, is one of far-reaching importance in acoustics.

The student of electricity may find it interesting to compare these equations, (20), (21) and (22) and their respective solutions with the corresponding cases of circuits containing resistance (R), inductance (L), and capacitance (C). For example, (22) has its counterpart in the equation for the quantity (Q) of electricity at any instant in such a circuit to which an alternating E.M.F. of $\frac{p}{2\pi}$ cycles per second is applied.

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E\sin pt.$$

The comparison shows L to be of the nature of an inertia, R of a viscous force, and $1/C$ of an elastic one. The solution of this is, *mutatis mutandis*, our equation (26), and the solution corresponding to the forced vibration (25), is worked out in textbooks on electricity.[1] In the absence of damping forces due to resistance, the equation becomes:

$$L\frac{d^2Q}{dt^2} + \frac{1}{C}Q = E\sin pt.$$

[1] *See e.g.* Starling, *Electricity and Magnetism*, 353.

In this case the natural frequency of the circuit is $\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$; $\left(\text{corresponding to } \sqrt{\frac{k}{m}} \text{ in (20)}\right)$; and resonance is produced when the applied E.M.F. alternates with this frequency, i.e., when $p = \sqrt{\frac{1}{LC}}$.

Resonance. If the effect of viscosity is small, then the amplitude under the action of the driving force is a maximum when the denominator in (25) and (26) is a minimum, i.e., when $k = mp^2$ or $p = \sqrt{\frac{k}{m}}$, or, the frequency of the driver is equal to that of the driven; in fact, if $\mu = 0$, the amplitude would become infinite at this frequency. This enhanced oscillation when the periods coincide is known as "resonance." The amplitude at resonance is (with friction) $\frac{F}{\mu p} = \frac{F}{\mu}\sqrt{\frac{m}{k}}$. A quantity of importance in sound is the "sharpness of resonance," expressing the fall of amplitude with change of frequency on each side of the maximum. When the damping is great this drop of amplitude is very slow; on the contrary, the resonance is sharp when the frictional losses are small. The truth of this apparently paradoxical statement—it appears at first sight as though friction should prevent the attainment of large amplitudes except at coincidence of periods—is verified by the following calculation.

Sharpness of Resonance and Variation with Pitch. Following Rayleigh,[1] we may gauge the response of the particle to sustained forcing, by working out the kinetic energy possessed by the particle at the instant of its passage through the undisturbed position. This is equivalent to finding the maximum velocity in the motion defined by (25), i.e., the velocity when $\cos(pt - \delta) = 1$.

$$\left(\frac{dy}{dt}\right)_{max} = \frac{pF}{\sqrt{\mu^2p^2 + (k - mp^2)^2}}$$

$$\text{Kinetic energy} = \tfrac{1}{2}m\left(\frac{dy}{dt}\right)^2 = \frac{\tfrac{1}{2}mp^2F^2}{\mu^2p^2 + (k - mp^2)^2}.$$

Dividing out by $\frac{1}{2}\frac{F^2}{m}$, the mean square of the driving force per

[1] *Sound*, ', 47, 1894.

unit mass during the period, we get the kinetic energy per unit forcing, which Barton[1] calls the response (R);

$$R = \frac{p^2}{\frac{\mu^2 p^2}{m^2} + \left(\frac{k}{m} - p^2\right)^2}. \quad . \quad . \quad . \quad . \quad (27)$$

Now $\sqrt{\frac{k}{m}}$ represents the natural frequency of the system in the absence of damping, so that the expression in the brackets represents the extent to which the natural frequency of the system deviates from the forced frequency, and has been called the "mistuning." When driver and driven are exactly in tune, the response $R = \left(\frac{m}{\mu}\right)^2$, i.e., is inversely as the frictional coefficient μ. As the tuning gets

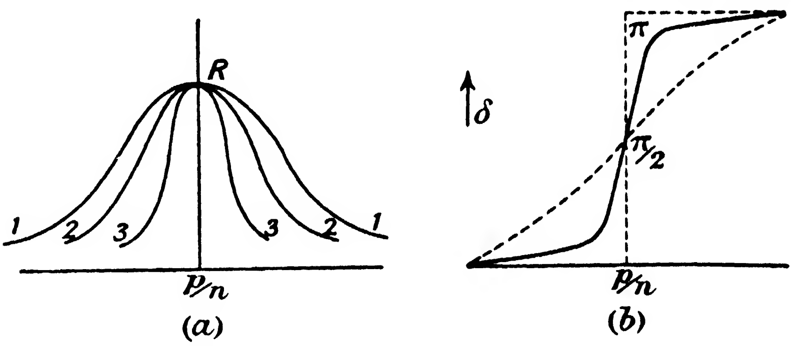


FIG. 17.—Response and Phase at Resonance.

worse, R naturally falls, giving a curve of the type shown in Fig. 17a, when R is plotted against the ratio of the frequencies $\frac{p}{n}$. On the other hand, keeping to one natural frequency and supposing μ to vary, the curves will drop less rapidly as μ increases, since μ occurs in the denominator of (27), and their maxima will be successively less. This was first pointed out by Helmholtz[2] who calculated the mistuning necessary to reduce the response to one-tenth of its maximum. Thus the sharpness of resonance, represented roughly by the steepness of the resonance curve (Fig. 17a) increases inversely as the damping of the system, and the maximum response is thereby reduced.

In practice, the systems we deal with are capable of performing in addition natural vibrations corresponding to a number of mem-

[1] *Phil. Mag.*, 26, 111, 1913.

[2] *Sensations of Tone*, App. 10, 1865. The references to this book are to Ellis' translation, 1885.

bers of the harmonic series, i.e., with frequencies which are multiples of n. Provided the damping represented by $\frac{m}{\mu}$ is unchanged for such a harmonic, for a given mistuning, the response will be inversely as p^2 (cf. 27), and therefore will diminish as we go up the harmonic series. In other words, although the maximum response under these conditions will be constant, the resonance curve will droop more sharply at the higher natural frequencies (see Fig 17*a*).[1]

Phase of Resonance. Having considered the amplitude in equation (25) we will now consider the other factor δ, representing the phase lead of the forced vibration in front of that of the driving force. If p is small, then is δ also small, but grows with p until at resonance, $p^2 = \frac{k}{m}$, $\tan\delta = \infty$, therefore $\delta = \frac{\pi}{2}$, so that the lead has now reached a quarter period. Beyond the resonant

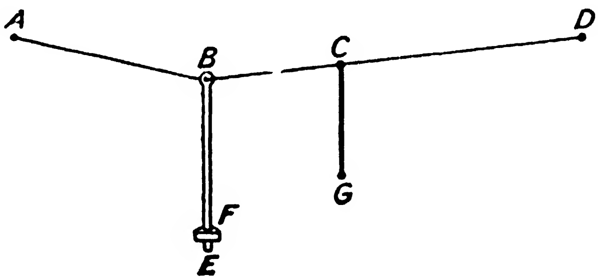


FIG. 18.—Demonstration of Resonance.

frequency this lead grows until, when $p = \infty$, δ has reached the value π. If the friction is large, the numerator of the expression for $\tan\delta$ is always considerable. On the other hand, if μp is a small quantity δ keeps low values until quite close to resonance.

These extreme cases are shown in the accompanying Fig. 17*b*. The dependence of amplitude and phase on the relative frequencies of driver and driven may be exhibited by the arrangement shown in the next figure.

BE is a metal stick on which a bob *F* can be adjusted by sliding. *CG* is a simple pendulum of string (rather shorter than *BE*) having a small bob *G*. The points of suspension are secured on the string tied at *A* and *D*. When *F* is pushed to the bottom of the rod and the driving pendulum set in oscillation through a small angle, *CG* oscillates a little nearly in phase with *EF*. As *F* is pushed

[1] Barton, *loc. cit.* Waetzmann, *Phil. Mag.*, **27**, 467, 1914.

higher, resonance is eventually reached, and now G moves through a large amplitude in time with F, but a quarter of a period later, so that when F has reached its maximum displacement on one side, G is just moving through its lowest point towards the same side. When F is pushed further up, it will be found that the phase difference gradually increases, and the amplitude of G's oscillation diminishes rapidly. The experiment may be made still more striking by placing a series of light pendulums of progressively increasing length between D and B. If as before G is the resonant pendulum, those to the left of G whose frequency is higher will lead G by progressively increasing phases, while those to the left of low frequency will lag behind G.[1]

In such a system it will be found that the resonance though strong is not sharp [2]; as an example of strong and sharp resonance under external forces, the tuning fork (Chap. IV) may be instanced. The strange fact should be noticed that, at resonance, the driving force is exerting its maximum influence at the moment when the amplitude of the subsidiary system is the greatest, and not when it is passing its mean position.

Reaction on the Driver: Coupled Systems. When, at resonance, the amplitude of the resounding system attains unusual dimensions, it is natural to look for the source of the energy required to maintain it. This must evidently lie in the driving system itself; and unless this system possesses some external maintaining energy, such as that of an electric battery or engine, its vibrations must be strongly damped, owing to loss of energy to the driven system. This may be shown, for example, when a tuning fork mounted on a resonance box, i.e., a vessel of air whose natural frequency corresponds to that of the fork, is strongly bowed. Owing to the assumption of the vibration by the air in the box the tone of the combined "coupled system" is much more intense than that of the fork alone; but, for the same initial impulse, the vibrations die away more rapidly.

When the two systems are equal, or nearly equal, in mass or inertia, the rôles of driver and driven may be interchanged. Of course, either of the components of two coupled systems will react on the other, and it is only in view of practical systems that we

[1] Barton and Helen Browning, *Phil. Mag.*, **36**, 169, 1918 and **37**, 453, 1919 and **44**, 573, 1922. *See also* Wachsmuth and Schütz, *Phys. Zeits.*, **26**, 75, 1925.

[2] I suggest "broad" for the converse of sharp, as applied to resonance.

usually speak of the smaller as the resonator; but the distinction is less obvious when the systems are nearly equal.

A striking experiment illustrates this. Two equal pendulums AB, CD are suspended from two points A and C and are connected by a thread or light rod EF (Fig. 19). If one pendulum D be given a small displacement and allowed to swing, it will begin to act on B, which now takes up the oscillation, taking energy from D until it has reduced D to rest. And now the position of affairs is reversed, B becomes the driver and D again takes up the oscillation until B has come to rest. So the interchange goes on, till friction has damped out all vibration. The position of EF determines the extent to which the pendulums can react on each other. If EF

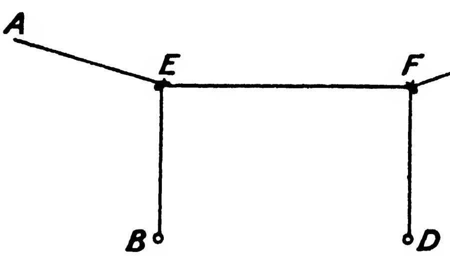


FIG. 19.—Coupled Cord Pendulums.

occupies the position AC this influence is a minimum, and the coupling is said to be "loose"; as EF is lowered each body has more control over the other, i.e., the coupling is tightened.

The effect of one system on the other is that of an added force due to the mass-acceleration of the other. The equations of the two systems (neglecting damping which considerably complicates the analysis) are, cf. (20):—

$$m_1\frac{d^2y_1}{dt^2} + M\frac{d^2y_2}{dt^2} + k_1y_1 = 0 \quad . \quad . \quad . \quad . \quad (29)$$

$$m_2\frac{d^2y_2}{dt^2} + M\frac{d^2y_1}{dt^2} + k_2y_2 = 0 \quad . \quad . \quad . \quad . \quad (30)$$

where the second terms represent the mutual reaction. Differentiating twice:—

$$m_1\frac{d^4y_1}{dt^4} + M\frac{d^4y_2}{dt^4} + k_1\frac{d^2y_1}{dt^2} = 0 \quad . \quad . \quad . \quad (31)$$

$$m_2\frac{d^4y_2}{dt^4} + M\frac{d^4y_1}{dt^4} + k_2\frac{d^2y_2}{dt^2} = 0 \quad . \quad . \quad . \quad (32)$$

Substituting in (31) the value of $\frac{d^4y_2}{dt^4}$ obtained from (32), we get :—

$$m_1\frac{d^4y_1}{dt^4} - \frac{M^2}{m_2}\cdot\frac{d^4y_1}{dt^4} - \frac{k_2M}{m_2}\cdot\frac{d^2y_2}{dt^2} + k_1\frac{d^2y_1}{dt^2} = 0\,;$$

again substituting in this equation the value of $\frac{d^2y_2}{dt^2}$ obtained from (29), and dividing out by m_1, we get :—

$$\left(1 - \frac{M^2}{m_1m_2}\right)\frac{d^4y_1}{dt^4} + \left(\frac{k_1}{m_1} + \frac{k_2}{m_2}\right)\frac{d^2y_1}{dt^2} + \frac{k_1k_2}{m_1m_2}y_1 = 0\,; \qquad (33)$$

with the same equation for y_2. Now $\frac{k_1}{m_1} = 4\pi^2n_1{}^2$, and $\frac{k_2}{m_2} = 4\pi^2n_2{}^2$, where n_1 and n_2 are the natural frequencies of the respective systems removed from mutual influence. Further, if the combined systems follow a S.H.M. of frequency N, y_1 (or y_2) $= a \sin 2\pi Nt$;

$$-\frac{d^2y_1}{dt^2} = 4\pi^2N^2y_1\,; \quad \frac{d^4y_1}{dt^4} = 16\pi^4N^4y_1.$$

Putting finally $\frac{M^2}{m_1m_2} = \kappa^2$ (33) becomes, on dropping the common factor $16\pi^4y_1$:—

$$(1 - \kappa^2)N^4 - (n_1{}^2 + n_2{}^2)N^2 + n_1{}^2n_2{}^2 = 0$$

$$\therefore N^2 = \frac{n_1{}^2 + n_2{}^2 \pm \sqrt{(n_1{}^2 - n_2{}^2)^2 + 4n_1{}^2n_2{}^2\kappa^2}}{2(1 - \kappa^2)} \qquad (34)$$

κ is called the "coefficient of coupling."

This equation has in general two roots, showing that the complex system has two natural frequencies. In the special case of $\kappa = 0$, $N = n_1$ or n_2; otherwise the effect of coupling is to add to the expression under the root in the numerator, and reduce the denominator of (34), so that, assuming n_1 greater than n_2, the roots of N become respectively greater than n_1 and less than n_2 as the coupling gets closer. If

$$n_1 = n_2 = n,\ N = \frac{n}{\sqrt{1 - \kappa}} \text{ and } \frac{n}{\sqrt{1 + \kappa}}.$$

This property is utilized in what is called "multiple resonance," by coupling two resonators in order to get a system having two resonance peaks, which are in general, respectively above and below the upper and lower resonance peaks of the separate systems (cf. p. 218).

Again, an analogy may be shown with two coupled electric circuits whose equations, when resistance can be neglected, are

$$L_1\frac{d^2Q_1}{dt^2} + M\frac{d^2Q_2}{dt^2} + \frac{1}{C_1}Q_1 = 0$$

$$L_2\frac{d^2Q_2}{dt^2} + M\frac{d^2Q_1}{dt^2} + \frac{1}{C_2}Q_2 = 0$$

corresponding to (29) and (30), the coefficient of coupling being $\sqrt{\frac{M^2}{L_1L_2}}$. This M is called the "mutual inductance," and can be found experimentally, and can be calculated for simple configurations. On the contrary there is no means of finding a value of M in the case of two acoustic systems; we can only say that the coupling is "tight" or "loose." M has been calculated for certain pendular and similar systems.[1]

Resonance is both a useful and an obnoxious phenomenon in sound. On the one hand, it is useful in reinforcing feeble sounds, but on the other hand it upsets the delicacy of response of a system which is intended to reproduce or amplify tones of all frequencies indiscriminately. From the conglomeration of tones that fall upon it, a vibrating system picks out those which correspond to one or other of its natural vibration frequencies, and exaggerates them out of all proportion to the rest. There are two remedies for this; either the natural period of the body must lie outside the range of those to which it is submitted, or, better, by a suitable arrangement of multiple resonance frequencies it must be made to resound, more or less, to the whole range. The amplitude-frequency curve of such a system consists of a series of overlapping resonance peaks (cf. p. 218).

Sub-synchronous Maintenance. In our treatment of resonance we have assumed Simple Harmonic Motion in both systems. The driver may, however, provide impulses for a small fraction of the period, and in this case the frequency of the impulse may be a sub-multiple of that of the driven system. In this way Raman[2] has maintained pendulums in vibration by an electro-magnet which, by its attraction on the steel bob, has the effect of momentarily increasing the gravitational force at the bottom of its swing. The frequency of the interruption of current is equal to, or is a sub-multiple of, that of the pendulum, and if a fork interrupter is used, the frequency of the latter can be accurately calculated if that of the pendulum is known.[3]

[1] *See* Lees, *Phil. Mag.*, **48**, 129, 1924 *and the references therein.*

[2] *Phil. Mag.*, **29**, 15, 1915 and **34**, 129, 1917. *Phys. Rev.*, **95**, 449, 1912.

[3] Raman and Dey, *Roy. Soc. Proc.*, **95**, 533, 1919.

A peculiar type of resonance curve is obtained if the maintaining impulse occurs at frequencies somewhat above or below that of the driven system. The curve is quite unlike Fig. 17*a*, since it has no peak, the amplitude falling steadily on one side of resonance and rising on the other, so that, as the frequency of the driver is lowered through resonance and beyond, the forced amplitude continuously increases.

The absence of the peak has been ascribed to breakdown of the simple theory of forced vibrations, so that terms involving y^3 need to be introduced into (21) and (22). In such a case, the amplitude, even of the free system, would be dependent on the frequency, so that change of amplitude of the freely swinging pendulum would alter its natural frequency; perhaps these changes mask the true resonance phenomenon in the observed curve.

Relaxation Oscillations. A somewhat similar phenomenon has been analysed by van der Pol.[1] First suppose that the damping μ in (21) is a negative quantity so that the solution involves a factor $e^{+\alpha t}$. This will not in fact be a physically realizable case since the amplitude would *increase* with time to infinity. But now suppose that the amplitude is limited by an additional damping factor which grows with it. Van der Pol chooses an equation of the type:

$$\frac{d^2y}{dt^2} = 2\alpha(1 - y^2)\frac{dy}{dt} - \omega^2 y \quad . \quad . \quad . \quad . \quad (21a)$$

in which the net damping is negative until $y^2 = 1$. He shows that if α is not too small, the square of the frequency is determined by the quantity $\omega^2/2\alpha = k/m \div \mu/m = k/\mu$. The period, $T = 2\pi\sqrt{\mu/k}$ is known as the " time of relaxation " of the system since it determines the rate at which the stress disappears as it is taken out in the resulting shear. The term was introduced by Clark Maxwell[2] in an attempt to relate the viscosity of a system to its elasticity.

In sound, these " relaxation oscillations " occur when a system is subjected to a considerable distorting force, is displaced and then relaxes until the force can again displace it. The method of exciting vibrations in a walking-stick by pushing the point in front of one over a rough surface with which it from time to time engages, and the way in which the breath escapes through the tensioned vocal cords during speech (cf. p. 270) are examples of this type of oscillation.

Interference. We have already spoken of this subject in dealing with the superposition of small motions. Here will be cited some practical instances in which mutual destruction of out-of-phase components is noteworthy. The first of these is the interference-tube, based on a principle first envisaged by Herschel,[3] but put into practical form by Quincke.[4] Sound of any pitch is conducted

[1] *Phil. Mag.*, **2**, 978, 1926.
[2] *Phil. Mag.*, **35**, 33, 1860.
[3] *Phil. Mag.*, **3**, 111, 1833.
[4] *Ann. d. Physik*, **128**, 177, 1866.

from opening A (Fig. 20) of the tube to the ear at B. The orifice at B must be tightly plugged into the ear, and the other ear stopped in order not to hear the note by way of the open air. Beyond A the tube divides; the sound reaches B both by the short tube C and by the longer tube D, which can be lengthened by sliding it out in the rubber connections E, in the same way as the slide of a trombone works.

If the difference of path *viâ* C or *viâ* D is exactly half the wave-length of the note, the two components arrive at B with a phase difference of π, and no sound is audible at B provided they are of equal intensity. To ensure this, C is made a little narrower than D in order that friction may dissipate energy at a greater rate, per unit length of tube, in C than in D. Fig. 20 shows another arrangement in which the alternative path is provided by reflection

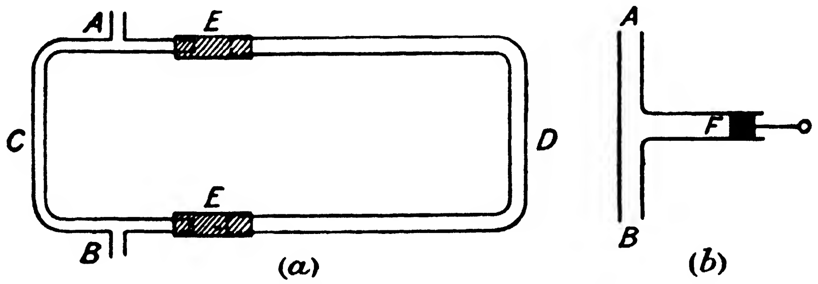


FIG. 20.—Quincke's Interference Tubes.

at the adjustable stopper F. Knowing the frequency and wave-length of the sound, its velocity in the tubes could be estimated but that it will be different in each tube if these are not of the same bore. More must not be expected of the apparatus than a qualitative demonstration of interference. Instead of the ear, manometric and sensitive flames (p. 157) have been used at B.[1]

Interference produced by reflection on a large scale may be observed. If we imagine a series of plane waves impinging on a plane non-absorbent wall and being there reflected, there will be zones of no motion in the air (nodal planes), in fact, stationary vibration between the source and the wall, if the position of the source with respect to the wall is suitably adjusted. In actual fact the waves sent out by the source are nearly spherical, but there will be, nevertheless, *points* of no vibration instead of the planes. These points can be detected by any of the instruments, sensitive flame, resonator, etc., described later. In a building,

[1] Waetzmann, *Ann. d. Physik*, **31**, 837, 1910; Schaefer, *Ann. d. Physik*, **50**, 700, 1916; Tricca, *N. Cimento*, **16**, 63, 1918.

owing to multiple reflection in several directions, the maxima and minima caused by superposition become very complex, and may be detected by an auditor moving about in the building while the tone is sounding.

An interesting use of the interference between a progressive wave travelling along a pipe and its reflection from the stopper at the end has been made by G. D. West.[1] The source of sound is a high-pitched whistle which can be moved along the axis of the pipe from the open end. Complete interference will occur whenever the returning waves arrive at the source exactly out of phase with those emitted, and nothing will be heard at the mouth of the pipe. By moving the whistle slowly into the pipe and measuring the distance between successive positions of silence, the velocity of sound may be found. The whistle must be blown from a large reservoir of compressed air, otherwise its pitch will change slightly as the pressure falls.

Reflection Tones. When, in place of a simple tone, there falls upon a wall a conglomeration of tones at various frequencies from a complex source, an ear moved to different distances from the wall will hear most strongly that tone, or series of tones, which has a node at the point where the ear is placed. This phenomenon seems to have been first observed between a waterfall (as a complex source) and a rocky wall, by Baumgarten. Above the roar of the fall those "reflection tones" are heard whose frequency, in accordance with this theory, is peculiar to the distance of this point of hearing from the wall.[2] The phenomenon has lately again received attention, as it manifests itself in the reflection from the ground of the sounds made by an aeroplane. These are noises due to the engine, its turbulent wake, the rotation and vibration of its propeller blades, etc.,[3] one or other of which noises is exaggerated to the listener on the ground by interference. Prandtl noticed that the reflection tone rose in pitch as he stooped, and also as the angular elevation of the aeroplane changed, since this change alters the angles of incidence and reflection of the sound rays impinging on the ground.[4]

Other demonstrations of interference will be noted under the particular systems to which they are applicable.

[1] *J. Sci. Inst.*, 6, 354, 1929.

[2] Starke, *Ver. deut. phys. Ges.*, 6, 285, 1908.

[3] Fage, *Roy. Soc. Proc.*, 107, 451, 1925; Waetzmann, *Zeits. f. tech. Physik*, 2, 166, 1921; Lübcke, *ibid*, 4, 99, 1923; Fassbender and Kruger, *ibid*, 8, 277, 1927; Paris, *Phil. Mag.*, 13, 99, 1932 and 16, 50, 1933.

[4] Prandtl, *Zeits. f. tech. Physik*, 2, 244, 1921, *but see* Dévé, *Comptes Rendus*, 174, 1,010, 1922.

Beats. A quite special result of interference arises when two systems of nearly (but not quite) equal frequency react on one another, of which a theoretical study has been made by Helmholtz.[1] Let the displacement of the greater system acting alone be given by $y = A \sin pt$, and of the other alone by $a \sin (qt + \delta)$ where $(p - q)$ is small compared with p. Then the resultant displacement of the particle is given by :—

$$\begin{aligned} y &= A \sin pt + a \sin (qt + \delta) \\ &= A \sin pt + a \sin \{pt - [(p - q)t - \delta]\}. \end{aligned}$$

Put $y = C \sin (pt - \Delta)$, and equate coefficients of $\sin pt$ and $\cos pt$

$$\begin{aligned} A + a \cos [(p - q)t - \delta] &= C \cos \Delta \\ a \sin [(p - q)t - \delta] &= C \sin \Delta. \end{aligned}$$

Squaring and adding, we find the amplitude of the resultant given by :—

$$C^2 = A^2 + a^2 + 2Aa \cos \{(p - q)t - \delta\}.$$

The amplitude thus fluctuates between $(A + a)$ when the cosine $= + 1$, and $(A - a)$ when the cosine $= - 1$; with a long period given by $\frac{2\pi}{p - q}$. The phase also fluctuates, for :—

$$\tan \Delta = \frac{a \sin \{(p - q)t - \delta\}}{A + a \cos \{(p - q)t - \delta\}}.$$

Thus the resultant vibration represents a vibration of the same period as the larger force $A \sin pt$, with fluctuating amplitude reaching a maximum equal to the sum of the component amplitudes, with a frequency equal to the difference $\left(\frac{p}{2\pi} - \frac{q}{2\pi}\right)$ of the component vibrations; and phase lag varying from 0 to $\frac{a}{A}$ (fractions of 2π) behind the larger force. These fluctuations in intensity are readily detected by ear, when two systems, whose frequencies differ by a few vibrations per second, are sounding together, and are called "beats." Beats may also occur between a forcing vibration and the natural frequency of the driven system, until the latter is damped out. The fact that the number of beats per second equals the difference of the frequencies of the component vibrations, forms a ready method of measuring small differences of frequency, and is accordingly used in tuning instruments, or of

[1] *Sensations of Tone*, App. 14.

testing a unison.[1] The displacement-time diagram of a system producing beats is shown below.

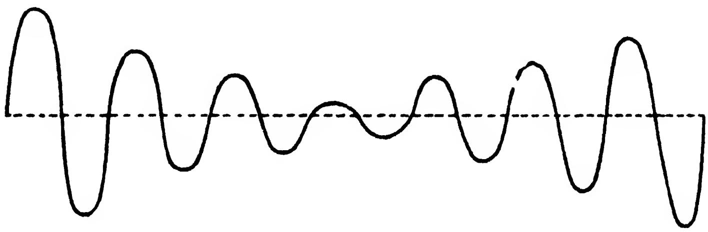


FIG. 21.—Beat Diagram.

There are several instruments capable of showing that these fluctuations actually exist in the air surrounding two bodies of nearby frequency; they usually consist of membranes which copy the motion of the air, the motion of the membrane being exhibited by one or other of the methods described in the succeeding chapters.

Combination Tones. We now treat of the cases where, at the simultaneous sounding of two tones, a quite new tone is formed, a subject round which a whole literature has collected in the last thirty years. It was noticed almost at the same time by three musicians at the beginning of the eighteenth century (Tartini[2] the violinist seems to have been the first), that when two notes a fifth apart in the middle of the scale were played on the organ, together with them a low note was heard, whose frequency was the difference of the two higher-pitched notes. These are "difference tones," and there have also been observed "summation tones," having a frequency equal to the sum of the primary tones.

Such tones, beside being subjectively studied on the organ as in the original investigation, can be demonstrated objectively by means similar to those used for beats. In the aural examination, using an organ or harmonium, the existence of the difference-tone may be made more apparent, by first sounding the low note which the two primaries should combine to give, and then sounding these alone. The original low note will then be recognized in the combination.

With smaller apparatus the difference tone may be heard on sounding (*a*) two tuning forks of moderately high frequency and some fifty vibrations apart, (*b*) the double whistle consisting of two Pan pipes of high pitch, blown by a blast of air directed across their mouths, or (*c*), Rudolf König's[3] double glass rods, in which longi-

[1] Rayleigh, *Nature*, **19**, 275, 1879. [2] *Trattato di Musica*, 1754.

[3] *Ann. d. Physik*, **69**, 721, 1899.

tudinal vibrations of high pitch are excited by a wheel having a wetted cloth rim. To make the experiments effective it is generally necessary that the same mass of air should be violently agitated by the two generators.

The best way to demonstrate that the difference tone can exist outside the ear is to use a suitably adjusted resonator (p. 213) tuned to the expected tone. It will be found on sounding the two primaries that the resonator responds to the (low) resultant tone, reinforcing it. We are here speaking of objective combination tones as against certain tones which exist apparently only in the ear—the so-called subjective combination tones. In point of distinction, this is accepted as the *experimentum crucis* of objectivity, that every vibration which can be picked up by a suitably tuned resonator, actually exists in the air.

Beat Tone Theory of Combination Tones. It was suggested, first by Lagrange [1] and by Young [2] soon after Tartini's discovery, that the difference tones had the same origin as beats, i.e., that here we have primary tones so far apart that the beats they produce follow in such rapid succession as to blend into a new tone of low pitch. Beats and difference tones are, on this theory, physically identical; it is the ear which makes the distinction between them, recognizing regular impulses of more than 16 per second as musical notes, and impulses fewer per second as distinct beats. The beat tone theory, or, as it is sometimes called, König's theory, from his exhaustive practical study of these tones,[3] presents, however, grave difficulties from the standpoint of the ear as analyser, which will be noted in that connection.

Helmholtz Intensity Theory of Combination Tones. The equivocal existence of combination tones except when the primary generators are intense suggested to Helmholtz [4] that we have here to deal with a case where the simple superposition of two tones can no longer be applied. The indicator, whether artificial membrane, ear-drum or other form, no longer responds to the double forcing of the primaries in a way which is the vector sum of the vibrations they would impress upon it if acting alone. In fact, the restoring forces are not now proportional merely to the displacement. Helmholtz added a term proportional to the square of the displacement. Under the action of the two forces $a \sin pt$

[1] *Misc. taurinens*, 1759. [2] *Roy. Soc. Trans.*, 1800.
[3] *Ann. d. Physik*, **157**, 177, 1876.
[4] *Sensations of Tone*, p. 152 and App. 12.

and $a' \sin(qt + \delta)$, the equation for the forced vibrations of the system then becomes :—

$$m\frac{d^2y}{dt^2} + ky + k^1y^2 = a \sin pt + a' \sin (qt + \delta) \quad . \quad (35)$$

The solution of this equation, the working of which must be sought in the original,[1] contains terms involving $\sin 2pt$, $\sin 2qt$, $\sin (p - q)t$, $\sin (p + q)t$, but the amplitude in the difference-tone term is a fraction of the product aa', and so requires large values of the primary amplitudes a and a' in order to be appreciable. There should therefore be heard the octave of each primary as well as the difference, and the summation tone, when the responding system possesses a non-linear response characterized by the term in y^2. Such a condition implies that a system is asymmetric, for, in addition to the usual restoring force which changes sign with the displacement, this new force inherent in the system does not so change sign ; so that the free vibrations of such a system are asymmetric with respect to the equilibrium position. Helmholtz' theory depends on a *special* species of asymmetry, characterized by elastic forces, $ky + k^1y^2$.

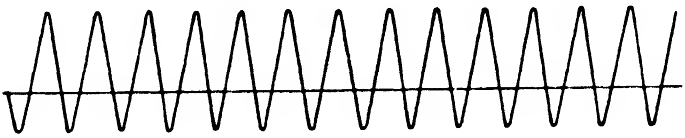
Waetzmann's General Asymmetry Theory. It has frequently been pointed out that the large intensity of the primaries which Helmholtz' theory requires for the production of a difference-tone is not borne out by practice, the difference-tone often being produced by comparatively weak primaries. Moreover the mathematics of Helmholtz has not escaped criticism. In an attempt to reconcile theory with facts, Waetzmann [2] pictures a *general* asymmetry in a system reproducing combination tones ; any asymmetry, in fact, which will make the response of such a system one-sided in the sense that the displacements are no longer symmetrical about the equilibrium position. Such a system Waetzmann realized by loading a membrane with a central weight on one side of the membrane only, and found its free vibrations given by a curve like that in Fig. 22*a*.

He next led two simple tones from two tuning forks to this asymmetric membrane, and recorded the vibration of the latter

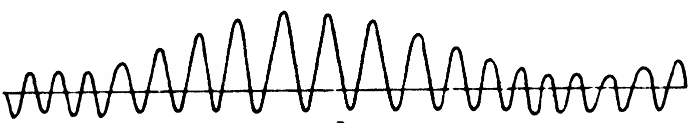
[1] *Or* Lamb, p. 299.

[2] *Zeits. f. Physik*, **1**, 271 and 416, 1920 contains a good summing-up. Also in the *Ann. d. Physik*, 1907–1913 *passim* and **62**, 371, 1920 ; *Phys. Zeits.*, **8**, 346, 1907 and **21**, 122, 1920 ; *Verh. Deut. Phys. Ges.* (with Mücke), **15**, 59, 1913.

under the simultaneous double forcing. The curve in Fig. 22*b* represents the result. Comparing this with the record of beats (Fig. 21), we observe that the curves are similar, but that the loaded membrane possesses a " rectifying " property, pushing that part of the curve near the maxima to one side of the zero position, as compared to the minima. Performing the Fourier analysis of such a curve, Waetzmann finds the original tones (p, q), a difference tone ($p - q$), of several times greater amplitude than either primary, a weak secondary difference tone ($2q - p$), and occasionally a summation tone ($p + q$). These superposed in the ordinary way make up the recorded curve. This loaded membrane may be taken as a type of the ear drum with its attached ossicles producing



(*a*)



(*b*)

FIG. 22.—Vibrations of Asymmetrically Loaded System.

" subjective " combination tones, but similar rectifying properties have been found in other systems, so that we may expect to find " objective " combination tones in the external air due to the same cause, though less definite than at the ear drum.

The theory thus satisfactorily explains the phenomenon, both as regards amplitude and frequency, and, in a way, combines and reconciles the " beat " and the " intensity " theories which failed in generality.[1]

Doppler Effect. This is the name given to the apparent change in frequency of a moving source, or the apparent change in frequency of a stationary source received by a moving hearer. A

[1] *See also* Hermann, *Ann. d. Physik*, **25**, 697, 1908 and **37**, 425, 1912; Schaefer, *Ann. d. Physik*, **33**, 1,216, 1910 and **41**, 871, 1913; Schulze, *Ann. d. Physik*, **34**, 817, 1911; Peterson, *Ann. d. Physik*, **40**, 815, 1913; Schaefer and Juretska, *Ann. d. Physik*, **41**, 581, 1913; Budde, *Phys. Zeits.*, **19**, 321, 1918 and **21**, 274, 1920 and *Verh. Deut. Phys. Ges.*, **21**, 70, 1919; Morton and Mary Durragh, *Phys. Soc. Proc.*, **27**, 339, 1915; Stumpf, *Zeits. f. Psych.*, **55**, 1, 1910.

stationary source gives out n waves per second of wave-length λ, and these reach the stationary hearer with velocity c. If the source moves away with velocity V, still sending out n waves per second, the wave-length or distance between successive maxima is increased in the ratio $\frac{V+c}{c}$, so that the frequency of the waves received by the hearer at velocity c cm. per second is $\frac{nc}{V+c}$. Similarly, if the source is kept still and the hearer moves at a velocity U towards the source, the apparent frequency is $\frac{n(c+U)}{c}$. For both in motion, it is $n\frac{c+U}{c+V}$. This change of frequency was first worked out by Doppler for the optical case. Acoustically it may be observed under the same conditions as the original investigation of Buys-Ballot [1] in Holland. The pitch of a note of a whistling locomotive falls as the locomotive passes, owing to the sudden change of sign of V in the formula. An identical result is heard on the locomotive as it passes a signal bell, also when a projectile passes overhead.[2] Mach produced the same effect indoors by a rapidly whirled bar on the end of which a source is placed so as to approach and recede alternately from the hearer.

[1] *Ann. d. Physik*, **66**, 321, 1845.

[2] Esclangon, *Ann. de Physique*, **8**, 186, 1917; *see also* Zenneck, *Phys. Zeits.*, **29**, 343, 1928; Müller and Kraefft, *ibid*, **33**, 305, 1932.

CHAPTER THREE

LONGITUDINAL AND TORSIONAL VIBRATIONS IN SOLIDS

Longitudinal Vibrations in Rods. The rod in question will be of uniform material and section, i.e., a cylinder, but not of such small diameter as to fall under the classification "strings." As regards longitudinal vibrations without bending, the shape and size of the section are not of importance, though in practice the rod, whether solid or hollow, is circular, for ease in producing the tone. The formula for velocity of propagation of longitudinal waves in a solid is identical with that (3) obtained earlier for similar waves in air, but of course in deducing the value of the elasticity coefficient to be used, the gaseous relations between volume and pressure no longer apply. In the actual rods employed, every elongation δl produces a lateral contraction $-\delta r$. Now :—

$$\frac{v + \delta v}{v} = \frac{(r - \delta r)(l + \delta l)}{rl} = \frac{rl - l\delta r + r\delta l - \delta r\delta l}{rl}$$

$$= \frac{rl + r\delta l - \delta r(l + \delta l)}{rl} = \frac{l + \delta l}{l}$$

approximately, neglecting the term in δr. Under these circumstances, i.e., when changes in length are allowed to take place at the expense of lateral movements, change of volume per unit original volume $\frac{\delta v}{v}$ in the elasticity formula (1) becomes change in length per unit original length $\frac{\delta l}{l}$, provided the appropriate coefficient of elasticity is used. This coefficient is that determined by stretching the wire statically so that lateral contraction can take place and keep the density unchanged, as in the dynamical conditions of vibration, and is known as Young's modulus.

A like method may be used to derive the formula for the longitudinal velocity in rods to that for the speed in gases. Let the

longitudinal displacement at section P of co-ordinate x (Fig. 23) be ξ; at Q a little further along the rod, i.e., at $x + \delta x$, it will be, at the same instant, $\xi + \frac{\partial \xi}{\partial x}.\delta x$, taking $\frac{\partial \xi}{\partial x}$ as the mean change of displacement with x, over the slice between P and Q. Then the total elongation of the layer is $\left(\xi + \frac{\partial \xi}{\partial x}.\delta x\right) - \xi$, and since $PQ = \delta x$,

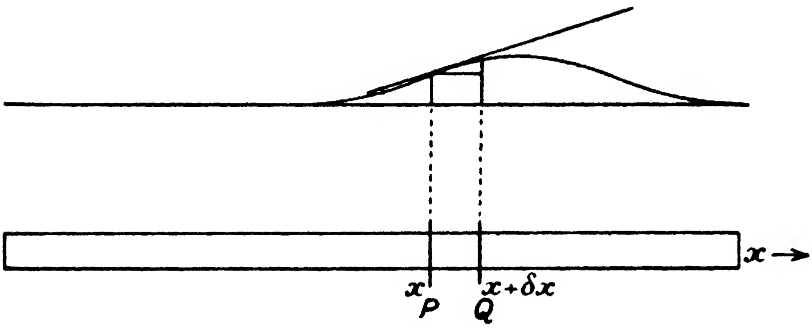


FIG. 23.—Velocity of Longitudinal Vibrations.

the elongation per unit original length is $\frac{\partial \xi}{\partial x}$, so that, if Young's modulus for the material is $\mathbf{E}$, then $p = \mathbf{E}\frac{\partial \xi}{\partial x}$ (p being the total applied tension per unit area, or negative pressure on the slice). There is a force p per unit area at P directed to the left, and $p + \frac{\partial p}{\partial x}.\delta x$ at Q directed to the right, making a total of $\frac{\partial p}{\partial x}\delta x$ on unit area of the material of the slice, but since above

$$p = \mathbf{E}\frac{\partial \xi}{\partial x},$$

then
$$\frac{\partial p}{\partial x}\delta x = \mathbf{E}\frac{\partial^2 \xi}{\partial x^2}.\delta x.$$

Now we may also express the resultant force on the material as the product mass $\times$ longitudinal acceleration, i.e., $\rho\delta x\frac{\partial^2 \xi}{\partial t^2}$,

so that
$$\rho\delta x\frac{\partial^2 \xi}{\partial t^2} = \mathbf{E}\frac{\partial^2 \xi}{\partial x^2}\delta x,$$

or
$$\frac{\partial^2 \xi}{\partial t^2} = \frac{\mathbf{E}}{\rho}.\frac{\partial^2 \xi}{\partial x^2} \quad . \quad . \quad . \quad . \qquad \text{(cf. 2)}$$

A solution of this equation is :—

$$\xi = f_1(x - Vt) + f_2(x + Vt),$$

where V^2 is put for $\frac{E}{\rho}$, and f_1 denotes any function of $(x - Vt)$, and f_2 denotes any function of $(x + Vt)$. Now $(x - Vt)$ represents a point whose distance from the origin of x is decreasing at the rate of V cm. per sec., so that $f_1(x - Vt)$ is some form of displacement or distortion wave, which is propagated in the positive direction of x with the velocity V., suppose the displacement ξ is produced at the instant t_0 at the section x_0 of the rod. At a later instant t_1 the function has the same value if x is increased by $V(t_1 - t_0)$, for then :—

$$f_1\{[x + V(t_1 - t_0)] - Vt_1\} = f_1(x - Vt_0)$$

showing that the same displacement ξ is now to be found at the section $x + V(t_1 - t_0)$, representing a transmission of this displacement, in fact, of the whole wave-form with velocity V in the direction of increasing x. Similarly, $f_2(x + Vt)$ represents a wave moving in the direction of diminishing x.

Now if f_1 and f_2 be simple sine functions of the variable, we have seen (p. 42) that two such waves travelling in opposite directions will produce stationary vibration in the rod, with places of maximum vibration (antinodes) and of no vibration (nodes). In particular a clamped section will form a node, and a free end will form an antinode. The position of the clamp will determine the possible modes of vibration of the bar. In practice, we find two types of " end conditions " as they are called, in the longitudinal stationary vibration of a bar. Firstly, the centre may be clamped and the ends left free, which is the state of affairs in the Kundt's tube experiment. Secondly, the bar may be clamped at two points, either at the ends or at certain positions symmetrically placed with regard to the centre.

Since the wave-length of stationary vibration is to be taken as twice the distance between two nodes (p. 42), we should have :—

$$n = \frac{V}{\lambda} = \frac{1}{2l}\sqrt{\frac{E}{\rho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

for the fundamental or lowest tone of the rod, l being its total length. This gives the frequency of the tone shown diagrammatically in Fig. 24(a and b).

At (c), $$n = \frac{1}{l}\sqrt{\frac{E}{\rho}}$$

and at (d) $$n = \frac{3}{2l}\sqrt{\frac{E}{\rho}}.$$

The full "harmonic series" of tones corresponding to frequencies 1, 2, 3, 4, etc. times the fundamental, can thus be obtained by suitably adjusting the position of the two clamps.

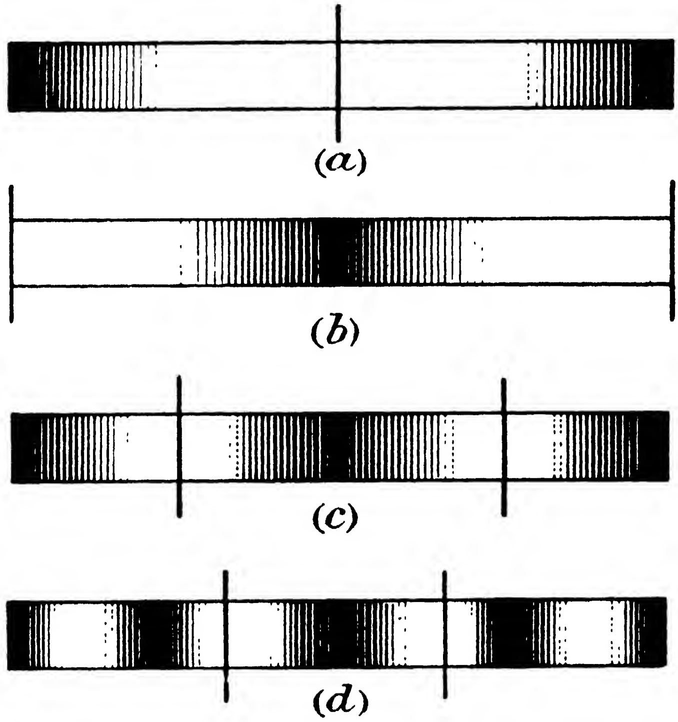


FIG. 24.—Stationary Longitudinal Vibrations of a Bar.

Experimental Methods. Beside metal, glass and wooden rods or tubes of square or round section, wires and rubber cords may be used for the study of longitudinal vibrations. Apart from the Kundt's tube method of determining the velocity of the waves (using air as the standard), it is possible to investigate the position of nodes and antinodes in the solids themselves and to verify the conclusions of the preceding section.

To produce the tones, the usual method is to surround the rod by a cloth (resined for a wooden rod, damp for a glass or metal rod), held not too tightly in the hand and then drawn along the rod in the neighbourhood of an antinode. In "strings" it is possible to get these tones by rubbing a resined bow along the string, in cords by a simple longitudinal displacement. In the latter cases the material must perforce be clamped at the ends, and the tones are usually adulterated with transverse vibrations. These are sometimes present as low notes when the longitudinal tones of rods are carelessly produced by friction—the low inharmonic note resulting is called "*son rauque.*" Unlike the transverse, the

pitch of longitudinal vibrations is independent of the tension along the string. Altberg[1] has shown that it is possible to maintain these tones in a rod by a revolving wheel whose rim is covered with resin, and which continually rubs upon the rod near one end.

The nodes on a rod of square or flat section can be shown by strewing dust, which collects at the places of no motion on the rod. When the rod is of transparent material the vibrations can be studied by a much finer method based on a discovery of Biot.[2] He placed the rod in the polarized light between crossed mirrors, so that the light on reflection from the second mirror was completely extinguished. On exciting longitudinal vibrations in the rod, the light reappeared and remained as long as the tone lasted.

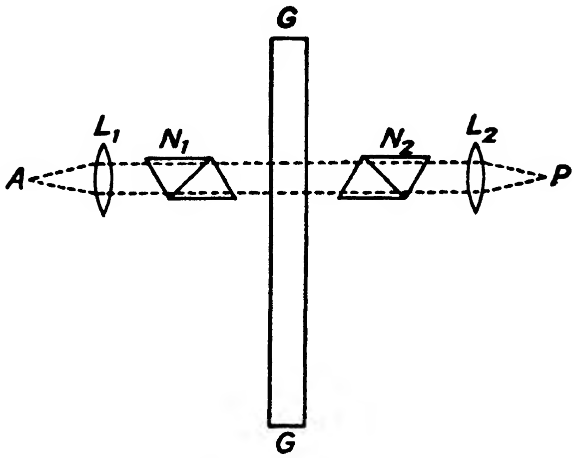


FIG. 25.—Longitudinal Vibrations in a Rod, by Polarized Light.

Mach[3] used the light which passed between two crossed Nicol prisms, one on each side of the rod, obtaining in the spectrum of the transmitted light interference bands, which oscillated with the changes in density of the section when the rod was sounding. If the material can be shaped in the form of a long rod, a pulse may be sent along it by a sudden application of tension or pressure at one end. If two polarized beams of light pass through the rod at distant points, they will be successively disturbed by the passage of the pulse, so that a chrono-photographic record of the disturbances will give the time interval between them. In this way Herbolsheimer[4] measured the speed of longitudinal waves in gelatine. Fig. 25 shows schematically the arrangement of the Biot phenomenon. Light from an arc A, made parallel by the

[1] *Ann. d. Physik*, **11**, 405, 1903. *See also* Quimby, *Phys. Rev.*, **25**, 558, 1925. [2] *Physique*, **2**, 15, 1828.

[3] *Ann. d. Physik*, **146**, 316, 1872. [4] *Zeits. f. Physik*, **3**, 182, 1920.

lens L_1, passes through the first Nicol N_1, then through the rod GG, the second Nicol N_2, and on to a screen or plate at P. N_2 is first turned to extinguish as much as possible the light getting through N_1 and GG. On exciting longitudinal vibrations in the rod, light periodically reappears at P.

Davis[1] and also Clark[2] have investigated the tones of continuously rubbed strings by observing the motion of bright points on the strings under a microscope, the string being rubbed by a wheel. The patterns observed conformed to those of strings bowed in the usual manner, i.e., transversely (cf. p. 89), discontinuities in lateral displacement being replaced by changes in density along the strings.

Torsional Vibrations of Rods. The formula which we have already deduced for the velocity of propagation of longitudinal compressions in bodies can also be applied to vibrations produced after giving a twist to one part of an elongated solid, provided the appropriate elasticity be introduced. This elastic force is commonly known as the rigidity (N), and is determined from the twist (ϕ) produced at an end by an applied torsional couple, just as Young's modulus is determined from the extension (ξ) produced by an applied longitudinal tension. The formulæ are therefore:

$$\frac{\partial^2\phi}{\partial t^2} = \frac{\mathrm{N}}{\rho}\cdot\frac{\partial^2\phi}{\partial x^2},$$

$$V_{\text{tors}} = \sqrt{\frac{\mathrm{N}}{\rho}}$$

for the velocity, and for the fundamental tone (centre fixed)

$$n = \frac{1}{2l}\sqrt{\frac{\mathrm{N}}{\rho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Similar divisions into nodes and antinodes corresponding to Fig. 24 can be obtained by suitable clamping, and shown by sand, if the top is plane. To obtain the tones from such square-section rods one usually employs two violin bows, drawn across in opposite directions, one above and one below the section. As may be imagined, this gives an admixture of transverse and possibly of longitudinal vibrations, therefore Grögor[3] employed a rotating wheel rubbing the end of the rod. With care it was possible to

[1] *Am. Acad. Proc.*, **41**, 691, 1906. [2] *Phys. Rev.*, **7**, 561, 1916.
[3] *Phys. Zeits.*, **15**, 788, 1914.

get a note sufficiently pure for pitch estimation. Comparing the fundamental tones of the same rod for longitudinal (36) and torsional (37) vibrations the ratio of the extensional and torsional elasticities of the material can be found. Torsional tones are of little importance in practice.

There is renewed interest in the longitudinal and torsional tones of short quartz rods (excited by the piezo-electric effect) and steel or nickel rods (excited by magneto-striction).[1] Further details will be found in Chap. XI. These tones may also be excited in rectangular rods by a blast of air directed on to a suitably chosen sharp edge, the " edge tone " (cf. p. 169) so produced being tuned to the desired tone of the rod.[2]

[1] Ruedy, *Canad. J. Res.*, **5**, 149, 1931; **12**, 10, 1935; Field, *ibid.*, **5**, 619, 1931; Röhrich, *Zeits. f. Phys.*, **73**, 813, 1932; Giebe and Blechschmidt, *Ann. d. Physik*, **18**, 417 and 457, 1933; Straubel, *Phys. Zeits.*, **34**, 894, 1933; Swigart, *Rev. Sci. Inst.*, **7**, 252, 1936; Allan, *Phil. Mag.*, **26**, 609, 1938; Schœneck, *Zeits. f. Phys.*, **92**, 390, 1934; Foch, Lindsay and Wilks, *Acoust. Soc. J.*, **9**, 348, 1938. (The last two deal with velocities in single crystals.)

[2] Kröncke, *Zeits. f. tech. Phys.*, **13**, 196, 1932.

CHAPTER FOUR

TRANSVERSE VIBRATIONS OF STRINGS AND RODS

Properties of the Theoretical String. The string is ideally a body having length only, infinitely thin, able to be bent laterally in transverse vibration without bringing into play viscous forces in the material. In so far as natural wires and threads fail in this last respect, we say that they possess "stiffness." In a rod or bar this resistance to bending is the actual cause of the vibration, as it constitutes the restoring force to a displacement. In the string, the tension between the particles, due to the applied tension at the ends of the wire, takes the place of this resistance to bending. In the absence of applied tension, vibration will not take place on displacing laterally a point on the string.

Velocity of Transverse Waves in a String. A string may execute stationary vibration having nodes at the fixed ends, and any number of loops between. The fundamental will be that vibration corresponding to a single loop stretching from end to end. As we have seen, this type of motion may be regarded as made up of two oppositely directed transverse progressive waves moving along the string. It remains to calculate the velocity of such a wave.

Let AB (Fig. 26) represent a portion of the displaced string δx in length, the end A being at a distance x from one end of the string, and its displacement from the undisturbed position A', being y. In the absence of stiffness the stretching force F will be the same throughout the string, but as it acts tangentially at every point its inclination will vary along Ox. If the tangent to the string at A makes an angle θ with Ox, the component of the tension in the direction AA' will be $F \sin \theta = F \tan \theta = F\dfrac{\partial y}{\partial x}$ when θ is small. The component at B along $B'B$ will then be

$$F\left[\frac{\partial y}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)\delta x\right]$$

F

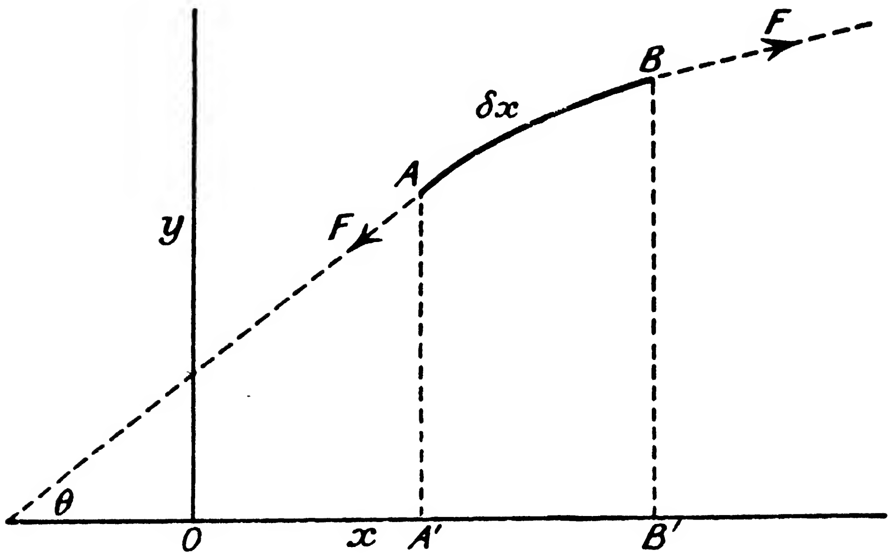


FIG. 26.—Velocity of Transverse Waves in a String.

The net force on AB tending to increase its displacement is therefore $F\frac{\partial^2 y}{\partial x^2}\delta x$ which can be equated to the mass × acceleration, i.e. to $m\delta x\frac{\partial^2 y}{\partial t^2}$, where m is the mass of unit length of the string.

Finally
$$\frac{\partial^2 y}{\partial t^2} = \frac{F}{m}\frac{\partial^2 y}{\partial x^2}$$

in which we recognize, by comparison with equation (2), p. 3, the velocity of the waves as:

$$V = \sqrt{\frac{F}{m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Knowing the velocity of transverse waves in the string, we can determine the frequencies corresponding to the wave-length of each possible segmental division of the string of length l, from $\lambda = 2l$ for the fundamental, through all the harmonics or "partial" vibrations of the string. The number and magnitude of these partials is determined by the manner of exciting the string.

Apparatus for Strings.[1] The laws expressed by the formula (38) were known in part to the ancients, but the first quantitative verification appears to have been made by Mersenne[2] in 1635. At the beginning of the eighteenth century Sauveur[3] stretched a

[1] *See* Allen, *Phil. Mag.*, 4, 1324, 1927; Carrière, *J. de Physique*, 8, 365, 1927.

[2] *Harmonicarum*, 2, 1635. [3] *Mém. de l'Acad. Paris*, 1700 *to* 1711.

string between two bridges, applying tension by a weight at the end which hung over a pulley. This experimenter observed the nodes and antinodes ("nœuds et ventres") which were formed when the string was vibrating in aliquot parts so as to produce an overtone.

His apparatus was the precursor of the modern monochord or sonometer, which generally holds two wires stretched either horizontally or vertically between two bridges. One wire of fixed length, screwed up to a certain tautness, gives a standard tone when gently displaced at the centre. In the horizontal form, the other wire passes over a pulley to a scale-pan in which weights are placed. This wire passes over a movable bridge, and the sounding length between the fixed and movable bridges is adjusted to unison with the standard wire. Two methods are available to test whether this condition has been attained. Either the two wires may be sounded together, and the adjustment made so that the beats between them vanish; or use may be made of the resonance principle in the following manner. The standard wire alone is excited, and the second wire, if in tune, vibrates sympathetically so strongly as to throw off light paper riders hung on it for that purpose. In order to increase the "volume" of the sound, as in most stringed instruments the sound-board consists of a hollow box, the air in which is forced into vibration to a certain extent by the string. The vertical form of monochord is perhaps to be preferred, as the friction on the pulley is banished and that on the bridges lessened.

Stiffness of Wire. The ideal string has no stiffness; a rod is a body in which this quality is of prime importance. In an actual wire stiffness plays a definite though subordinate part. Savart[1] first observed that such stiffness could permit a vibration (n_0) even in the absence of tension, and proposed an empirical formula for the actual frequency (n_1) in terms of the ideal frequency (n); $n_1^2 = n^2 + n_0^2$. Rayleigh[2] has pointed out that stiffness alters the "end conditions," the conditions $\frac{\partial y}{\partial x} = 0$ and $\frac{\partial^2 y}{\partial x^2} = 0$ being no longer completely fulfilled at a rigidly clamped end. The various formulæ proposed agree in ascribing to the stiffness a rise in frequency above that given by the simple theory, a rise

[1] *Ann. de Chim. et Phys.*, 6, 1, 1842. *See also* Schaefer, *Ann. d. Physik*, **62**, 156, 1920.

[2] *Sound*, **1**, 239, 1896.

which is proportionally greater as the frequency grows, so that the partial tones no longer form a harmonic series.

It may be pointed out here that another cause may lead to the violation of the end conditions: the yielding of the end supports, which has the effect of increasing the length of the wire.

Experimental Study of Transverse Vibrations. Before proceeding with the detailed description of the types of vibration of strings under different modes of excitation, it will be as well to describe experimental methods of accurately examining the frequency and amplitude of such vibrations. The methods to be described can be adapted with but little alteration to all transverse vibrations of solid bodies; in particular to rods. The methods divide themselves into two classes; graphic and stroboscopic. The former are better adapted to complex vibrations containing a number of partials; the latter to a simple harmonic motion.

Graphic Methods. In the simplest of these, the vibrating body carries a style which traces a mark on a piece of paper, which is moved along at right angles to the direction of vibration. The trace is therefore a wavy line, which in the simplest case of S.H.M. corresponds to the sine wave of Fig. 14, if the paper moves past the style at constant speed. Either the speed of the paper is known, or else a subsidiary time-marker marks dashes at constant intervals of time on the paper alongside the trace of the vibration. In the former case two methods are in vogue. In the one, the strip of paper is wrapped round a drum which is turned by a motor rotating at a constant known speed (usually one of the so-called "phonic motors," cf. p. 116). If then m waves are traced on a length l of the paper, wrapped on a drum of circumference $2\pi r$, rotating at n revolutions per second, these m vibrations occupy a time of $\frac{l}{2\pi rn}$ seconds, whence the frequency of the wave motion may be calculated. In the other, a smoked glass plate is allowed to fall under gravity past the style, which is made to vibrate horizontally by the action of the fork. In consequence of the accelerated fall of the plate, the waves traced are crowded together at first, but gradually open out. If the number of waves between any two points be counted, the distances l_1, l_2 measured from the start, the times t_1, t_2 taken by the plate to fall these distances are found from the formulæ $l_1 = \frac{1}{2}gt_1^2$, $l_2 = \frac{1}{2}gt_2^2$. Hence

the time taken for the plate to fall the distance over which the waves have been counted

$$= t_2 - t_1 = \sqrt{\frac{2}{g}}(\sqrt{l_2} - \sqrt{l_1}).$$

When the speed of the paper or plate cannot be kept constant by some automatic device, it is preferable to use a time-marker. This may be a suitably arranged metal pendulum, adjusted to beat seconds or some convenient and accurately known interval, the time-period of the pendulum having been determined by comparison with a chronometer. The bob of the pendulum P just dips into mercury in the trough T as it crosses its lowest position, and so establishes a current through the electro-magnet M (Fig. 27) which attracts a spring marker S. The marker leaves momentarily its place of rest, and makes a "jag" in the straight line which it has been tracing on the drum. A number of cross

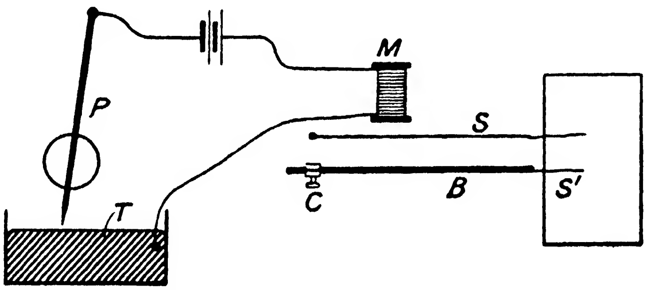


FIG. 27.—Graphic Method Applied to Vibrating Bar.

lines are thus marked on the drum corresponding to intervals equal to half the time of swing of the pendulum. The figure shows diagrammatically this method applied to the transverse vibrations of a bar B clamped at one end C. A style S' has been fixed to the other end of the bar, which traces a sinuous line upon the drum when the latter is rotated, and the bar lightly displaced. As might be expected, the graphic method is more readily applied to a body of large mass, and is not much used for thin strings, where the friction and inertia of the style would appreciably affect both the amplitude and frequency of the motion it is desired to study.

Photographic Method. When a more permanent record is desirable, a camera is resorted to, and this device has the advantage that the apparatus, if any, which has to be attached to the vibrating body is small and offers the minimum of damping to the motion of the body. In the simplest type, a small mirror is

stuck to the part of the body where the movement is large, and this reflects a beam of light from a powerful and concentrated source—like the Pointolite lamp—on to a camera of the "moving plate" variety. Such a camera, which can be simply made and worked, is shown, with the back removed, in Fig. 28.

P is the plate holder and dark slide from an ordinary quarter-plate camera. To this are attached two long loops of elastic cord of equal length, passing over hooks at each end. Under the tension of these cords the holder is normally held in the centre of the camera behind the shutter *S*. Just before taking a photograph, *P* is pulled to one end and held by a clip; the shutter *S* is adjusted to one of three positions, so that the opening will be over one-third of the plate as it shoots past. The reflected spot of light is adjusted to vibrate vertically across this hole. The back of the camera is clipped on, the slide pulled out so as

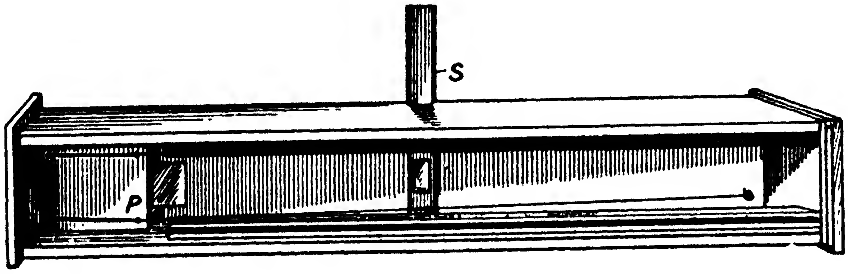


FIG. 28.—Sliding Plate Camera.

to expose the plate (it is possible to do this by introducing the hand through a flap in the back), the catch is released and the plate shoots along grooves, past the shutter, until it is brought violently up against the far end with sufficient force to reclose the slide upon the plate; the holder can then be removed from the camera by taking off the back. If the cords are of equal length there should be no force acting on the holder as it passes *S*, and consequently its velocity at this part of its path should be constant. As this velocity is not known, however, it will be necessary to mark the time in some way; this is most conveniently done by allowing the bob of a pendulum to cut off the light periodically, leaving a gap in the record of the plate; the width of this gap is governed by the size of the pendulum bob which is suited to the speed of movement of the plate, which speed is, in its turn, determined by the tension put upon the elastic cords. The *tout ensemble* is shown in Fig. 28.

If the camera can be set up vertically, and the deflection of

the beam of light be horizontal, it is often better to do without the cords, and let the plate holder fall through the guiding grooves under gravity. A plate such as is used for flashlight photography is suitable.

When the vibration to be studied is that of a thin wire it is better to dispense with a mirror, and to place the wire directly in the path of the light, so that it casts a shadow on the plate. The technique of this device has been considerably extended in connection with the Einthoven string galvanometer, where the forced vibrations of a wire carrying an unsteady electric current in a magnetic field have to be recorded. It is obvious that in any photographic apparatus of this kind the instantaneous or steady projection of the vibrator on the plate must be either a bright spot on a dark background, or a black dot on an illuminated ground. To transform the shadow of our wire from a line

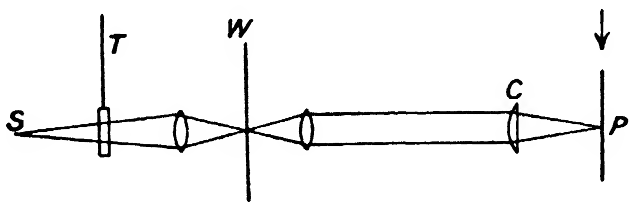


FIG. 29.—Optical System for Photographic Record of Transverse Vibrations.

to a point, cylindrical lenses are necessary. The arrangement is shown in Fig. 29.

The light from the source S is concentrated upon the wire W, and the beam made parallel by a second spherical lens; this would give a shadow of the wire in the plane of the paper on the plate, but the cylindrical lens C having its axis perpendicular to the paper reduces the shadow to a spot at P. The wire is arranged to vibrate in a plane perpendicular to the paper, so that if the plate P be shot in the direction of the arrow, the required trace will be made upon it by the spot. The time-marker T interrupts the light periodically, as explained.

The Stroboscopic Method. A method which has been of the greatest assistance in studying periodic motion of all kinds, seems first to have been conceived by a number of scientists early in the nineteenth century, but not used for exact measurements till the time of Toepler.[1] The method can be employed in two ways. Either, the vibrating body is illuminated intermittently, or else

[1] *Ann. d. Physik*, **127**, 556, 1866.

glimpses of the motion are obtained intermittently. If now the intermittence coincides with the period of the motion, the body will always be seen in one phase, and so will appear motionless. If we can determine the frequency of the intermittence of the light, this will give us that of the motion. If the period of the intermittence is slightly longer than that of the motion, each glimpse will be a little later in phase than the last, and show the body in a somewhat later epoch of its period. In other words, in spite of its actual rapid vibration, the body appears to move slowly to and fro, enabling the vibration to be viewed at leisure.

Intermittent vision and illumination can be furnished by a disc with radial slits (equally spaced round the circumference), passing in front of the eye, or in front of a light shining on the body. The former is the more usual arrangement. The disc (στρόβος, a top) is fixed to the shaft of an electric motor whose speed is regulated by a resistance. The speed of the disc in revolutions per second multiplied by the number of slits when the stationary appearance is attained, gives the required frequency. The speed of the disc can be read on a cyclometer fastened to the motor shaft, or, more accurately, by another stroboscope. To this end, a pattern or set of patterns is inscribed on the disc, and the pattern is viewed through an interrupter of standard frequency, or is illuminated at fixed intervals. The former may be adapted from a tuning fork, or may be specially designed. One such—the "Crompton-Robertson Stroboscopic Vibrator"—is shown in Fig. 30.

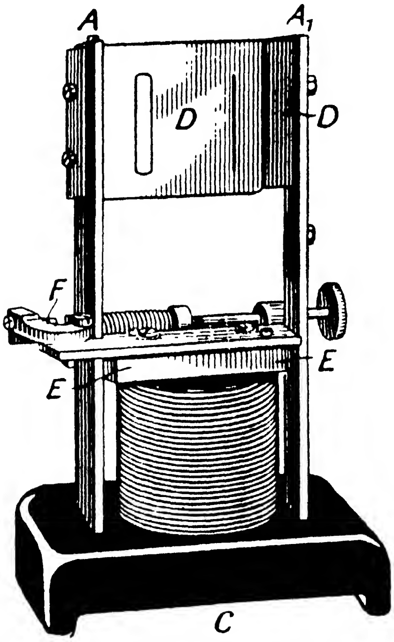


FIG. 30.—"Crompton-Robertson" Stroboscopic Vibrator.

Two thick iron bars AA_1 have their ends clamped tightly into the iron base C of the instrument. On the free ends are fixed two light aluminium vanes D, D, leaving a narrow chink through which the observer may look, or a light may shine. Above the base is an iron core and pole pieces E, E. A coil is wound on the core and the circuit completed between the terminals viâ two platinum contacts, one on A, and one F permanently fixed to the base. On connecting the terminals through a battery, the

coil is energized, A and A_1 attracted by E and E which closes the view-slit, breaks the circuit at F, and the bars spring back to their original position. The slit is thus opened at the natural fundamental frequency of the equal bars A and A_1 vibrating transversely. Pairs of bars of different lengths provide a means of changing this frequency. This stroboscopic vibrator is placed in front of the disc with the slits the rate of revolution of which is to be determined. The disc recommended for use with the vibrator has inscribed on it a number of concentric rings and equi-spaced black triangles in each ring, whose number increases from 15 to 20 as the rings range outwards. Suppose that the vibrator gives n glimpses per second, and that through it the ring containing p triangles appears stationary. Then between two openings of the slit of the vibrator, the disc has moved on one p'th of a revolution, or makes one complete revolution in p glimpses, occurring in $\frac{p}{n}$ seconds; therefore the rate of revolution is $\frac{n}{p}$ per second. Now n is a standard frequency, and p is

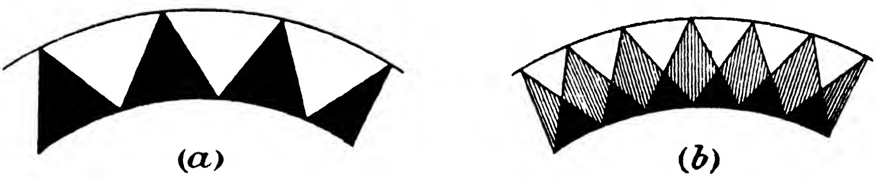


Fig. 31.—Stroboscopic Appearances.

known, so that we have found the rate of revolution of the disc. It is not necessary that the disc shall have moved on only one p'th revolution for this ring to appear stationary; the disc might have moved on $2/p$'ths, $3/p$'ths or any integral number of p'ths between the opening and re-opening of the slit, but in that case the disc will be moving at the corresponding multiple of $\frac{n}{p}$ revolutions per second; if, on the contrary, the disc is travelling half as fast as $\frac{n}{p}$ revolutions per second, then each triangle moves only half the distance between two teeth, so that there appear to be $2p$ triangles standing still in this ring, but overlapping as in Fig. 31b. At a speed $\frac{n}{3p}$ revolutions per second there will appear $3p$ triangles stationary. Thus, concentrating our attention on the p

ring and starting with the stationary appearance when the disc has moved on only $1/p$'th between two glimpses (this we may call the 1st position), accelerating the disc we shall come to the 2nd position at double this speed, the 3rd position at treble speed, etc. Decreasing the speed from the first position we come to the $\frac{1}{2}$ stationary position at $\frac{1}{2}$ speed, $\frac{1}{3}$ position at $\frac{1}{3}$ speed, etc. Stationary patterns can also be observed at the $\frac{3}{2}$, $\frac{5}{4}$, position etc. Thus if m be the number or order of the position, the speed is $\dfrac{mn}{p}$ revolutions per second.

It appears that if we restrict ourselves to the observation of stationary positions only, we are limited as to accuracy by the relative closeness of the number of teeth per ring. Usually we shall find no particular set of rings standing still on looking through the slit of the vibrator, but that the outer rings are moving (apparently) in one direction, and the inner rings in the opposite; the stationary position would correspond to some number of teeth lying between the adjacent rings which are moving slowly in opposite directions. We can however work out accurately the speed of the disc by noting the rate at which one of these slowly moving rings "slips" round the disc. For example, one of the triangles on the p ring may be moving in the same direction as the disc and pass completely round in q seconds. This number q may be counted and called the "slip." Instead of the former movement of $\dfrac{mn}{p}$ revs. per sec., we have now an increased movement of $\left(\dfrac{mn}{p} + \dfrac{1}{q}\right)$ revs/sec. Note that if $q = \dfrac{p}{n}$ we have moved up one position in the scale of stationary rings. Also if a number of rings of teeth are provided, two sets may be simultaneously still, but at different orders. Thus the 4th position in the 20 teeth ring will appear stationary with the 3rd position on the 15 teeth ring, because $\frac{4}{20} = \frac{3}{15}$. This often gives a cue as to the appropriate value of m.

Let us return to the complete apparatus, and take a numerical example. Suppose the stroboscopic disc has 4 slits and is adjusted to the fastest speed at which a vibrating string, seen through the slits appears motionless and single. The vibrator, giving 100 glimpses per second, is now set in motion in front of the disc, and one sees, on looking at the disc through the chink between the vanes, that the 16 teeth ring is moving slowly in the backward direction at the rate of one revolution in 50 seconds. What is the frequency of the string? We must first find which order of position on the 16 teeth ring we have. On temporarily decelerating the disc, we find it passes through 3 more positions at which this ring is motionless, before it appears as 32 teeth overlapping (half position). We, therefore, had the 4th order of the 16 ring; this may be checked by noting that the 20 ring is slip-

ping slowly backward in the 5th position ($\frac{4}{16} = \frac{5}{20}$). Neglecting the slip, the speed of the disc would have been $\frac{4}{16} \times 100 = 25$, but with the slip of 50 back, the speed is less than this by 1/50 rev., giving 24·98 revs/sec. as the speed of the disc. Finally, as there are 4 slits on the disc, the frequency of the string = 99·92 vibns./sec.

Other methods for obtaining intermittent illumination may be used. The vibrator described above may itself be so used if a beam of light is focussed on the aperture between the vanes, and then allowed to fall upon the disc, which it illuminates. In the other type, originally due to Michelson,[1] the lamp itself flashes intermittently, being of the discharge-tube pattern, such as the well-known Osglim lamp. Connect one of these across the secondary coil of a small induction coil, whose primary coil has a make-and-break of a vibrating reed or tuning fork electrically maintained, and it will provide sufficient light for observing the disc, if the latter is shaded from direct daylight by a hood, or if the whole apparatus is in a darkened chamber. Lately attempts have been made to improve the intensity of such stroboscopic illuminations.[2]

When it is desired to study a periodic motion in detail, it is necessary, as explained above, to have a slow "slip" between the motion and the illumination. An instrument which obviates the necessity of suitably controlling the speed of the stroboscopic disc for this purpose, is the "Oscilloscope"; in this apparatus a motor is made to run in synchronism by the motion to be studied itself. This motor controls the flashing of the lamp, but the phase of the flashing is continuously advancing on that of the motor, producing the necessary slip, and giving the "slow motion" appearance required for detailed study.[3]

Stroboscopes without Intermittent Light. A number of instruments have been designed, which rely on the peculiar properties of the eye to give a stroboscopic effect; in these, there is no vibrator or flashing lamp. Such an instrument is the apparatus of Mikola,[4] particularly suited to vibrating strings. An image of the centre of the string is projected on to a revolving drum, which contains a number of white stripes perpendicular to

[1] *Phil. Mag.*, **15**, 84, 1883.

[2] Günther, *Phys. Zeits.*, **22**, 369, 1921; Bairsto, *Phys. Soc. Proc.*, **36**, 349, 1924. [3] *Machinery*, **23**, 1923.

[4] *Ann. d. Physik*, **20**, 619, 1906. *See also* Lampa, *Akad. Wiss. Wien. Ber.*, **123**, 161, 1914.

the wire, so that as the latter vibrates, a dark point representing its centre moves up and down the stripes. If the drum is set in rotation, the position of this image on successive stripes follows the motion of the wire, but if the frequency of the stripes (meaning the number of stripes which pass a given point in a second) coincides with that of the wire, successive stripes will always receive the shadow of the wire in the same position. In consequence of this synchronism, and of the after-image effect in the eye, the wave form of the wire will then appear stationary on the cylinder, whereas, if the latter is turned a little slower or faster, the wave will appear to progress slowly round the cylinder.

Methods based on Rectilinear Vibrations. In Chapter II it was shown that two vibrations compounded together at right angles produce figures whose form can give us information with regard to the periods of the components. Lissajous himself applied the figures formed by a standard tuning fork together with another transverse vibrator, e.g., a wire or another fork, to the elucidation of the wave-form of transverse vibrations; but, as the figures can be recognized only when a simple relation exists between their component frequencies, they are of very limited application, and the method of obtaining them will be briefly dismissed. Lissajous[1] fixed the object glass of a microscope to the standard fork and oscillated it, keeping the eyepiece firmly fixed. A bright point then appeared drawn out into a line; if the bright point were on a body moving to and fro in a direction at right angles to the motion of the object glass, one or other of the figures was formed in the microscope. This apparatus Lissajous called the Vibroscope. An earlier apparatus on the same principle, the Kaleidophone of Wheatstone, consisted of a metal strip twisted at its mid point so that it formed two parts in planes at right angles, one of them being clamped firmly at the end. The twisted strip was pulled aside and the two parts allowed to swing in rectangular directions; the remaining unclamped end showed the figures, as it was constrained to follow two vibrations at right angles, its own vibration, and that imposed by the clamped part of the strip.

Chattering Method. This method, due to W. H. Bragg,[2] is particularly useful for measuring very small amplitudes such as that of a stretched membrane in vibration. Let *DD* (Fig. 32) represent

[1] *Comptes Rendus*, **45**, 48, 1857. [2] *J. Sci. Inst.*, **6**, 196, 1929.

the body under investigation and let a system consisting of the mass M on the end of a spring S, whose other extremity is fixed in the massive block BB, be pushed up until the tip of M just touches the body DD. "Chattering" will ensue, the noise being due to periodic contact between the two systems. The acceleration in the motion of DD—whose maximum value is $a\omega^2$ if the equation of its motion is $y = a \sin \omega t$—will throw the mass off, unless the restoring force exerted by the spring is equal to or greater than the ejecting force. The restoring force can be increased by pushing the block BB through a further distance A towards DD, thus bending the spring. In the position shown in the figure, the mass M would, if the obstruction of DD were removed, execute vibrations given by $y = A \sin \omega_0 t$ in which the acceleration at maximum amplitude, i.e., the actual position, would be $A\omega_0^2$. If then the vibrating system just fails to throw the mass off so that the two move without gap, we must have

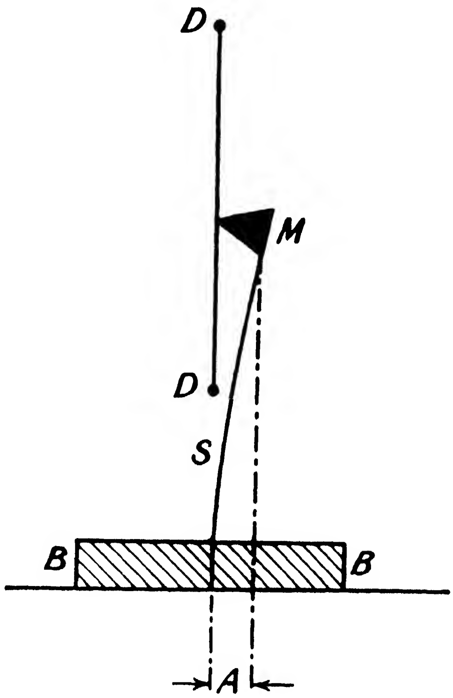


FIG. 32.—Chattering Method.

$$a\omega^2 = A\omega_0^2 \text{ or } a = A(\omega_0/\omega)^2.$$

The shift A and the ratio of the natural frequencies must therefore be known in order that the amplitude a can be determined; ω_0 will, of course, be small compared to ω.

Yet another method for vibrations of small amplitude is that used by Backhaus [1] for an examination of the vibration of sound boards, violin bellies, etc. This consists in sticking a strip of metal foil on the part to be examined and combining it with the nearby metal plate to form a small electrical condenser. The oscillatory changes in capacity caused by the changes of thickness of air gap, while the body is vibrating are measured in a valve circuit such as that of Fig. 49.

Plucked Strings. This is the simplest of the modes of exciting transverse vibrations in strings, which will now be considered

[1] *Zeits. f. tech. Phys.*, **8**, 509, 1927, and **9**, 491, 1928.

in detail. The string is pulled aside by a finger or by a "plectrum" at a particular point, and then let go. From the formula already obtained for the velocity of small transverse vibrations in strings $V^2 = \frac{F}{m}$ (cf. eq. 38) we can set down for the differential equation of the motion :—

$$\frac{\partial^2 y}{\partial t^2} = V^2 \frac{\partial^2 y}{\partial x^2} = \frac{F}{m}\frac{\partial^2 y}{\partial x^2}.$$

The solution can be written in the form $y = f_1(x - Vt) + f_2(x + Vt)$ representing the two waves travelling in opposite directions, which compose the stationary vibration of the string (cf. p. 68). More explicitly, the equation of the vibration of the string executing a fundamental vibration can be written (after the style of equation (15) Chap. II)

$$y = a \sin \frac{2\pi}{2l} x \cos 2\pi n t \quad . \quad . \quad . \quad . \quad . \quad (39)$$

$$= a \sin \frac{\pi}{l} x \cos \frac{\pi V t}{l},$$

or for a partial tone of the j^{th} order :—

$$y_j = a_j \sin j \frac{\pi}{l} x \cos \frac{j\pi V t}{l},$$

since the distance between successive nodes $\left(\frac{\lambda}{2}\right) = \frac{l}{j}$, and $V = n\lambda$.

For a note of complex quality, the complete Fourier series is :—

$$y = \sum_{j=1}^{j=\infty} \sin j\frac{\pi}{l} x \left(a_j \cos \frac{j\pi V t}{l} + b_j \sin \frac{j\pi V t}{l}\right), \quad . \quad . \quad . \quad (40)$$

the maximum amplitudes of each partial $\sqrt{a_j^2 + b_j^2}$ being obtained from the Fourier integrals representing a_j and b_j. The quality of the note of a plucked string—i.e. the number and variety of the harmonics—depends on the form into which the string is bent before being released, and this again depends on the form of the plectrum. To take an extreme and unpractical case, if the string were bent into a bow of sine-wave pattern and suddenly let go, it would execute the simple fundamental tone of frequency $\frac{1}{2l}\sqrt{\frac{F}{m}}$, according to equation (38). A type which lends itself to scientific study and approaches closely to conditions in certain musical instruments is that in which the string is pulled

aside at a single point. An extensive experimental study of this case has been made by Krigar-Menzel and Raps,[1] while it has received theoretical treatment at the hands of Helmholtz.[2] In the experimental research, light from a narrow slit fell across a wire and was projected on to a rotating drum carrying a sensitized paper strip. The wire executed transverse vibrations along the line of the slit, while the sensitized paper moved at right angles to the slit. Thus when the wire moved the shadow of a given point moved across the bright image of the slit on the drum, and when the drum revolved a sinuous trace was made on the paper by the dark spot, representing the transverse motion of

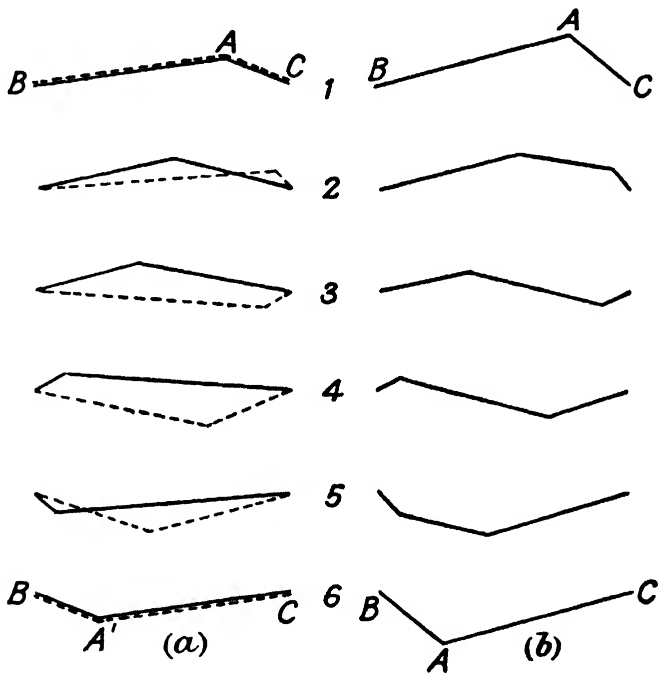


FIG. 33.—Displacement Curves of Plucked String.

the point on the string, extended on a time axis. Both point of plucking and point of observation were varied. From a set of photographs, dealing with a fixed point of plucking, it was deduced that the wire went through a form of stationary vibration, corresponding to the figure above, representing half a complete period.

The string is plucked initially at A into the triangular shape BAC (Fig. 33a). On releasing the string, the bend at A is levelled out by two discontinuities travelling down AB and AC respec-

[1] *Ann. d. Physik*, **50**, 444, 1893; Carrière, *J. de Physique*, **8**, 365, 1927.

[2] *Phil. Mag.*, **21**, 393, 1860. *See also* Sizes, *Comptes Rendus*, **156**, 1,234, 1913; Ghosh, *Ind. Assoc. Proc.*, **6**, 51, 1920; Das, *Ind. Assoc. Proc.*, **7**, 13, 1921.

tively; on reaching the ends of the string, these are reflected as "humps" on the other side of the mean position, meeting again at A', such that $BA' = CA$; and $CA' = BA$. Finally the cycle is completed by a return to the position BAC. At no moment does this string occupy its undisplaced position; this will only occur when all the partials which make up the vibration are in phase, when, e.g., the string is plucked at its mid point. If X is the distance of the point measured from B, and Y the extent, of the plucking, we have initially $y = \frac{Yx}{X}$ for all values of x up to X, and $y = Y\frac{(l - x)}{(l - X)}$ for values of x between X and l.

We can represent the subsequent motion as the superposition of two discontinuities given by equations having half the amplitudes of the above, and reflected with complete change of phase of π at B and C, the fixed ends of the string. The manner in which these add up to the observed vibration is shown in Fig. 33*b*. It is possible to obtain coefficients for the Fourier series to represent a "bent" wave of the type of one of these. Such analysis shows that the series contains all the higher partials from $j = 1$ to $j = \infty$, but with amplitudes rapidly diminishing in the inverse ratio of the square of the order of the partial, i.e., as $\frac{1}{j^2}$. Owing to this rapid diminution of amplitude a finite number of partials will in practice suffice to represent the motion.

Krigar-Menzel and Raps found that the effect of plucking the string over a considerable length—instead of at a single point, as by a sharp edge—was to "round off" the sharp corners in the zig-zag form of wave. The more the corner is rounded off the less the number of Fourier partials required to make up the wave; in the extremes, a true zigzag, with a discontinuity at the corner requires an infinite series of components, whereas the true sine wave requires only one. It was observed that the wave-form changed slightly from period to period after releasing the wire. This was ascribed to movement of the ends of the string, due to the bridges not being rigid. The yielding of the bridges of a stringed instrument has another effect; instead of the sounding length of the string ending at the bridge, it is continued a little beyond, in other words, the string vibrates about a point a little beyond the bridge, instead of about a point on the bridge itself, when the latter partakes to a small extent in the vibration.

Consequently, all the tones of such a string are slightly lowered by the yielding of the bridge, but all to the same extent, so that the series forming the " note " remains harmonic.[1]

Harp and Banjo. Stringed instruments excited by plucking may be divided into two classes; those, of which the harp is the architype, having strings of fixed length between two stops or bridges. Others, of which the banjo is a common example, in which the pitch is varied by the player " stopping " the strings at different points so as to change the sounding length.

The harp consequently requires a separate string for every note of the musical scale. To reduce space and expense in the modern instrument, strings are provided for playing in one " key " only (C flat major). To enable the player to play in other keys, Erard added a device by which each string can be shortened or lengthened by a small fraction, so raising or lowering the fundamental by a tone or semi-tone; the alteration being applied simultaneously to each of the seven lowest notes and to all its octaves by one of seven pedals. The strings are plucked by the fingers; and if the octave of the fundamental is to be elicited they are touched at the same time in the middle with part of the palm of the hand.

The strings of the banjo are usually plucked by the finger-nail or by a pointed piece of horn (plectrum). As a consequence the resulting waves contain a large number of higher partials, and the tone is more brilliant, but also more metallic, than that of the harp. The points at which the finger is to be placed for shortening the effective length of the wire are regulated by a series of frets, placed along the finger-board over which the strings are stretched. These instruments are provided with a sound box, which by forced vibration amplifies the sound of the strings alone. In spite of this the tone is weak, so that the notes are generally reiterated by repeated touch.[2]

Bowed Strings. The first theoretical and practical investigation of the bowed string came from Helmholtz,[3] who by means of the vibroscope observed the Lissajous figures produced by the motion of the mid-point of the string in combination with a tuning fork. He found that the form of stationary vibration showed discontinuities similar to those shown in Fig. 33 by a

[1] Rayleigh, *Sound*, **1**, 200, 1877.

[2] Banerji and Ganguli, *Phil. Mag.*, **7**, 345, 1929 and **9**, 38, 1930; Obata and Ozawa, *Phys. Math. Soc. Japan Proc.*, **13**, 1, 1931.

[3] *Phil. Mag.*, **21**, 393, 1861.

plucked string. A complete experimental study of the vibration curves was undertaken by Krigar-Menzel and Raps,[1] with the apparatus already described. They found that only in special cases were these of the pure zig-zag form; in most cases bending of the theoretical straight lines is apparent.

There are three laws which arose from their investigations, summarized by Davis [2] as follows.

Young's [3] *Law.* "No overtone is present which would have a node at the point of excitation."

Helmholtz' Law. "When a string is bowed at an aliquot point $\left(\frac{1}{k}\right)$, the part of the string immediately under the bow moves to and fro with [two] constant velocities, whose ratio is equal to the ratio $\frac{1}{k-1}$ of the segments into which the string is divided by the point in question. The smaller of these two velocities has the same direction as that of the bow, and is equal to it."

Krigar-Menzel's Law. "When a string is bowed at any rational point p/q, where p and q are prime to each other, the part of the string immediately under the bow moves to and fro with [two] constant velocities whose ratio depends only on q, and is $\frac{1}{q-1}$."

These laws were established mainly on an empirical foundation, but serve to determine the motion of a bowed string in most circumstances. Young's law applies to strings stretched between sharp bridges in whatever way excited.[4] The other laws deal with the motion of the bowed point, the third being more general, as Helmholtz considered the motion only for the case where the string was bowed at $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., of its length from one end. The sudden velocity changes at the bowed point determine discontinuities in the velocity waves which pass over other parts of the string, and so determine the motion of all points in the string, in a manner which will be considered in detail presently.

The action of the bow upon the string as envisaged by Helmholtz and substantiated by experimental tests of the necessary consequences, is then as follows. During part of the period, the bow pulls the string aside with it, so that they have no relative

[1] *Ann. d. Physik*, **44**, 623, 1891.
[2] *Am. Acad. Sci. Proc.*, **41**, 691, 1906.
[3] *Phil. Trans.*, **90**, 106, 1800.
[4] *See* Raman, *Ind. Assoc. Proc.*, **7**, 29, 1921.

velocity, and the full static frictional force is exerted between the string and the resined hairs of the bow. The tension in the strings is initially at right angles to the frictional force, but as the bowed point is pulled aside, the two parts of the string make a diminishing acute angle with the direction of movement of the bow, and consequently the tensions in the two parts have increasing components in this direction, opposing the frictional force. When these components become together greater than the friction, the string ceases to adhere to the bow. String and bow now possess a relative velocity, and the dynamical friction involved is less than the static. The components of tension now drag the string back against the bow to the original position, and, owing to inertia, beyond this, until the reversed components of tension bring the string to rest, when it is suddenly caught by the bow, and again dragged with it.

From this theory of the action of the bow, Helmholtz was able to calculate the requisite coefficients of the Fourier analysis of the tone of the bowed string, in the form :—

$$y = A \sum_{j=0}^{j=\infty} \frac{1}{j^2} \sin \frac{j\pi x}{l} \sin 2\, j\pi nt, \quad . \quad . \quad . \quad . \quad (41)$$

corresponding to the solution for the plucked string, but with the proviso that in accordance with Young's law, all partials having a node at the bowed point are to be omitted from the summation. In this equation A is the maximum amplitude of the fundamental at its antinode, i.e., in the centre of the string, and depends on the velocity of the bow. The frequency n of the fundamental is given by the usual formula $n = \frac{1}{2l}\sqrt{\frac{F}{m}}$: the amplitudes of the partials, obtained by putting $j = 1, 2, 3$, etc., diminish as the square of the order of the partials, save that those having a node at the bowed point have zero amplitude.

It may be noted that, in practice, the string is bowed near one end, so that q in Helmholtz' law is a large quantity. Consequently, the partials of order q, $2q$, $3q$, etc., only are missing, and as anyway these would have a very small amplitude $\left(\frac{1}{q^2}, \frac{1}{4q^2},\right.$ etc., of the fundamental) the Young proviso may be ignored in this case.

So much for the Helmholtz theory of the bow. Experiment suggests that it is virtually correct; the forward velocity is con-

stant and equal to that of the bow. The constancy of the return velocity does not seem to be so clearly established, but it does seem that we have sudden reversals of the velocity of the string at each end of the forward dragging of the bow.

Raman's Analysis of the Bowed String. Starting from this basis, of two constant velocities at the bowed point, Raman[1] has built up an analysis not requiring the Fourier series, giving the transverse velocity, amplitude and their variation at each point of the string. He treats the stationary vibration of the string as made up of two progressive waves, reflected from end to end of the string, but finds it easier to consider the changes in transverse velocity $\frac{dy}{dt}$ produced by these waves at each point of the string, instead of the displacement. We can then formulate these velocity waves, so that, as they cross the bowed point, they satisfy the known conditions there, i.e., that the resulting velocity of the string $\frac{dy}{dt}$ jumps between two finite and constant values U_1 and U_2. In consequence of the constant velocities at the bowed point, $\frac{d^2y}{dt^2} = 0$ here, except for certain infinitesimal fractions of the period when the jumps occur, and then $\frac{d^2y}{dt^2} = \pm \infty$.

Consequently, if $\frac{dy_1}{dt}$ represents the velocity of the string due to the positively travelling wave, and $\frac{dy_2}{dt}$ that due to the negative wave, their sum at the bowed point should remain constant except at the instants at which the jumps of velocity occur. It is possible to secure this condition, if on one side of the bowed point and distant l from it, the slope of the velocity wave (not that of the string itself) due to the positive wave alone, would be $\tan^{-1}\alpha$, that due to the negative wave only, at the same distance l on the other side of the bowed point is $\tan^{-1}(-\alpha)$ (Fig. 34).

When the waves are thus related to each other, the values of $\frac{dy_1}{dt}$ and $\frac{dy_2}{dt}$ at the bowed point change to equal but opposite extents in the displacements occurring in any small interval of time, and their sum therefore remains constant. It is possible

[1] *Ind. Assoc. Bull.,* **15**, **1918**; Mitra, *Indian J. Phys.,* **1**, **311**, **1927**.

to draw any number of curves to represent the positive and negative velocity waves satisfying this condition. But there are two additional conditions to be satisfied, viz., that $\frac{dy_1}{dt} + \frac{dy_2}{dt}$ should be always zero at two other points on the string, i.e., its two fixed

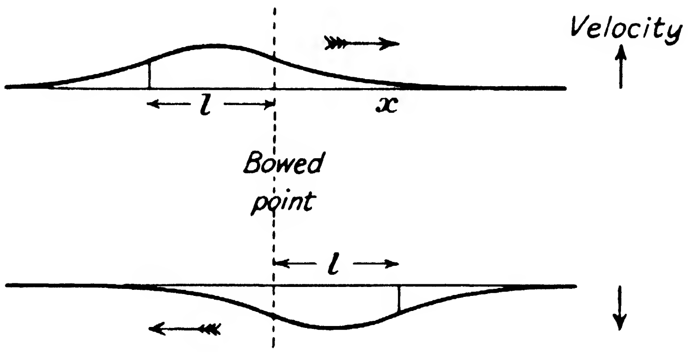


Fig. 34.—Velocity Waves on a String (Smooth Type).

ends. Such a velocity wave-form as that in Fig. 34 therefore cannot be the solution, in fact, in order to avoid this *impasse* it is necessary to postulate that the waves should have constant slope throughout, with discontinuities where the velocity suddenly drops. The waves are accordingly of this zig-zag type (Fig. 35), in which the velocity wave of amplitude represented by the continuous line is travelling to the right; that represented by the dotted line of equal amplitude, to the left. The note is made up of such waves, having a number of discontinuities proportional to the order of the partial. In our figure the second harmonic is pictured, since there are two discontinuities in twice the length of the string, twice the length of the string being the wave-length of the fundamental tone in the string.

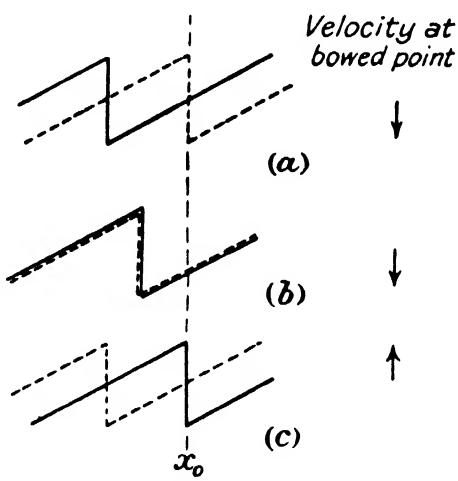


Fig. 35.—Velocity Waves on a String (Zig-zag Type) Forming the Second Harmonic.

A study of these successive phases shows that the velocity at the bowed point (x_0) due to the superposition of these two waves, is always a constant negative quantity between the epochs represented at (a) and (c) (Fig. 35). At the instant figured at (c) occurs a jump to a positive velocity which remains constant between

the epochs (c) to (a), where we return to the original circumstance —it is easier to see what happens at the bowed point if templates are cut out of card to the shape of the velocity waves, and moved past the point in opposite directions. The superposed displacement due to these waves is similar to that shown in Fig. 33 for the plucked string.

In order that the velocity at x_0 may jump from U_1 to U_2 at stated instants during the period, the simplest case to imagine is that in which a discontinuity due to one wave passes over the point at the instant when the other wave would produce, by itself, no velocity of the point. When the point is a node of one of the harmonics of the string, the non-appearance of this harmonic in the complex note requires a number of discontinuities, at intervals equal to half the wave-length for this harmonic, passing over the bowed point simultaneously in pairs, and so having no part in the formation of discontinuities in the velocity of the bowed point. Apart from these absent harmonics, each of the discontinuities involves a jump in the velocity from U_1 to U_2, as they pass over the bowed point.

If therefore there are j discontinuities in twice the length of the string, the inclination of these waves to the x axis is given by α, where :—

$$\tan \alpha = j\frac{(U_1 - U_2)}{2l}$$

The superposition of these velocity waves causes the resultant velocity graph along the string to consist of j discontinuities between $(j + 1)$ straight lines inclined at an angle $\tan^{-1} 2\alpha$ to the x axis.

Now the points where the inclined lines of this graph cut the

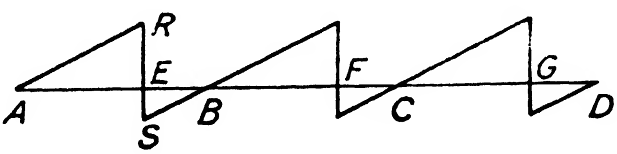


FIG. 36.—Resultant-Velocity Along Bowed String.

x axis are the nodes of the j'th partial, for as two of them (the ends of the string) are points of continual zero velocity, so also must the others be. Thus A, B, C, D, etc. (Fig. 36), are nodes of the stationary vibration; E, F, G, etc. are points where the discontinuous jumps from U_1 to U_2 occur. These points, E, F,

G, etc., thus form possible bowing points for the maintenance of the motion, and further,

$$\frac{U_2}{U_1} = \frac{ES}{ER}.$$

Therefore
$$\frac{U_2}{U_1 + U_2} = \frac{EB}{AB}.$$

Putting $EB = x_j$, the distance of the bowed point from the nearest node of the j'th harmonic, and $AB = \frac{l}{j}$ we find :—

$$\frac{U_2}{U_1 + U_2} = \frac{jx_j}{l}.$$

This is also the fraction of the period for which the string moves with the bow, i.e., with velocity U_2, since the bowed point moves between two definite points with two definite and constant velocities. From the velocity graph we can calculate the configuration of the string at each instant, since the displacement at any point is the time integral of the velocity wave. The displacement wave for the single discontinuity case consists of the two straight lines meeting at the point which the velocity discontinuity has reached (see Fig. 37). The positive and negative displacement waves superposed then give the instantaneous configuration of the string, as in Fig. 33.

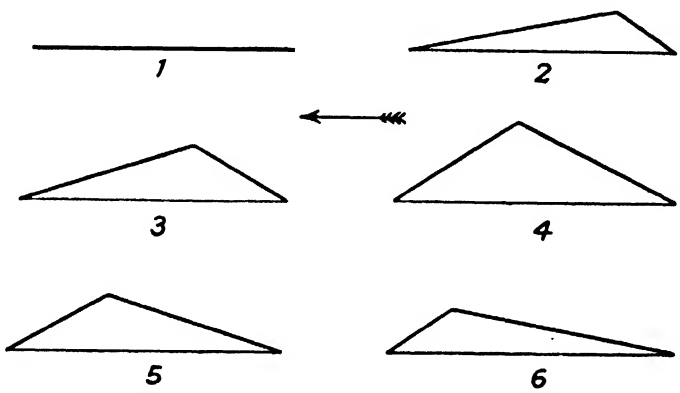


FIG. 37.—Displacement Wave of Bowed String.

The above examples and figures apply to bowing at an irrational point. When the bow is applied at a node, Raman shows that the absence of several harmonics requires discontinuities separated by straight lines parallel to the x axis; for the proof of this the original should be consulted.[1] The velocity waves and superposition for the string bowed at the centre are shown in Fig. 38.

[1] *Loc. cit.*, p. 45.

The resultant velocity wave contains two discontinuities travelling in opposite directions and crossing at the bowed point, the

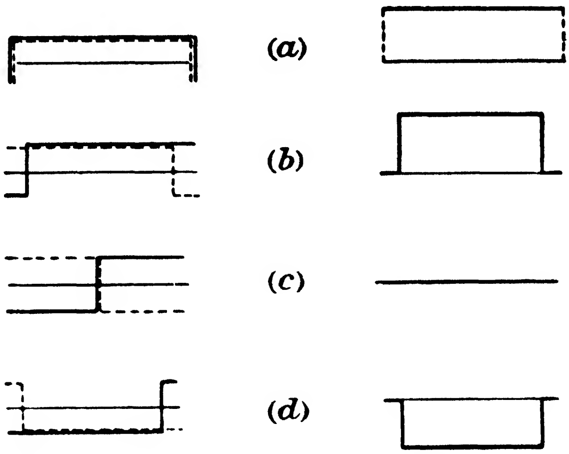


FIG. 38.—Velocity Waves of String Bowed at Centre.

resultant displacement curve (Fig. 39) contains in general two corners travelling opposite ways which meet at the centre, and is identical with that of a string plucked at this point.

When the string is bowed at a point of trisection, quadrisection,

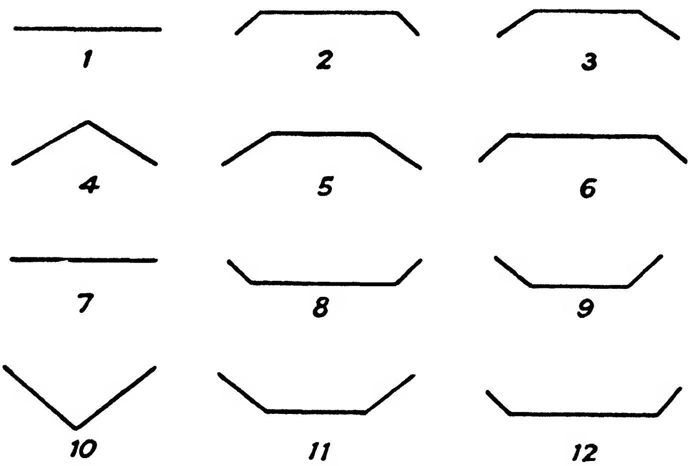


FIG. 39.—Displacement Curves of String Bowed at Centre.

etc., the 3rd, 4th, partial, as the case may be, is absent; the resultant discontinuity in the velocity curve is then repeated at the other nodes of the absent harmonic. Consequently the string may be bowed at the other nodes without change of the velocity relation, i.e., $U_1 - U_2$ depends only on the denominator of the fraction p/q; this is Krigar-Menzel's law.

Transitional Forms of Vibration. It should be repeated that the above velocity graphs were obtained by Raman on the assumption

of only two constant velocities U_1 and U_2 for the bowed point. This investigator has himself experimentally verified the first assumption, i.e., that the string moves for a fraction of the period with the velocity of the bow, under all circumstances. In addition, when the bow is applied at a nodal point $\frac{l}{j}$, the return velocity is constant. If however the bow, while still in action, be moved away from this point a little, the progressive reappearance of the j'th, $2j$'th, etc., partials involves irregularity in the return motion of the string, i.e., U_2 is no longer constant. This modifies the simple Helmholtzian zig-zag wave forms, introducing a crumpled appearance to the steep descending line. The essential characteristic of these "transitional forms" is, that the return of the bowed point is not executed with a uniform velocity, but consists of two or three stages, in which the velocities are different. After the disappearance of these distinct steps, the motion settles down to the Helmholtz type, corresponding to a point of irrational division of the string, with a full series of harmonics.[1]

Pressure and Width of the Bow. If the pressure of the bow is increased or its velocity decreased, the fraction of the period during which the string adheres to the bow is increased, so that there may be a change in the type of vibration. If the bowing point is near the node of an important harmonic, it will obviously require a greater pressure to produce this harmonic in the subsequent motion than it would require at an antinode of the same harmonic. Raman found experimentally that the pressure needed to produce the fundamental, by bowing near the end of the string, decreased as the square of the distance of the bowing point from this node. Violin teachers usually give instructions that the speed of the bow is to be increased when greater intensity is required, the bow being moved slightly nearer the bridge at the same time. Lippich[2] asserts that the increase of either bowing speed or pressure raises the general intensity, so that, for example, it is difficult to press lightly and play *forte*, or to press heavily and play softly at the same time. The effect of the finite width of the bow will be similar to that of the finger when the string is plucked. It tends to remove some of the higher harmonics, for all those having a node under the bow will be executed with difficulty. From this cause only, the quality of the note produced will be varied according as the bow is near or far from one end. With the bow of a violin, for example, close

[1] *Loc. cit.*, p. 95; *also Phil. Mag.*, **38**, 573, 1919.

[2] *Akad. Wiss. Wien. Ber.*, **123**, 1,071, 1914. *See also* Kar, *Phys. Rev.*, **20**, 148, 1922.

to the bridge, the note will be richer in the first half dozen harmonics than if bowed at the usual place, i.e., between $\frac{l}{15}$ and $\frac{l}{7}$. Playing near the bridge of a violin is sometimes introduced by composers as a special effect (*sul ponticello*). The vibration forms shown in Figs. 37 and 39 will also be complicated by the fact that only one point on the string can move with the bow; the other parts of the string under the bow will have a positive or negative velocity relative to the bow in the outward motion.

The Instruments of the Viol Family. In these instruments played by a bow, the four strings are stretched between two saddles along an unmarked finger-board. The higher saddle is known as the "bridge," the lower on the finger-board, is known as the "nut." The bridge stands upon the body, a hollow wooden box which is of a peculiar shape necessitated by the movement of the bow, and is provided with two openings known from their shape as *f* holes. The purpose of the body is to reinforce the sound of the strings by the forced vibrations of the box and the air inside it. That the air in the box does execute vibrations similar to, yet more complex than that of the string, has been shown by Barton and his pupils,[1] who covered one of the openings with a membrane and attached mirror, so that photographs of the corresponding motions could be made on the same plate. The vibrations are communicated to the air viâ the bridge and the wooden case of the body, and they have shown that these parts also are set in forced vibration, by means of an optical lever rocked by the bridge or the body. The motion of the air was found to be more complex than that of the intermediate vibrators, showing that the air more nearly followed the forcing vibration of the string. It is mainly in the construction of this sounding system that a good violin with its smooth tone is distinguished from a poor one, or from the sonometer. The best violins were and are those made by the Italians of the seven-

[1] Barton and Penzer, *Phil. Mag.*, **12**, 576, 1906; Barton and Richmond, *Phil. Mag.*, **18**, 233, 1909; Barton and Ebblewhite, *Phil. Mag.*, **20**, 456, 1910, and **23**, 885, 1912. *See also* Raman, *Phil. Mag.*, **21**, 615, 1911; Helen Browning, *Phil. Mag.*, **2**, 955, 1926; Ghosh, *Indian J. of Sci.*, **1**, 141, 1926; Backhaus, *Zeits. Tech. Phys.*, **8**, 509, 1927; *Zeits. f. Phys*,, **62**, 143, 1930 and **72**, 218, 1931; Chambers, *Phil. Mag.*, **5**, 160, 1927; Seiffert, *Zeits. f. Inst.*, **49**, 116, 1929 and **76**, 405, 1932; Karapetoff, *Frank. Inst. J.*, **207**, 645, 1929.

teenth century. The niceties of their construction, the dependence of the *timbre* on the construction of the body, even on the type of varnish, are not understood thoroughly after centuries of violin making, not even in an empirical fashion; so that it is not strange that a great deal remains to be scientifically elucidated on this subject.

The natural tones of the air in the resonance box might be expected to have considerable influence on the resultant notes of the violin. As a matter of fact, Barton found that in general the vibrations of the enclosed air were due to pure forcing, but Hermann,[1] on the contrary, found on analysis of the vibration curves of the air in the body distinct partials due to that enclosed air (the second harmonic stood out especially in the violin examined, the fundamental in a violoncello). To the incidence of these body-tones Hermann attributes the noticeably different quality of violin and 'cello tone in different parts of their pitch range.[2] The strings also exert a mutual influence on each other. Morton and Vinycomb [3] found that if two strings were tuned to the same pitch, and one of them was excited, the fundamental only was generally transferred to the second string.

Wolf Note. When the pitch of the tone elicited from the string coincides with the fundamental (or with an important harmonic) of the wood or air of the body, one would expect a large reinforcement of the former, the energy being furnished by the increased bowing pressure required to maintain the same amplitude of the string. In fact, when the pitch is that of certain harmonics of the wooden structure a quite special and undesirable effect is produced, in which the control of the string seems to pass out of the player's hands. The howling effect produced at this pitch has given this note the name of "wolf." With the object of correlating the vibrations of belly and string, White [4] obtained simultaneous records by the method of Barton at the wolf pitch. The belly showed a S.H.M. of large amplitude, whereas at frequencies a little above or below, its motion was quite complex. A fact which does not seem to be explained is that the wolf-note is usually observed at the upper harmonics of the wooden system

[1] Thesis, Königsberg, 1908.

[2] Thesis.

[3] *Phil. Mag.*, **8**, 573, 1904.

[4] *Camb. Phil. Soc. Proc.*, **18**, 15, 1915. *See also* Hewlett, *Phys. Rev.*, **55**, 359, 1912; Raman, *Ind. Assoc. Proc.*, **6**, 19, 1920. Deodhar, *Phys. Soc. Proc.*, **36**, 379, 1924.

of the violin, and not at the fundamental. The wolf-note can also be excited by plucking; in this case the vibration curves of the string and body show simple resonance.

Raman[1] prefers a more detailed explanation of the bowed "wolf-note." His theory of the bowed string leads him to expect a relatively higher bowing pressure for the fundamental than for the octave, when the string is bowed at a point near its end. Accordingly, at the wolf pitch the string starts sounding its fundamental, but resonant vibration of the wood drawing energy from it, the pressure between the bow and the string falls, until the fundamental cannot be maintained, and the vibration changes to a type in which the octave predominates; on the consequent cessation of the vibration in the wood, the fundamental reappears. These cyclic changes between octave and fundamental are apparent in the records of the movements of the string.

The Mute. In order to deaden the intensity of the sound it is customary to load the bridge of viol instruments by a metal clamp which grips it, the size and weight of this "mute" depending on the size of the instrument. Not only does it reduce the general amplitude of the vibrations of the instrument, but it gives new quality to the note, a fact long made use of in orchestration. The motion of the bridge is mainly lateral, it is pushed and pulled in the direction of the string. There is also a smaller motion in its own plane about one or other of the feet of the bridge. Giltay and de Haas[2] think that the mute probably damps the first motion more than the latter. These lateral oscillations of the bridge may sometimes be at twice the frequency of the string owing to the double forcing by the latter[3] (cf. Melde's experiment, p. 118). A further point is that the additional load will alter the natural resonances of the instrument including the wolf pitch, but it is not sufficient to treat the bridge as a rigid addition to the body. The bridge is capable of executing vibrations on its own account; we have, in fact, to deal with coupled vibrations of string and bridge. Hence the importance, for the sound of the violin, of loading the bridge, and hence also the importance of the position of the added load, i.e., whether the centre of the added mass is high or low above the bridge.

[1] *Phil. Mag.*, **32**, 391, 1916, and **35**, 493, 1918.

[2] *K. Akad. Amsterdam Proc.*, **12**, 513, 1910. *See also* Broca, *Comptes Rendus*, **180**, 507, 1925.

[3] Morton and Florence Chambers, *Phil. Mag.*, **50**, 570, 1925.

Struck Strings. It might be expected that the general form of vibration peculiar to a plucked string would be exhibited by the same string when struck by a hammer, since this is merely another method of displacing a given point or region of the string. The motion of the string is in fact similar in the two cases, but more complex in the latter because the struck point is displaced a little before the rest of the string. We could describe the initial conditions of the plucked string as "static," that of a struck string as "kinetic." We have in fact more variables at our disposal in the latter case, e.g., the relative masses of hammer and string, the striking velocity of the hammer, which have no counterpart in the plucked string.

Theoretical solutions have been attempted from two directions. Helmholtz [1] and others equate the instantaneous force of the hammer on the string to the product mass × acceleration of the struck part of the string; a solution for the subsequent motion of this point is obtainable, if an expression representing the variation of the force on the string with time be assumed. The first named assumed a form:

$$m\frac{d^2y}{dt^2} = P = A \sin 2\pi\frac{t}{\tau}, \quad . \quad . \quad . \quad . \quad . \quad (42)$$

where τ is the time of contact between hammer and string and is dependent upon the elasticity of the hammer. The theory assumes that τ is a very short time, and that, as a result of the sudden initial velocity imparted to the struck point, waves move in opposite directions along the string and are reflected at the ends.

As a result of his experiments Kaufmann [2] rejected the Helmholtz theory, as experiment showed that the time of contact was always considerable compared with the periodic time. In its stead, Kaufmann based a theory on an assumption that the hammer acts as a massive unyielding particle striking the string. A number of other assumptions and approximations were necessary, in order that he might finally obtain a solution for a striking point near one end of the string, giving expressions for the time of contact, motion of struck and other points on the string.

The theory has been developed and made to fit the results

[1] Sensations of Tone, App. V. *See also* Delemer, *Annales Soc. Sci., Bruxelles*, **30**, 299, 1906.

[2] *Ann. d. Physik*, **54**, 675, 1895.

more exactly,[1] but as the simplest, the original form of Kaufmann will be outlined here. Let m_0 represent the mass of the system moved, i.e., the mass of the hammer plus a mass equivalent to the string or part of the string moved, imagined to be collected at the struck point. Taking this as the origin of co-ordinates, we imagine the string to be bent into a form similar to that in Fig. 40. The short piece of string a, to the right of the struck

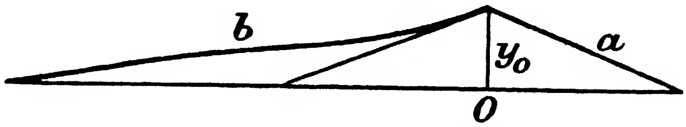


FIG. 40.—Displacement Form of Struck String.

point, is assumed to remain straight during the impact, whereas the longer piece b is bent into some form of wave given by $y = f(x + Vt)$ such that there is a definite displacement y_0 at O, and a definite slope $\left[\frac{\partial y}{\partial x}\right]_{x=0}$ at the same point on the negative side of O. Now the forces which tend to restore the string to its undisplaced position are the resolved tensions in b and a at O, i.e.

$$-F\left[\frac{\partial y}{\partial x}\right]_{x=0} - F\frac{y_0}{a} = m_0\frac{\partial^2 y_0}{\partial t^2}.$$

As $y = f(x + Vt)$ on the left of O, $\left[\frac{\partial y}{\partial x}\right]_{x=0} = \frac{1}{V}\frac{\partial y_0}{\partial t}$, so that the equation of motion of this point can be written :—

$$m_0\frac{d^2 y_0}{dt^2} + \frac{F}{V}\frac{dy_0}{dt} + \frac{F}{a}y_0 = 0 \quad . \quad . \quad . \quad . \quad (43)$$

This equation is of the form corresponding to damped motion (21), and its solution (as long as the reflection of the wave $f(x + Vt)$ has not reverted to O from the distant end, i.e., while $t < \frac{2b}{V}$) is :—

$$y_0 = \frac{U}{\sqrt{\frac{F}{m_0 a} - \frac{F^2}{4m_0^2V^2}}} e^{-\frac{Ft}{2m_0V}} \sin\left(\sqrt{\frac{F}{m_0 a} - \frac{F^2}{4m_0^2V^2}}\right)t.$$

[1] Raman and Banerji, *Roy. Soc. Proc.*, **97**, 99, 1920; Bhargava and Ghosh, *Phil. Mag.*, **47**, 1,141, 1924 and **49**, 121, 1925; Datta, *Indian Assoc. Proc.*, **8**, 107, 1923; Ghosh and Dey, *Indian Assoc. Proc.*, **9**, 193, 1925; Das, *Indian Assoc. Proc.*, **9**, 297, 1925 and **10**, 75, 1926; Ghosh, *Phys. Rev.*, **24**, 456, 1924 and **28**, 1315, 1926, and *Phil. Mag.*, **1**, 875, 1926.

where, at $t = 0$, $y_0 = 0$ and $\frac{dy_0}{dt} = U$, the striking velocity of the hammer (*cf.* p. 47). Kaufmann put this result into a more practical form by putting in the fundamental time-period $\left(\mathrm{T} = \frac{2l}{V}\right)$, thus $F = V^2 m = \frac{4lM}{\mathrm{T}^2}$, and M = mass of whole string.

$$y_0 = \frac{UT}{\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)}} e^{-\frac{M}{m_0}\cdot\frac{t}{T}} \sin \frac{t}{\mathrm{T}}\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)},$$

assuming that the hammer is thrown back off the string at the instant the latter comes to rest, which occurs when the opposing force due to the string equals the pressure exerted by the hammer. By differentiating the above and putting $\frac{d^2y_0}{dt^2} = 0$, we obtain the time of contact (τ).

$$\frac{\tau}{T} = \frac{1}{\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)}} \tan^{-1}\left[\frac{\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)}}{-2\frac{l}{a} + \frac{M}{m_0}}\right].$$

When $\frac{M}{m_0}$ is small compared with $\frac{l}{a}$, as Kaufmann claims it is in practice, the angle in the above expression is approximately π radians, and

$$\frac{\tau}{T} = \frac{\pi}{\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)}}, \text{ and}$$

$$y_0 = \frac{UT}{\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)}} e^{-\frac{M}{m_0}\cdot\frac{t}{T}} \sin \pi\frac{t}{\tau} \quad . \quad . \quad . \quad (44)$$

It is assumed, as stated above, that the motion at y_0 has not been disturbed by the reflected wave during the time τ.

Though Kaufmann himself gave a few experimental vibration curves for the string, the exhaustive treatment which Raman devoted to the bowed string has been undertaken for the struck string by George [1] in order to discriminate between the Helmholtz

[1] *Phil. Mag.*, **47**, 591, 1924; **48**, 34 and 48, 1924; **49**, 92, 1925; **50**, 491, 1925; George and Beckett, *Roy. Soc. Proc.*, **114**, 111, 1927; **116**, 115,

and allied theories on the one hand, and the Kaufmann theory and its developments on the other. His hammer consisted in principle of a compound pendulum, so that the velocity of striking, U, could be increased by giving the pendulum a larger swing. The time of contact τ was found by letting the contact make an electrical circuit containing an Einthoven galvanometer, or oscillograph. Certain of the oscillograms showed evidence of a second contact before the hammer finally left the string. By photograms of the Krigar-Menzel and Raps type, George obtains values of τ and y_0 to test the formulæ. Both theories give closest agreement with practice at a ratio of string and hammer masses = 1·7, when the percentage errors are about the same; with a light hammer however (e.g., ratio of masses = 0·267) the discrepancies are greater, but the Kaufmann formulæ fit the experimental values fairly well over the whole range, whereas the Helmholtz theory is very wide of the mark with a light hammer. From the example shown, which illustrates the motion of the struck point before and after being hit by the hammer (Fig. 41), reproduced from George's photographs, the principal assumption of Helmholtz that the time of contact is inappreciable in comparison with the period, is unjustified. George's results may be summarized as follows:—

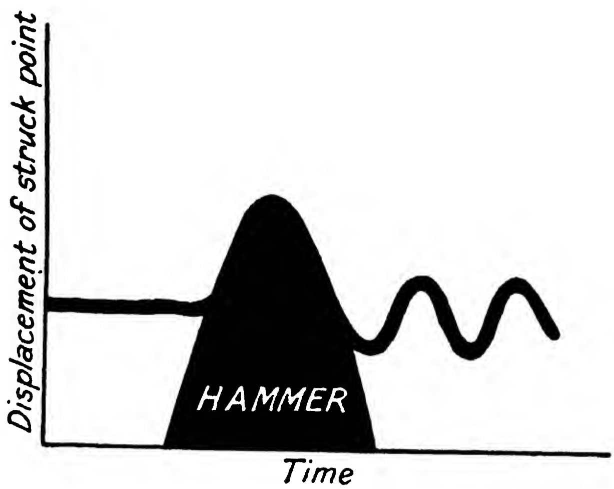


Fig. 41.—Motion of struck Point.

(1) The amplitude of the fundamental is greater when the string is struck near one end, and when the relative mass of the hammer is great.

(2) This amplitude varies in a discontinuous fashion, showing maxima and minima as the hammer is moved towards the centre, and the number of maxima and minima becomes less as the mass

1927; Das, *Indian J. Phys.*, **1**, 437, 1927; *Phys. Soc. Proc.*, **40**, 29, 1927; *Phil. Mag.*, **6**, 479, 1928; Ghosh, *Phys. Soc. Proc.*, **40**, 224, 1928 and *Phil. Mag.*, **9**, 306 and 1,174, 1930; Kar, Ganguli and Laha, *Phil. Mag.*, **5**, 547, 1928.

of the hammer is reduced, until there is only one maximum and two minima.

(3) Early workers in the subject thought it necessary to discriminate between a hard and a felt-covered hammer, but it appears that the distinction is merely one of the finite length of the string struck, similar to that between a narrow and a wide bow.

The Pianoforte. This instrument contains a large number of "strings" in the form of steel wires, one or more for each note of the musical scale, stretched between bridges, one set of which rests upon a "sound-board" and the other set is attached to the frame of the instrument. The strings are struck by hammers at a point distant from one-seventh to one-ninth of their length from one bridge. The vibrations of the strings are rapidly damped, but in order that the vibrations may be stopped as soon as the finger is lifted from the key which operates a hammer, a "damper" comes into action on the majority of the wires in the form of a felt wedge which presses on the string. If desired, all the dampers may be held off by a pedal. Improvement in steel wires and the introduction of the iron frame has made possible greater tensions than formerly, to the enhancement of the intensity of the notes. Nevertheless, three wires are usually given to each note—one or two only when the "soft-pedal" is brought into action. It was formerly thought that the choice of striking-point was governed by the expediency of removing the dissonant 7th and 9th partials—the makers unconsciously following the principle stated in Young's law—and this is no doubt accomplished to a certain extent, but the above experiments on the struck string show that this region is, in general, one of maximum fundamental amplitude for a string so excited, and for this reason is a desirable location for the striking point. Besides, Berry[1] has found that the natural vibrations of the sound-board (which the maker desires to eliminate) are a minimum when the string is struck in this region, the wood copying the vibrations of the wires. The hammers are covered with felt, but those which strike the short wires forming the upper octaves of the instrument are sharper and more pointed so as not to occupy a relatively larger part of the string than those that act on the long strings. It may be

[1] *Phil. Mag.*, **19**, 648, 1910, and **20**, 652, 1910, and **22**, 113, 1911. *See also* Kaeser, *Phys. Zeits.*, **8**, 123, 1907; Datta, *Ind. Assoc. Proc.*, **8**, 107, 1923; Ghosh, *Ind. Assoc. Sci.*, **11**, 29, 1927; Mahajan, *Ind. J. Phys.*, **4**, 515, 1930 and **7**, 539, 1933; Sawada, *Zeits. f. tech. Phys.*, **14**, 353, 1933.

imagined from what has been said about the violin that the quality of the instrument depends upon the material and construction of the sound-board. If the sound is not transmitted quickly over the entire board, different sections of it will be vibrating out of phase, and will impair its effectiveness. This is obviated by the bridges, and by a number of bars fitted tightly to the back of the board to increase its rigidity. Attempts have been made to fit a resonant column of air behind the sound-board, but this is unusual. Initiated by a paper of Bryan [1] a discussion took place in 1913 on the physical significance of that mystic quality called "touch," by which a player attempts to vary the quality of the notes, but the result which seemed to emerge from the discussion was, that the velocity of striking was all that could be varied by the player; whether the time of contact is *ipso facto* altered, seems doubtful.

Electro-magnetically maintained Wire. Apart from the use of a bow, transverse vibrations in an iron or steel wire may be maintained by an electro-magnet in which the current is made and broken with the same frequency as the fundamental vibration of the string, so that a part of the wire is attracted towards the magnet once in each period, and then springs back.[2] It is possible to arrange that the wire itself breaks the current which supplies the electro-magnet, when the wire has reached its maximum displacement in the direction of the magnet (cf. the electro-magnetically maintained tuning fork, p. 114). The tones of such wires have been examined by Klinkert [3] by the usual photographic methods. They correspond more or less to the tones produced in the wire when plucked, at the point where the magnet is placed, by an object of considerable extent, i.e., the resulting wave forms exhibit rounded-off discontinuities, except that the amplitude is maintained, instead of dying away.

When the wire acts as its own current-breaker, the fundamental tone is maintained. The system obeys Young's law, in that the electro-magnet cannot maintain a partial when it is opposite one of the relevant nodes. Even if, with the magnet at an irrational point of division, a harmonic be started in the wire by lightly touching at a rational point, the form of vibration is unstable, and tends

[1] *Phys. Soc. Proc.*, **25**, 147, 1912; *see also* White, *Acoust. Soc. J.*, **1**, 357, 1930; Hart, Fuller, Lusby, *ibid.*, **6**, 80, 1934.

[2] Eustis, *Proc. Boston Soc.*, **7**, 218, 1880.

[3] *Ann. d. Physik*, **65**, 849, 1898.

to pass over into one in which the fundamental predominates. It is possible to study the phenomena of resonance in strings by using for the interrupted current in the magnet, that obtained from a contact breaker on an auxiliary wire similarly maintained, or by using a rotary make-and-break. By varying the length of the auxiliary wire or the speed of the rotary make-and-break, one can impress different frequencies on the wire, and study the forced vibrations which ensue. The results verify the phase relationships deduced theoretically (in Chap. II, p. 52) for frequencies near resonance. When the natural frequency of the wire is a little less than that of the impressed force, the phase difference is $\pi/2$. and the amplitude of the forced vibration becomes very large.

Transverse Vibration of Bars. Just as we obtained a general differential equation for the transverse vibration of a stretched string by equating the product mass $\times$ acceleration of any element to the restoring force due to the tension in the string, so we can obtain a differential equation for the bent bar vibrating transversely ; only now the restoring force is the bending moment which is a *fourth* order function of the displacement at the point in question. The method of deriving the equation follows Barton's [1] simplification of Rayleigh's [2] treatment. If an isolated force f act on the bar at right angles at any point distant x from the clamped end, it will be equivalent to a bending moment fx about the end. Now, owing to the mutual action of the molecules of the bar, the application of such a force on any part of it will introduce forces on the other parts, or, in other words, it is impossible to bend one small part of the bar by such a single force without bending the whole bar more or less. The effect then of bending down one end of such a clamped horizontal beam, is to introduce various bending moments at various points of the beam, so that the total bending moment about any point can be represented in the form of a sum of products of forces into their distances from the point in question; $B = \Sigma fx$, or $\dfrac{\partial B}{\partial x} = \Sigma f = F$, say. Further, owing to the mutual interaction of particles of the bar, the value of F at any point will depend on that at neighbouring points, so that F continuously increases along the bar. Writing F as a function of x, and differentiating again :—

$$\frac{\partial^2 B}{\partial x^2} = \frac{\partial F}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

[1] *Phil. Mag.*, **14**, 578, 1907. [2] *Sound*, **1**, 267, 1896.

It is assumed that the bar is uniform, and free from forces parallel to its length; and the vibrations so small that rotary effects can be neglected. Consider an element δx of the rod (Fig. 42) having a force F at right angles to its length, which "shears" one face; the corresponding force on the other face will be $F + \frac{\partial F}{\partial x}.\delta x$. The difference between these forces, i.e., $-\frac{\partial F}{\partial x}.\delta x$ represents the force tending to straighten the element if the impressed forces were suddenly removed, and may therefore be equated to the mass × acceleration of the element.

We now require an expression for B in terms of the configura-

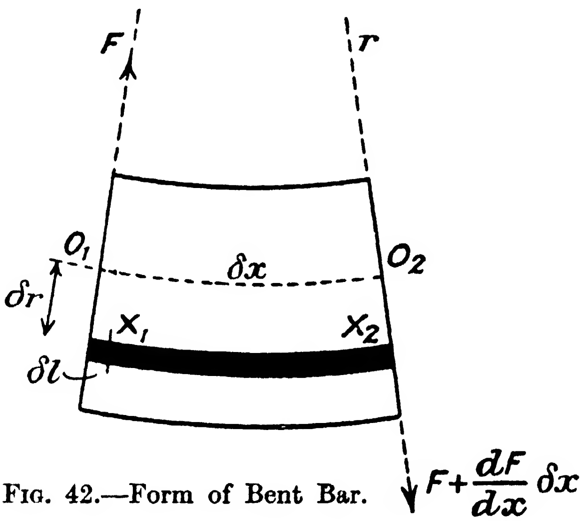


FIG. 42.—Form of Bent Bar.

tion of the element. Let it be bent into an arc of a circle of large radius r, so that we may take $\frac{\partial^2 y}{\partial x^2}$ to equal the curvature. Owing to the bending, the original slice X_1X_2 (Fig. 42) at a distance δr from the "neutral surface" O_1O_2 (which remains unchanged in length) becomes stretched by the fraction $\frac{\delta l}{\delta x} = \frac{\delta r}{r}$ (by similar triangles, Fig. 42). This is the strain on the slice; the stress is found by multiplying this by Young's modulus $\mathbf{E}$. Then the total force on this slice of cross section δS, becomes $\mathbf{E}\frac{\delta l}{\delta x}.\delta S = \mathbf{E}\delta r\frac{\partial^2 y}{\partial x^2}\delta S$. Multiplying this by δr to obtain the bending moment about the neutral surface, and integrating over the whole area of the element

$$B = \mathbf{E}\frac{\partial^2 y}{\partial x^2}\Sigma(\delta r)^2\delta S = \mathbf{E}\kappa^2\frac{\partial^2 y}{\partial x^2}S \quad . \quad . \quad . \quad . \quad (46)$$

where κ is written for the "spin-radius" of the section about the neutral surface. Returning to (45), the restoring force on the element

$$= -\frac{\partial F}{\partial x}.\delta x = -\frac{\partial^2 B}{\partial x^2}.\delta x = -\mathrm{E}\kappa^2\frac{\partial^4 y}{\partial x^4}S\delta x.$$

Equating this to the mass $\times$ acceleration of the element $S\rho\delta x\frac{\partial^2 y}{\partial t^2}$ we obtain finally :—

$$-\frac{\partial^2 y}{\partial t^2} = \kappa^2\frac{\mathrm{E}}{\rho}\frac{\partial^4 y}{\partial x^4} = \kappa^2 V^2\frac{\partial^4 y}{\partial x^4} \quad . \quad . \quad . \quad . \quad (47)$$

where V is the speed of longitudinal waves in the bar.

This differential equation combined with the appropriate end conditions will enable us to calculate the frequency of the bar vibrating in fundamental or partial modes. The possible end conditions are :—

(1) Clamped end. Never any displacement; slope of bar always *nil*; $y = 0$, $\frac{\partial y}{\partial x} = 0$.

(2) Free end. Displacement and slope arbitrary, but no force beyond the end to produce curvature or shear. $\frac{\partial^2 y}{\partial x^2} = 0$, $\frac{\partial^3 y}{\partial x^3} = 0$.

(3) End supported on an edge. No displacement, but slope may vary, whereas curvature is nothing, as at a free end. $y = 0$, $\frac{\partial^2 y}{\partial x^2} = 0$. Various combinations of "ends" are possible, but only two are of practical importance, i.e., (1) one end free and the other fixed, (2) both ends supported.

On the assumption of simple harmonic and undamped motion, a solution of (47) is :—

$$y = a \sin(\omega t + \delta).$$

giving :—

$$\frac{\partial^4 y}{\partial x^4} = \frac{\rho\omega^2}{\mathrm{E}\kappa^2}y = \beta^4 y,$$

so that :—

$$y = A'e^{bx},$$

where b is a root of $b^4 = \beta^4$, i.e. :—

$$b = \pm\beta \text{ or } \pm i\beta,$$

whence :—

$$y = (A\cos\beta x + B\sin\beta x + C\cosh\beta x + D\sinh\beta x)\sin(\omega t + \delta).$$

Bar Clamped at One End. Taking the fixed end as the origin of x, and the free end at $x = l$, we find the multiplier of $\sin(\omega t + \delta)$ in the last equation to be given by values of βl which satisfy :—

$$\sec \beta l = - \cosh \beta l.$$

The readiest way of solving this equation is to plot the graphs of $y = - \cosh \beta l$, and of $y = \sec \beta l$; the solutions will then be given by the values of the intersections of the two curves. From such values the corresponding relative frequencies can be found, since these are proportional to β^2.

TABLE OF βl.

1·88	4·69	7·85	11·00	14·14	17·28	etc.

The fundamental of such a bar is therefore given by

$$\beta l = 1{\cdot}88$$

or $$\sqrt{\frac{\rho}{\mathrm{E}}}\cdot\frac{\omega l^2}{\kappa} = (1{\cdot}88)^2 \text{ ; whence } n = \frac{\omega}{2\pi} = \frac{3{\cdot}53\kappa}{2\pi l^2}\sqrt{\frac{\mathrm{E}}{\rho}}.$$

The frequency is inversely as l^2. In other words $n\lambda$ is no longer a constant ; the velocity of the transverse waves in a bar is a function of the frequency. This is why we cannot employ Fourier analysis to find the amplitude of the partials. The ratios $\frac{\beta^2}{\beta_1{}^2}.n$ show that the partial tones are inharmonic ; their frequencies are not in the ratio of the natural numbers as on a string, nor are the partial nodes equidistant. For this reason vibrating bars or reeds are rarely used alone to form musical instruments ; without a resonator to qualify them, their tones are harsh, and in fact are described as "reedy." Note that a bar fixed or free at both ends has a different series of tones, given by :—

$$\sec \beta l = \cosh \beta l,$$

the fundamental being 2·67 octaves higher.

Bar Supported at Both Ends. Considering again the amplitude expression :—

$$A \cos \beta x + B \sin \beta x + C \cosh \beta x + D \sinh \beta x.$$

$$\text{At } x = 0,\ y = 0 \qquad A + C = 0$$

$$\frac{\partial^2 y}{\partial x^2} = 0 \qquad A - C = 0$$

which reduces the expression to $B \sin \beta x + D \sinh \beta x$.

$$\text{At } x = l,\ y = 0 \qquad B \sin \beta l + D \sinh \beta l = 0$$

$$\frac{\partial^2 y}{\partial x^2} = 0 \qquad -B \sin \beta l + D \sinh \beta l = 0,$$

subtracting we find $\sin \beta l = 0$. Therefore βl is a multiple of π. The nodes of the partials are now equidistant, β has successive values in the ratio 1, 2, 3, etc., as for a vibrating string, but the partials have frequencies proportional to β^2, and therefore in the ratio 1, 4, 9, etc.

Experimental Work on Bars, especially Reeds. The bar may be encouraged to produce these partials by supporting it symmetrically in aliquot parts. Thus a vibration having three nodes with antinodes at the ends may be produced by supporting the bar at one-quarter of its length from each end (see Fig. 43).

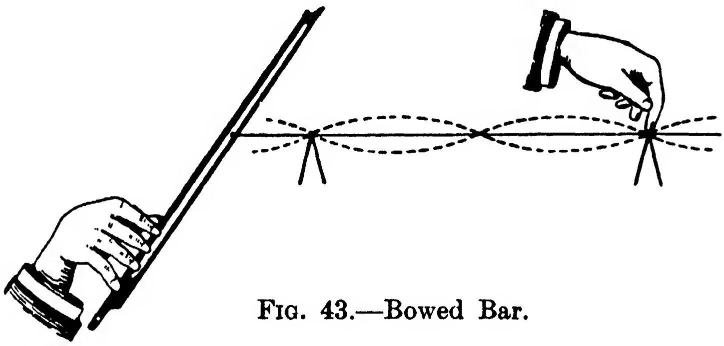


FIG. 43.—Bowed Bar.

The vibrations may be excited by bowing on the end, or by striking with a rubber-covered hammer on the overhanging portion, the bar being prevented from rising off the supports, by pressing it with the finger over the distant support.

Apart from the methods already described for examination of transverse vibrations, the position of the nodes on a flat-topped bar may be roughly indicated by sprinkling sand on the bar.

The use of strips of metal or wood, clamped at one end, in a number of wind instruments, where however the tones of the reeds are generally subordinated to those of a resonator, has led to this form of bar being widely used in experiments. Out of a number of researches on the tones of bars and reeds, having no special interest save to verify the theory outlined above, a few are worthy of selection.

Rayleigh[1] inquired whether it was not possible by means of a single force applied at some point along the bar to bend it statically into the form which it assumed when vibrating in its fundamental

[1] *Sound*, **2**, 254, 1896.

mode. He decided that a single force at one-quarter of the length from the free end would be capable of doing this, but a nearer approximation by Garrett [1] gave one-fifth as the point of application of the force. This was proved by means of instantaneous photographs. Hartmann-Kempf obtained "resonance-curves" for steel reeds by exciting transverse vibrations in them through the medium of an electro-magnet, actuated by alternating currents of various frequencies. When one or other of the natural frequencies of the reed approached that of the intermittent magnetic force, a large response of the reed was, of course, obtained. By means of a set of calibrated and numbered reeds arranged in front of an electro-magnet, he constructed an instrument which indicates the periodicity of the A.C. sent through the electro-magnet, by the reed or reeds which responded.[2] Relf and Cowley [3] have examined the natural frequencies of thick rods under applied forces of varied frequency, the forces in their experiments being mechanical. The end of the rod was given transverse oscillations through a crank movement connected with the hub of an electric motor, rotating at known speeds. The agreement with theory was quite good.

The Tuning Fork. This apparatus which now serves as a precision standard of frequency may be regarded as a bent steel bar, vibrating transversely. The effect on a straight bar, vibrating with two nodes, of gradually bending up the ends, is to bring the two nodes closer together, until, on reaching the familiar U shape of the tuning fork, the nodes lie close together at the centre of the bend where the stem of the fork enters. The two parallel bars vibrate with a frequency about two-thirds of that corresponding to the original partial. The rather complex form of the tuning fork does not lend itself to rigid mathematical treatment, but from what has been calculated for the straight bar we should expect the fundamental tone to be inversely as the square of the length, and directly as the breadth in the plane of bending. The size of the fork is determined empirically by the maker, the final tuning adjustment being done by shaving the ends of the prongs. It is very necessary that both prongs should be exactly alike. Mer-

[1] *Phil. Mag.*, **8**, 581, 1904.

[2] *Ann. d. Physik*, **13**, 124 and 271, 1904; **36**, 74, 1911, and *Phys. Zeit.*, **11**, 1183, 1910.

[3] *Phil. Mag.*, **48**, 646, 1924; Obata, *Frank. Inst. Jour.*, **203**, 647, 1927; Thomas and Warren, *Phil. Mag.*, **5**, 1125, 1928.

cadier [1] examined the tones of a number of similar forks of different size, and his results satisfy a formula of the type $n = k.\frac{b + b_0}{(l + l_0)^2}$, where b and l are the breadth and length, k, b_0 and l_0 being constants (k contains the velocity of longitudinal waves and a "form-factor"). He also showed that the thickness of the prongs perpendicular to the direction of vibration was without influence on the pitch. The prongs of low-pitched forks are accordingly made thin and long, those for high pitch, very short and thick. The pitch of the tuning fork is subject to a small negative temperature coefficient; Rudolf König [2] found the change to be of the order of 1 in 10,000 per degree Centigrade. Magnetization of the steel causes a very small increase in the frequency. The tone of the fork may be elicited by bowing, by striking, or by pressing the ends together, and unless the excitation is done violently, the tone elicited consists almost entirely of the fundamental, with a little admixture of high overtones which are of course inharmonic. The substantial construction and considerable elasticity of the fork discourages the production of the overtones, while some of these are rapidly damped because their nodes lie close together in the material. Chladni [3] found the following tones in a fork weakly excited: 128 (fundamental), 793, 2,340, 4,480, 7,824. The predominance and isolation of the fundamental, as well as its permanent quality, are the advantages which this instrument possesses as a standard frequency source, and may be still further ensured by mounting the fork on a resonance-box, containing a body of air, of size to vibrate with the fundamental frequency. The latter possesses overtones in the harmonic series, so that it will not respond to the inharmonic partials of the fork, if any are present. Nevertheless, under special conditions, a fork may appear to produce other tones than those mentioned above. The octave of the fundamental pitch of the fork may in general be heard. As the transverse overtones of the fork, as well as possible longitudinal or torsional tones would be inharmonic to the fundamental, Lindig [4] concluded that these harmonic tones were formed in the air itself,

[1] *Comptes Rendus*, **76**, 1198 and 1256, 1874, and **79**, 797, 1876.

[2] *Ann. d. Physik*, **9**, 394, 1880.

[3] *Akustik*, 1802. *See also* Struycken, *Ann. d. Physik*, **23**, 643, 1907; Waetzmann, *Phys. Zeits.*, **10**, 409, 1909; Stefanini and Gradeniger, *N. Cimento*, **15**, 131, 1908.

[4] *Ann. d. Physik*, **11**, 31, 1903; *see also* Derjaguin, *Soviet Phys. Zeits.*, **3**, 574, 1933.

were in fact terms which arise according to the calculation of Helmholtz when the relation between the restoring force on an air particle is no longer proportional to its displacement simply, but involves the square of the displacement (see Chap. II, p. 62). The vibrations communicated to the air particles are accordingly asymmetric, owing to the construction of the fork. To produce subharmonics of the fork, the latter is struck and the steel stalk pressed lightly on a table. Owing to intermittent contact between the steel and the wood, the former is out of contact for two or more periods of the fork. Seebeck by this explanation ascribed the sounds to successive taps on the table. By using a metal plate in place of the wooden table, and a string galvanometer connected in series with the contact, this conjecture has been verified.[1]

Electro-magnetically driven Tuning Fork. In order to maintain the vibrations of the tuning fork a source of electrical energy is necessary. It is usual to drive the fork by an electro-magnet, the current in which is made and broken intermittently by the motion of the prongs. The contact may be of the platinum point type, as in the Crompton-Robertson Vibrator or may consist of a short tungsten wire fixed to one of the prongs and dipping into mercury, and a similar form of drive may be used to keep a steel reed in vibration. The frequency of the bar or fork may be slightly altered from that of the free vibrations by this form of drive. This may be shown by applying, as Hartmann-Kempf[2] did, an independently interrupted or alternating current to the

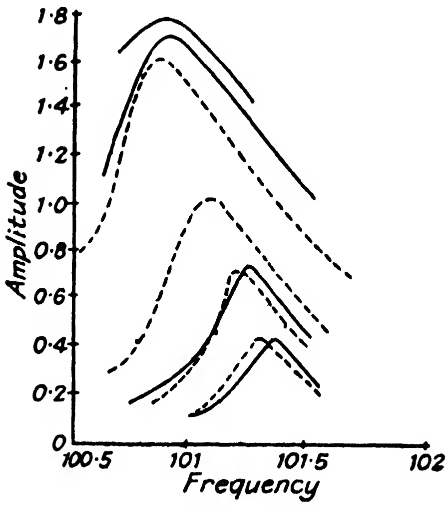


FIG. 44.—Resonance Curves of Electro-Magnetically maintained Tuning-Fork (Hartmann-Kempf).

[1] Knapman, *Roy. Soc. Proc.*, **74**, 118, 1904. *See also* Edelmann, *Phys. Zeits.*, **8**, 451, 1907 and **10**, 645, 1909; Sizes and Massol, *Comptes Rendus*, **145**, 872, 1907; **146**, 24, 1908; **148**, 1318, 1909; **150**, 1746, 1910; **151**, 437, 1910; Anderson, *Phys. Rev.*, **21**, 692, 1923; Bond, *Phys. Soc. Proc.*, **36**, 340, 1924; Idei, *Sci. Rep. Tohoku*, **16**, 436, 1927; Yamashita and Aoki, *Sci. Mem. Kyoto*, **16**, 367, 1933.

[2] *Ann. d. Physik*, **13**, 124 and 271, 1904. *See also* Rossi, *N. Cimento*, **16**, 97, 1908; Appleton, *Phil. Mag.*, **47**, 609, 1924; Mallett, *Phys. Soc. Proc.*, **39**, 334, 1927; Miles, *J. Sci. Inst.*, **5**, 152, 1928.

electro-magnet, keeping constant (maximum) current strength, but varying the frequency of the magnetic impulses on either side of that natural to the fork. From the photographic traces provided by mirrors on the fork, Hartmann-Kempf obtained the resonance peaks (for a number of current strengths) shown in Fig. 44, the dotted lines for interrupted current, the continuous lines for A.C. The effect of increasing the current strength is to lower the frequency of the fork, and to push the resonance maxima to the left of the figure.

Other experimenters have noticed that the natural frequency of such a vibrating system falls with the amplitude, approximately with the square root of the amplitude, which of course is governed by the attraction exerted by the magnet, and therefore by the strength of the current. Dadourian [1] pointed out that with this type of maintenance, increase of amplitude first lowered and then

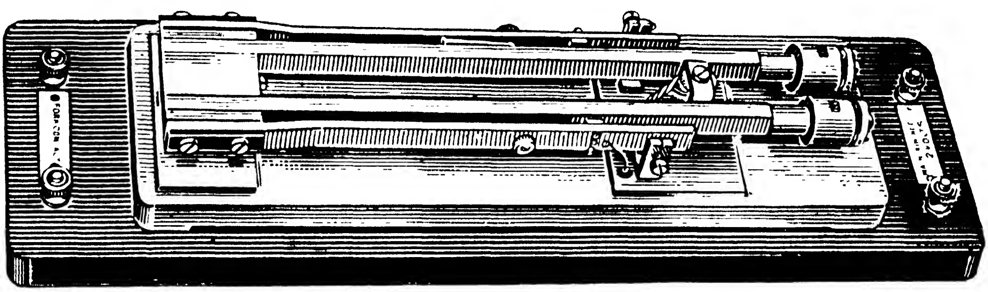


FIG. 45.—Tinsley Standard Tuning Fork.

raised the frequency, so that there is a certain minimum frequency corresponding to a certain amplitude, at which the fork should be run.

In order to use a fork as a precision time standard, the contacts must receive careful consideration, the "cut-off" must be very precise. Further, the fork tends to couple its vibrations, viâ the stem, to those of its surroundings, and if allowed to do so, these react on the frequency of the fork. Consequently the supports of the fork must be massive. These desiderata are incorporated in the Tinsley fork developed from that of Wood and Ford [2] (Fig. 45).

[1] *Phys. Rev.*, **13**, 337, 1919. *See also* Miller, *Phys. Rev.*, **11**, 497, 1918; Campbell, *Phys. Soc. Proc.*, **31**, 84, 1918; Lloyd Jones, *Phys. Soc. Proc.*, **34**, 66, 1921; Wood, *Journ. Sci. Inst.*, **1**, 330, 1923; Dye, *Journ. Sci. Inst.*, **1**, 340, 1923; Searle, *Phil. Mag.*, **1**, 738, 1926.

[2] *Journ. Sci. Inst.*, **1**, 166, 1923; *see also* Harrington, *J. Opt. Soc. Amer.*, **17**, 224, 1928 and **18**, 89, 1929; Holweck and Lejay, *Comptes Rendus*, **188**, 1541, 1929.

The two long bars form the fork proper, the stem having been replaced by a steel block to which they are fixed. The shorter bars merely support the contacts, which incorporate a stop device, which regulates the travel of the platinum points, keeping the time of contact constant from period to period, as long as the current remains constant. Loads permitting a slight adjustment are screwed on to the ends of the blades. It is claimed that this fork will remain constant to 1 in 10,000.

Valve-maintained Tuning Fork. The development of the triode valve has provided another possible way of utilizing electric energy to maintain a fork. A circuit on this principle was devised about the same time by Eccles [1] and by Abraham and Bloch.[2] The function of the valve is to magnify in the plate circuit, oscillations taking place in the grid circuit, when these are inductively coupled. It is arranged that the two prongs of the fork shall vibrate in front of two coils, one in each circuit, so that, on exciting the fork, the movement of the iron of one prong sets up induced currents in the grid circuit. The magnified oscillations in the plate circuit then act magnetically upon the other prong, with sufficient vigour to overcome the natural damping of the fork, and to maintain the vibration at the expense of the electric energy. For the coils, the receivers from a pair of headphones, with caps and diaphragms removed are useful. Valve-maintained vibrations will be further discussed in the next chapter.

The Phonic Wheel. This device, due to Rayleigh [3] and La Cour,[4] enables the speed of rotation of a motor to be kept at a constant rate controlled by a tuning fork, and is very useful for driving a stroboscopic disc. In principle, such a motor consists of a number of iron studs rotating past a corresponding number of electro-magnets. These magnets are energized only at certain definite and constant intervals. If the studs happen to lie opposite the soft iron magnets at the instant of their excitation we have a completed "magnetic circuit," and the studs will tend to move

[1] *Phys. Soc. Proc.*, **31**, 269, 1919. *See also* Butterworth, *Phys. Soc. Proc.*, **32**, 345, 1921; Dye, *Roy. Soc. Proc.*, **103**, 240, 1923, and Hodgkinson, *Phys. Soc. Proc.*, **38**, 24, 1923; Zahradnicek and Zak, *Ann. d. Physik*, **12**, 665, 1932; Dye and Essen, *Roy. Soc. Proc.*, **143**, 285, 1934.

[2] *J. de Physique*, **9**, 225, 1920.

[3] *Nature*, **18**, 111, 1878; *Phil. Mag.*, **13**, 331, 1907. *See also* Carrière, *J. de Physique*, **2**, 337, 1921; *Ann. de Physique*, **17**, 123, 1922; Wood and Ford, *Journ. Sci. Inst.*, **1**, 166, 1923.

[4] *La Roue Phonique*, 1878; *see also* Gall, *J. Sci. Inst.*, **6**, 18, 1929.

past the magnets at the same rate. If, owing to lagging or gaining by the system of studs, the excitation of the magnets occurs when the studs are a little behind or in front of the magnets, forces of attraction will be brought into play, tending to restore the *status quo.* The studs are formed on the rotor by cutting equidistant slots in a plain cylinder of soft iron. Corresponding soft iron bars line the hollow cylindrical stator, and these are magnetized by a single coil of wire, energized by a battery passing through a make and break on the fork. Such a phonic motor is not self-starting, therefore it must be run up to synchronous speed by hand, or by an auxiliary motor. By counting the revolutions of the motor when this condition has been attained, the frequency of the controlling fork may be accurately obtained.

Musical Instruments involving Bars or Forks. Many musical instruments involve a reed in their sound-producing parts, but in the majority the note of the reed is subordinated to that of the column of air to which it is coupled, so that these do not merit treatment here. Reeds, practically unassisted, are used in the harmonium. They are kept in vibration by the pressure of air which escapes from a reservoir, through an orifice which they cover. The action of the blast of air is rather like that of the bow of a stringed instrument. The pressure forces the free end of the reed outwards until it acquires sufficient potential energy to slip back past the stream, and then it is caught up again after passing its undisplaced position. The reed may be slightly smaller than the orifice, through which it can therefore pass on its return journey, or the orifice may be merely a small hole in a plate, against which the reed presses in its normal position; in the former case it is called a " free reed," in the latter, a " beating reed." The American organ employs free reeds, through which the air is sucked instead of being blown. Possibly because the discontinuities in the action are less abrupt, the latter arrangement produces a tone less rich in upper overtones, and so less harsh.[1]

Instruments containing " supported bars " are to be found in the " toy-shop " of the modern orchestra. The xylophone contains a number of bars of metal or wood, supported at two points near the ends, and struck with a hammer. In the *glockenspiel* a number of metal tubes are supported on a frame by strings passing through a point near the upper end. On being struck by a hammer near the point of suspension they give out a note intended to

[1] Taber Jones, *Rev. Sci. Inst.*, **5**, 192, 1934.

resemble that of a bell. They should approximate to "free-free bars," but the quality of their notes is complicated by the tones of the contained column of air; this interesting case appears not to have been treated experimentally. The note of the triangle—a continuous rod of steel of this shape, struck with a straight rod—contains many neighbouring partial tones forming an indefinite noise, and is not tuned to any definite pitch.

Coupled Vibrations of Bar and String—Melde's Experiment. Transverse vibrations may be maintained in a string by attaching one end to a bar of the same pitch, in which the vibrations are maintained by bowing or by electric means. This was first accomplished by Melde,[1] who gave the apparatus the following form (Fig. 46).

A massive tuning fork (n about 100) is securely clamped ver-

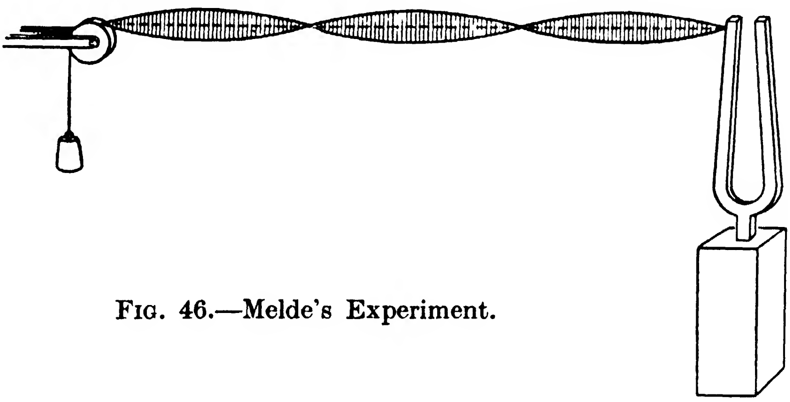


FIG. 46.—Melde's Experiment.

tically by its stem. A white thread passes from the end of one prong to a tension adjuster in the form of a screw-clamp some yards away, or in the modern form, to a pulley, over which the string passes to a scale-pan containing weights. If the fork is excited, and the length and tension of the string adjusted, presently a condition will be reached, in which one of the partials of the string coincides in frequency with that of the fork. When this state of affairs is reached, the string will exhibit strong resonant vibrations with nodes and anti-nodes corresponding to the particular partial elicited. These coupled vibrations are possible not only with the fork vibrating in a direction at right-angles to the string, but also with the fork's vibrations in the direction of the length of the string, as depicted in Fig. 46. In the latter case, as Melde showed, the frequency of the tone in the string is only half that of the fork, for every time that the string passes to its maximum displacement

[1] *Ann. d. Physik,* **109**, 192, 1859, and **111**, 513, 1860, and **24**, 496, 1885.

the prong moves to the left releasing the tension a little, while every time the string returns to its medial position, the prong moves to the right tightening the string again. By measuring the distance l between two nodes, we can in theory estimate the note in the fork, for that of the tone in the string is $\frac{1}{2l}\sqrt{\frac{F}{m}}$ (see p. 74), the value of F in dynes being obtained from the sum of the weights hung on the string. To make the nodes visible, a black cloth is hung behind the white string. When the adjustment for resonance is not exact, pseudo-nodes and antinodes will be observed, due to vibrations in the string "forced" by the fork. The resonance curves are always flat, for the light string accommodates itself to the period of the heavy fork.[1] Moreover in process of accommodation, the note in the string may be any sub-harmonic of that of the fork.[2] This makes Melde's experiment unreliable for estimating the frequency of the maintaining fork. Very interesting are the appearances when the fork is at an oblique angle to the string. These resemble Lissajous' figures as they are made up of vibrations corresponding to both cross and parallel types. In actual experiment, the nodes are only average positions at which the positions of the string in the two extreme phases cross. By stroboscopic examination Raman[3] showed that this crossing point may wander about between two successive loops.

Another point of subsidiary interest is the apparently continuous rotation of the pulley in one direction which usually accompanies the motion. Taber Jones[4] has shown that the motion of the pulley really consists of a series of jerks, and that during the period there are instants when the string will slip past the pulley. When a string is about to slip on a curved surface, there is a definite ratio between the tensions in the two portions of the string where they leave the surface, which ratio is dependent on the angle between the tangents. The string slips when the ratio of these two tensions exceeds a certain limit. This ratio reaches a maximum when the string in the loop next the pulley is uppermost, i.e.,

[1] Roberts, *Phil. Mag.*, **23**, 931, 1912. *See also* Kost, *Verh. Deut. Phys. Ges.*, **21**, 774, 1919; Schlesinger, *Zeits. f. tech. Phys.*, **12**, 33, 1931.

[2] Raman, *Phil. Mag.*, **24**, 513, 1912; *Phys. Rev.*, **35**, 449, 1912, and **5**, 1, 1915; Ghosh, *Ind. Assoc. Proc.*, **9**, 145, 1925.

[3] *Phys. Rev.*, **32**, 309, 1911, and **4**, 12, 1914. *See also* Taber Jones and Marion Phelps, *Phys. Rev.*, **10**, 541, 1917.

[4] *Phys. Rev.*, **11**, 150, 1918, and **27**, 622, 1926. *See also* Raman, *Phys. Rev.*, **32**, 307, 1911; Morton and McKinstry, *Phys. Rev.*, **29**, 192, 1927.

when the obtuse angle between it and the vertical part of the string leading to the scale-pan is greatest. It is at this part of the period that slip takes place; in the rest of the period the pulley is dragged with the string always in the one direction.

Vibrations under a double forcing action may be executed in the string if it is stretched between two tuning forks of different frequencies (n_1 and n_2). A large number of possible modes corresponding to $j_1n_1 \pm j_2n_2$, where j_1 and j_2 can have any integral values, have been detected. Coupled vibrations similar to Melde's form may be observed on rocking one end of a string by a crank of a reciprocating engine,[1] or by fixing a reed to one prong of a tuning fork.[2]

Decrement of Transverse Vibrations. All vibrations of solids of whatever type are damped by internal friction, but as transverse vibrations of solids involve a considerable motion of the surrounding fluid medium, external friction and resistance is a much more potent cause of the decay in amplitude, which ensues when the system is allowed to oscillate unmaintained. In order to estimate the rate of decay, the ratio of the amplitudes in two successive periods is obtained, a quantity which is known as the decrement. It is the logarithm of this quantity ("log. dec." Δ) which usually figures in calculations. In slowly damped motions, it requires a number of periods to elapse before an appreciable diminution of amplitude takes place. Thus if we measure the amplitude before and after a time equal to p periods :—

$$\frac{a_0}{a_1} = \frac{a_1}{a_2} = \frac{a_2}{a_3} = \frac{a_3}{a_4} = \frac{a_{p-1}}{a_p} = e^{\Delta},$$

$$\log_e \frac{a_0}{a_p} = p\Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

a formula which allows us to calculate Δ. If the period be accurately known, it is sufficient to measure, by the microscope, the amplitude at the beginning and end of any accurately measured time interval. Otherwise, a trace by photographic or other means of the vibration must be obtained, and the mean of a number of amplitude ratios over, say, 10 periods measured from the trace on

[1] Fleming, *Phys. Soc. Proc.*, **26**, 61, 1913; Raman, *Phys. Rev.*, **14**, 440, 1919.

[2] Lissajous, *Ann. Chim. Phys.*, **30**, 385, 1850; Oliver, *Phil. Mag.*, **14**, 318, 1932.

the paper. Valuable work was done by Hartmann-Kempf[1] by this method on electrically-driven reeds and forks.

In determining the dragging effect of a surrounding medium on vibrations, the decrement *in vacuo* is taken as standard as representing the decay due to internal friction alone. In this case, from equation (26) (p. 49) we have for the exponent representing the decay in amplitude $\alpha = \frac{\mu}{2m}$. The log. dec. being reckoned from half period to half period,

$$\Delta = \frac{\alpha T}{2} = \frac{\mu T}{4m} = \frac{\pi}{2}\frac{\mu}{\sqrt{mk}}$$

approximately. The surrounding medium may increase both the effective mass and the coefficient of viscosity. The increase in the mass to be set in motion by the vibration will, apart from the damping, lower the natural frequency of the vibrating solid; this is most marked when a bar or wire is maintained in vibration in a liquid. The theory is complicated by the fact that, as Hartmann-Kempf found for a tuning fork, the frequency changes somewhat as the amplitude diminishes, and along with this goes a diminution of the decrement. The correct delineation of the curve of decay for a tuning fork in air is of importance in connection with experiments on the sensitivity of the ear. A number of investigators have determined the decrement by obtaining resonance curves of the vibrating system, a method adapted from alternating-current technology.

The damping of a wire has been measured by Florence Chambers[2] by impressing on it the oscillations furnished by the plate circuit of a valve containing self-inductance and capacity. Instead of allowing these impressed oscillations to pass through an electromagnet placed near the wire, the current itself passed through the wire which was placed in a permanent magnetic field. Then the alternating current in the wire caused transverse oscillations of the same period in virtue of the deflection of a conductor, carrying a current in a magnetic field. From equation (25) (p. 49), we see that the amplitude of the forced vibration is

$$A^2 = \frac{F^2}{\mu^2 p^2 + (k - mp^2)^2}.$$

A is a maximum when $\mu^2 p^2 + (k - mp^2)^2$ is a minimum. Differ-

[1] *Ann. d. Physik*, **36**, 74, 1911. [2] *Phil. Mag.*, **48**, 636, 1924.

entiating this denominator with regard to p^2 and putting the result equal to zero, we find the minimum when

$$p^2 = \frac{k}{m} - \frac{1}{2}\frac{\mu^2}{m^2}.$$

This value of p is in fact the resonant frequency of the system when damping is included. The ratio $\frac{A^2}{A^2_{max}}$ was measured for different applied frequencies p by altering the inductance and capacity, and so the value of μ, the damping coefficient could be approximately found. It varied from 0·001 to 0·003, with the character of the sonometer-bed on which the wire was stretched. By a similar method Martin [1] found the decrement to vary inversely as the square root of the frequency of the natural tone of the wire, but that it was otherwise independent of the length or tension. Unexpected difficulties presented themselves when he used large amplitudes in order to render the observation of the amplitudes more convenient. Apart from the anomalous resonance peaks (cf. p. 114), observation in two directions at right angles showed that a string plucked or magnetically excited at large amplitudes possessed two slightly different frequencies, which loosely coupled together in the style of a Lissajous figure made up the inharmonic vibration of the string.

Wires and Bars in a Liquid. The lowering of pitch of a tuning fork when plunged into a liquid was mentioned by Chladni [2] in 1802. Auerbach [3] commenced an extensive study ; in his earlier experiments the vibration was not maintained, the lowering of tone produced in a vibrating wire by plunging it from air into water was estimated at 1·11 to 1·18, depending on the frequency. Since then, the subject has become the especial province of women physicists. Experiments in which the body was maintained in vibration and the frequency determined objectively were begun by Lizzie Laird [4] (using steel strings), and continued by Mary Northway and Mackenzie [5] (with steel bars) using a number of liquids. The arrangement for maintaining the vibration was similar to the electro-magnetic device of Eustis (see p. 106). The intermittent current through the electro-magnet also operated a style writing on a revolving drum (cf. Regnault's apparatus, Chap. I, p. 7), so

[1] *Ann. d. Physik*, **77**, 727, 1925.
[2] *Physik*, 1802.
[3] *Ann. d. Physik*, **3**, 157, 1878.
[4] *Phys. Rev.*, **7**, 102, 1898.
[5] *Phys. Rev.*, **13**, 145, 1901.

that the frequency of the intermittent current (= to that of the wire or bar) could be counted. The latter experimenters state that the density has considerable influence, but the viscosity of the liquid has a small effect on the period. The lowering is directly proportional to the width of the bar, but inversely as the thickness. It appears then that the string or bar is loaded with a column of liquid, proportional to the surface which it presents in vibration to the liquid. The reason for the diminished lowering when the thickness, i.e., the dimension in the plane of vibration, is large, is rather obscure; possibly it is connected with the fact that the relative mass of the liquid to that of the bar is less in this case. The statement in the penultimate sentence was deduced theoretically by Stokes, and later by Kalähne[1] who gives a formula in the form $\frac{n'}{n_0} = \left(1 - \frac{\rho'}{2\rho}\right)$, ρ' and ρ being the respective densities of solid and liquid medium, n' the frequency in the liquid, n_0 in vacuo.

If the resistance opposed by the liquid is very great, $n' = 0$, and the motion is no longer periodic. At or above this "critical damping," the wire, if plucked, will return to equilibrium without oscillation, a fact which may be demonstrated with a wire stretched in a bath of glycerin.

[1] *Ann. d. Physik,* **45**, 657, 1914, and **46**, 1, 1915. *See also* Ives, *Phil. Mag.,* **21**, 742, 1911; Giovanna Mayr, *N. Cimento,* **26**, 175, 1923.

Further References: (violin) Abbott, *Acoust. Soc. J.,* **7** 11, 1935; Backhaus, *Zeits. tech. Phys.,* **17**, 573, 1936, and **18**, 98, 1937; Saunders, *Acoust. Soc. J.,* **9**, 81, 1937; Meinel, *Zeits. tech. Phys.,* **19**, 297, 1938; Jarnak, *Frank. Inst. J.,* **225**, 315, 1938; (pianoforte) Hart, Fuller and Lusby, *Acoust. Soc. J.,* **6**, 80, 1934; Ghosh, *ibid.,* **7**, 27, 1935, and **8**, 254, 1936; Grützmacher and Lottermoser, *Akust. Zeits.,* **1**, 49, 1936; Vierling, *Zeits. tech. Phys.,* **18**, 103, 1937.

CHAPTER FIVE

MEMBRANES AND PLATES

We now turn to the transverse vibrations of bodies having extent in two dimensions, and just as we considered the transverse vibrations of the theoretically one-dimensional bodies under two headings, i.e., strings and bars, so we divide the two-dimensional bodies into membranes in which the restoring force is the tension alone and plates, where the restoring force is the bending moment. The former must have a fixed boundary along which the tension is applied, and the only mathematically simple and practically important case is that in which the tension is equal all along the boundary.

Supposing displacements to take place in the z direction, while the membrane at rest lies in the x, y plane, the equation of motion, by analogy with that of one-dimensional waves (2), may be written :—

$$\frac{\partial^2 z}{\partial t^2} = V^2\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

where the velocity of propagation of transverse waves is as for strings, $V = \sqrt{\frac{F}{m}}$, but m is now the mass per unit area $= \rho d$ ($\rho =$ density, $d =$ thickness in the z direction).[1] Equation (50) with the boundary condition that $z = 0$ at the boundary of the membrane, may be used to specify the motion at any point (x, y,) of the membrane.

Rectangular Membrane. If a, b are the lengths of the sides in the x, y direction respectively, and the origin of co-ordinates be taken at one corner of the rectangle, our boundary conditions are, $z = 0$, when $x = 0$ or a, and when $y = 0$ or b. These conditions and equation (50) are satisfied by :—

$$z = \sin\frac{j_a \pi x}{a}\sin\frac{j_b \pi y}{b}\sin\omega t \quad . \quad . \quad . \quad . \quad . \quad (51)$$

[1] Cf. Lamb, p. 141.

On differentiation and substitution it appears that :—

$$\omega^2 = V^2\pi^2\left(\frac{j_a^{\,2}}{a^2} + \frac{j_b^{\,2}}{b^2}\right).$$

The series of partials of the membrane accordingly have frequencies given by putting $j_a = 1, 2, 3$, etc., $j_b = 1, 2, 3$, etc., in the formula :—

$$n = \frac{\omega}{2\pi} = \frac{V}{2}\sqrt{\frac{j_a^{\,2}}{a^2} + \frac{j_b^{\,2}}{b^2}} = \sqrt{\frac{F}{4\rho d}\left(\frac{j_a^{\,2}}{a^2} + \frac{j_b^{\,2}}{b^2}\right)} \quad . \quad (52)$$

In consequence of (52) some of the partials are harmonic to each other. Owing to the lower tones all lying close together in the scale, the general note is a noise in which the lowest tone ($j_a = j_b = 1$) predominates. For the other partials, we have nodal lines instead of the nodal points of a string; their equation is given by putting one or other of the amplitude coefficients in (51) equal to zero, i.e., when $\frac{j_a x}{a}$ or $\frac{j_b y}{b}$ is a whole number.

Circular Membrane. Equation (50) can here be employed more readily in the polar (r, ϕ) form :—

$$\frac{\partial^2 z}{\partial t^2} = \frac{V^2}{r^2}\left(\frac{\partial^2 z}{\partial \log r^2} + \frac{\partial^2 z}{\partial \phi^2}\right) \quad . \quad . \quad . \quad . \quad (53)$$

with the boundary condition, $z = 0$, for $r = r_0$, the radius. This leads to a solution involving Bessel functions, and to the frequency of the lowest partial $\frac{\cdot 764}{r_0}\sqrt{\frac{F}{4\rho d}}$.[1] The nodal lines are composed of concentric circles and of radii. All the partials are inharmonic.

The existence of these nodes can be demonstrated, if the membrane in question is large enough, by Chladni's device of strewing fine sand on the membrane; the sand tends to collect in the still places. The membrane may conveniently be excited by tapping with a hammer. By comparing the emitted tones with a sonometer the nineteenth-century workers endeavoured to test the theoretical formulæ. For this purpose, membranes of paper, rubber, skin, etc., were employed, but all these substances possess stiffness in greater or less degree, involving departures from the condition of no vibration in the absence of tension. The difficulty of sustaining a constant tension along the boundary also troubled these experimenters. The nearest approach perhaps to the ideal membrane, and one whose movements may be optically projected, is the soap-

[1] *See* Lamb., p. 146.

film or glycerin-film. Owing to casual changes of density and tension, the natural tones of such membranes are very inconstant.

The formula (52) shows that the lowest partial of a membrane can be made high in the scale by making the tension sufficiently large, the radius and breadth on the contrary small. Membranes under these conditions are often used as accurate recorders of sound, since their resonant range can be placed beyond the average musical note, and at the same time, if the tension is not too great, they are sufficiently sensitive for the purpose. Examples in plenty will appear in the following chapters, in which the forced vibrations of a membrane are thus employed. The Western Electric Co. have made the diaphragm of their reproducing instrument so rigid that it has a natural frequency of 3,000. We have already mentioned the loaded "asymmetrical membrane" of Waetzmann designed to represent the ear-drum.[1]

A membrane of opposite characteristics is the Lumière pleated paper diaphragm, a foot or so in diameter, employed as a loud speaker for amplifying broadcast music; this is made large and fairly thick to give a marked resonance in the bass of the musical scale, and bring out those low notes to which the usual small diaphragm cannot resound.

Drums. It might be thought that this inharmonic series of overtones proper to a membrane would preclude its employment as a musical instrument. As a matter of fact, most types of drum are mere rhythm markers; they are not intended to take part in the harmony. Exceptionally, the kettle-drums, hemispherical shells containing a cavity of air which resonates to the fundamental of a skin stretched over the top, are tuned to a definite pitch. The tuning is done by altering the tension in the membrane, which is struck by a soft hammer at about a quarter of the diameter from one edge; this is the point at which experiment has shown the inharmonic overtones to be most stifled. The bass drum has two skins, one at each end of a cylindrical cavity. These skins may vibrate in or out of phase; only in the latter case when both move inwards together, is the air compressed and rarefied.[2]

[1] *See also* Waetzmann and Moser, *Deut. Phys. Ges. Verh.*, **19**, 13, 1917; Lewschin, *Zeits. f. Physik*, **33**, 155, 1925; Kucharski, *Phys. Zeits.*, **31**, 264. 1930; Franke, *Wiss. Veröfft. d. Siemens-Konzern*, **9**, 157, 1930; Strutt, *Ann. d. Physik*, **11**, 129, 1931; McLachlan, *several papers in Phil. Mag.*, **11** to **15**, 1931–3.

[2] Terada, *Math.-phys. Soc. Tokyo Proc.*, **4**, 345, 1908.

Ghosh [1] has drawn attention to certain Indian drums, in which, owing to progressive decrease of the mass per unit area of the membrane from circumference to centre, the partial tones are made to approximate to a harmonic series. Rayleigh [2] attributes to the drum proper another function besides resonance; it serves to spread its sound more uniformly, whereas an unassisted membrane would concentrate the sound in the direction of the vibration.

Plates. Theoretically the most complicated type of transverse vibration, the tones of plates represent an extension into two dimensions of the case of the transversely vibrating bar. Analysis [3] shows the analogy to hold good in the calculation of the lowest partial tone of a plate, which is directly proportional to the thickness, and inversely as the square of the diameter or other linear dimension, depending on the shape of the boundary.

For a circular plate, clamped at the rim, Lamb [4] deduces $\frac{\cdot 47 Vd}{r^2}$ for the lowest partial, V being the velocity of longitudinal waves in the material. When the material is in the form of a plate, as opposed to a rod, $V^2 = \frac{\mathrm{E}}{(1-\mu^2)\rho}$, where μ here is the ratio of lateral contraction to longitudinal elongation (Poisson's ratio). The upper partials, which are of course inharmonic, differ only in the numerical multiplier.

Experimental work on large plates originated with Chladni,[5] and the method of Chladni's Figures survives to this day. The plates used by Chladni were of brass, and fixed to a central pillar. This makes the central point a node; to obtain a vibration corresponding to a given partial it is sufficient to hold the plate lightly at some point on an appropriate nodal line, and to bow where an antinode intersects the edge. Both square and circular plates can be used, and the nodal lines shown by sand. Types of vibration are shown in Fig. 47. That in such cases, neighbouring segments vibrate in opposite phases was shown by Lissajous,[6] who held a Y-shaped interference tube over a pair of adjacent segments such as A and B; in this position little or no sound can be heard on

[1] *Phys. Rev.*, **20**, 526, 1922.

[2] *Phil. Mag.*, **7**, 149, 1879; *see also* Obata and Ozawa, *Phys. Math. Soc. Japan. Proc.*, **13**, 93, 1931; Hickman, *Acoust. Soc. J.*, **1**, 138, 1929.

[3] Cf. Rayleigh, *Sound*, **1**, 293, 1877.

[4] *Roy. Soc. Proc.*, **98**, 205, 1920.

[5] *Entdeckung. in d. Theorie d. Klanges*, 1787.

[6] *Comptes Rendus*, **40**, 133, 1855.

putting the ear over the main branch of the Y tube, the other ear being stopped. As segment *A* is moving up while *B* is moving down, their motions are in fact in dead opposition of phase, so that the resulting sound waves in the branch tubes completely interfere on combining. The Chladni Figures of plates of other shapes possess academic interest merely. Of special importance is the small thin metal plate clamped at its circular edge, on account of its widespread use in the telephone.

Chladni Figures of steel plates may readily be produced by a method first used by Hort and König.[1] An electro-magnet is held near the plate and alternating current of variable frequency

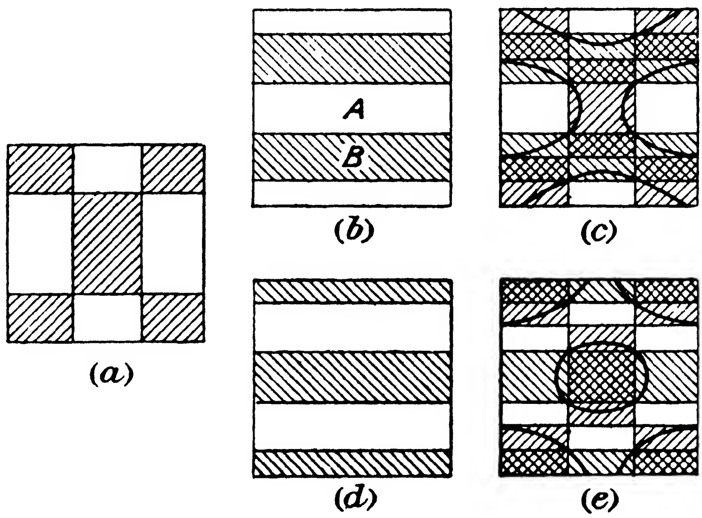


FIG. 47.—Vibrations of Plates (Zeissig).

fed to it. As each overtone frequency is reached the corresponding pattern appears in the dust on the plate. Alternatively a taut wire attached to the centre of the plate may have the alternating current passed through it. A telephone receiver held with the diaphragm horizontal may be dusted while the coils are supplied by the current. " Chattering " of the sand on the plate may be produced under certain conditions (cf. p. 84), but lighter particles may remain in permanent suspension over the plate following the vortex currents which are produced in the air above.[2]

[1] *Zeits. f. tech. Physik*, **9**, 373, 1928; *see also* Roussakoff, *J. de Physique*, **1**, 206, 1930; Andrade and Smith, *Phys. Soc. Proc.*, **43**, 405, 1931.

[2] Flügge, *Zeits. f. tech. Phys.*, **13**, 199, 1932; Colwell, *J. Frank. Inst.*, **213**, 373 and **214**, 199 and **216**, 763, 1932; Marty, *J. de Physique*, **4**, 557. 1933; Dimpfer, *Ann. d. Physik*, **19**, 225, 1934.

Zeissig[1] has given a simple explanation to the shape of the nodal lines on a rectangular plate. He considers the convolutions of the plate to be produced by the superposition of two sets of stationary waves on the plate; each set having linear nodes and antinodes parallel to one pair of edges. This is shown in the figure, where shaded parts of the plate are depressions, and unshaded parts are elevations; on adding the figures in (*a*) and (*b*), we obtain that in (*c*) with nodal lines approximately represented by the thick line. The original tones, in accordance with theory, have frequencies proportional to the thickness, and inversely as the square of the length of the edges. The complex vibration resulting from the superposition changes with the relative phase of the components, so that in general the nodal forms pass through a cyclic change, cf. Fig. 47*e*, formed by the superposition of (*a*) and (*d*).

Telephone Diaphragm. The thin steel plate commonly employed for telephony may be considered as a stage between the theoretical membrane and the theoretical plate, with a decided bias towards the latter. Without entering into all the electrical details it is sufficient to say that the steel diaphragm in the common form of transmitter is placed in the field of a magnet. Aerial waves impinging on the diaphragm cause it to vibrate and to produce changes in the neighbouring magnetic field, by its displacement. Enclosing part of the field is a coil forming part of an electric circuit, in which alternating currents are induced by the changing magnetic field. At a distant point in the same circuit is a similar coil, magnet, and diaphragm forming the receiver, where the reverse process takes place, the electric waves being more or less faithfully reproduced as aerial (sound) waves. Usually a battery is placed in the electric circuit, so that the induced currents are mere ripples on the steady current, but the battery is not essential. By suitable arrangements, either in the direction of amplifying the oscillatory current at the receiving end and so increasing the amplitude of vibration of the diaphragm, or by increasing the size of the latter so as to present a larger vibrating surface, or by adding a horn, or usually by a combination of all these methods, the "loud speaker" is produced, to the end that the ultimate air-waves may be of sufficient intensity to be heard at a distance from the receiver.

[1] *Ann. d. Physik*, **64**, 360, 1898; *see also* Franke, *Ann. d. Physik*, **2**, 649, 1929; Warren, *Phil. Mag.*, **9**, 881, 1930; Bekesy, *Zeits. f. Phys.*, **79**, 668, 1932.

The electro-magnetic effect is not the only one which has been used for telephones and loud-speakers. The "effects" which have been used are :—

(1) The electro-magnetic; currents set up in an electro-magnet by changes in the magnetic circuit.[1]

(2) The electro-dynamic; currents produced in a conductor by its movements across a magnetic field.[2]

(3) Magneto-striction; changes in length of a nickel wire in a magnetic field make changes in the latter.[3]

(4) Condenser; the diaphragm forms one plate of a condenser, whose capacity is changed by its movement, and causes current [4] oscillations in the circuit containing the condenser.

(5) Induction coil; one coil (incorporating the diaphragm) moves in and out of the other.[5]

Corrugations and conicality lower the natural frequency of the diaphragm. By using the second principle enumerated above, the Siemens-Halske firm has produced a loud speaker whose diaphragm, a very thin corrugated aluminium leaf, has a frequency (about 20 vibrations per second) below the audible limit of pitch. The familiar pleated paper diaphragm of Lumière has also a low frequency, and a cone-shaped membrane has similar properties.[6] A diaphragm of low frequency free from overtones can be attained by loading a plate with a central heavy boss.[7] It is to be noticed that the action of these instruments is reversible; if supplied with a sinusoidal current, they excite vibrations in the diaphragm, so that the instrument then acts as sound-emitter or signal; while if the aerial sound waves impinge upon them, they excite corresponding currents in the circuits to which they are connected, acting as receivers.[8]

Investigations on the acoustic side of the response of telephone diaphragms have been directed towards two desiderata, (1) faith-

[1] Bell, *Brit. Ass. Reps.*, 1876 and 1877.

[2] *See* Hewlett, *Phys. Rev.*, **19**, 52, 1922.

[3] Ader, *Comptes Rendus*, **88**, 575, 1879; Tieri, *Accad. Lincei. Atti.*, **22**, 484, 1913.

[4] Meissner, *Zeits. f. Physik*, **3**, 111, 1920.

[5] Meissner, *Zeits. f. Physik*, **3**, 11, 1920.

[6] Gaumont, *Comptes Rendus*, **175**, 1051, 1922.

[7] Gerdien, *Phys. Zeits.*, **22**, 679, 1921.

[8] *See also* Davis and Fleming, *Phil. Mag.*, **2**, 57, 1926; Hart, *Phil. Mag.*, **2**, 1282, 1926; Grant, *Phys. Soc. Proc.*, **34**, 104, 1922; Littler, *Journ. Sci. Inst.*, **4**, 337, 1927.

fulness of reproduction of the aerial waves by the diaphragm, and (2) sensitivity of response—two aspects which are however indissolubly connected. This point will be further elaborated in connection with the analysis of sound, but it should be obvious that it is the resonance curve for the diaphragm which must be obtained for a study of this question, which means to say that we have to apply forcing vibrations of constant intensity but progressively changing frequency to the diaphragm, and to measure the corresponding amplitude of the forced vibration. The forcing vibration is usually applied by an alternator of constant output producing oscillations in the electro-magnet over which the diaphragm is placed.[1] The response can be measured in a number of different ways.

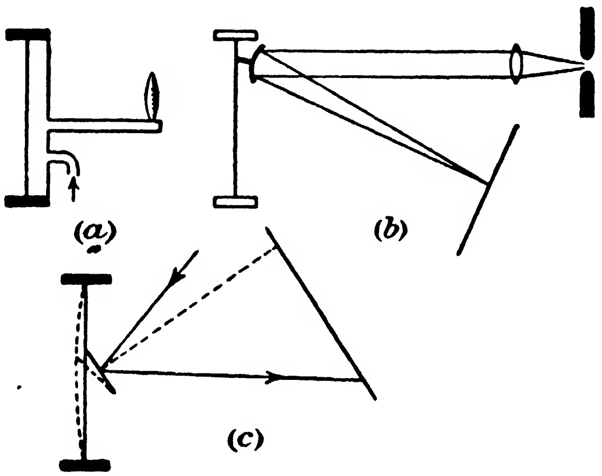


FIG. 48.—Methods of Recording Vibrations of Diaphragms.

(1) The diaphragm is covered with a capsule to which a supply of gas is admitted, and burnt at a pinhole burner (Fröhlich).[2] By this means a " manometric flame " is constructed (cf. p. 180) whose oscillations copy those of the diaphragm, but the manometric flame is unfortunately unreliable for quantitative measurements of amplitude (Fig. 48*a*).

(2) A mirror affixed at a point near the edge, so that movements of the diaphragm cause angular deviations in a beam of light reflected by the mirror on to a scale (Hartmann-Kempf)[3] (Fig. 48*b*).

(3) A mirror placed at the centre would move only parallel to itself and would not be tilted, but by connecting this point to

[1] Hahnemann and Hecht, *Ann. d. Physik*, **60**, 454, 1919 and **70**, 283, 1923.
[2] *Elekt. Zeit.*, **10**, 65, 1889.
[3] *Ann. d. Physik*, **8**, 481, 1902.

the leg of an optical lever, its oscillations may be recorded (Kennelly and Taylor)[1] (Fig. 48c).

(4) A peg may be arranged to stick out at right angles in the centre of the mirror, and to cut off periodically during the oscillation an exceedingly narrow beam of light passing over the edge of the peg (Siegbahn).[2]

All these methods (except 1) are open to the objection that they change in some degree the natural period of the diaphragm by adding inertia. For this reason Sell[3] preferred to measure the response in the neighbouring air itself by a resonator, but the introduction of this additional receiver of unknown sensitivity is a remedy probably worse than the disease.

The movements of the flame or spot of light are photographed in a camera of the sliding plate or revolving drum kind. In this way the frequency and maximum amplitude of the partial tones of the plate may be estimated and compared with the theory for a plate or membrane. If the recording mirror lies on a nodal line corresponding to a given partial, this particular partial may escape notice in the photographic record. Kennelly and Taylor accordingly explored the complete diaphragm with their tilted mirror, instead of keeping the recording mechanism fixed to one point, as did Hartmann-Kempf and Siegbahn.

Results tally with the theory of the circular plate as regards dependence on radius and thickness; the numerical factors given by theory are, however, more or less departed from, because of the stiffness and plasticity of the diaphragm. The amplitudes at the centre are of the order 0·00001 cm. (telephone) to 0·05 cm. (loud-speaker).

The Carbon Microphone. In Bell's original telephone both transmitter and receiver were of the type outlined above; Edison and later Hughes developed a new form of transmitter, which has remained in principle to the present day. Instead of producing the varying currents in the "line" by induction, they broke the circuit at the transmitting end, and completed it through pieces of carbon loosely packed together. The electric resistance between carbon blocks varies considerably with the pressure—since the granules are elastically deformed thereby[4]—and by allowing a

[1] *Amer. Phil. Soc. Proc.*, **54**, 96, 1915.

[2] *Ann. d. Physik*, **46**, 298, 1915, and **42**, 689, 1913.

[3] *Wiss. Veröfft. d. Siemens Konzern*, **2**, 349, 1922.

[4] Piola, *N. Cimento*, **22**, 278, 1921.

boss on the diaphragm to press upon the conglomeration of carbon, changes of the current from the line battery are set up by the oscillations of the membrane. These are re-converted into movements of the diaphragm at the receiving end in the original fashion, or, for experimental purposes, measurements of the original vibrations of the transmitting diaphragm may be made by studying the "wave-form" of the current in the line. In this way Powell and Roberts[1] have added a further method for the study of diaphragm vibrations by observing the oscillations set up in a circuit containing a carbon-contact microphone touching the diaphragm.

Sensitivity of Diaphragms. The main use of the diaphragm in sound recording has been indicated above, i.e., to give an accurate reproduction of aerial sound waves. The diaphragm is also used as a selective detector—mainly in connection with fog signalling—in which the object is to obtain maximum response at a given frequency—the fundamental tone of the diaphragm—and small response at frequencies above and below. Accordingly, a diaphragm having a steep resonance peak is required. As appears in the theoretical work of Chapter II, this means that the damping must be small. In experimental work on this problem, it is sufficient to use comparative methods. Two diaphragms are constructed, as nearly as possible identical, and the damping of one is varied without, if possible, altering the natural frequency, and its sensibility of response at different values of the damping coefficient is compared with the unaltered one as standard.[2] The comparison is effected by placing them in telephone receivers at equal distances from a source of constant frequency. The one which responds to the greater extent is "shunted"—part of the current induced in it is not allowed to pass through the detecting instrument—until the registered current response in the instrument is the same from both the shunted and the unmodified receiver. The relative response can be calculated from the value of the shunt required.[3]

[1] *Phys. Soc. Proc.*, **35**, 170, 1922. *See also* Bouthillon and Drouet, *Comptes Rendus*, **158**, 1568, 1914; Kennelly and Pierce, *Am. Acad. Proc.*, **48**, 111, 1912; Kennelly and Affel, *Am. Acad. Proc.*, **51**, 421, 1915; MacGregor Morris and Mallett, *Phys. Soc. Proc.*, **36**, 139, 1924; Stenzel, *Zeits. f. tech. Phys.*, **10**, 567, 1929; Goucher, *Frank. Inst. J.*, **217**, 407, 1934.

[2] Powell, *Phys. Soc. Proc.*, **36**, 84, 1924. *See also* Soret, *Comptes Rendus*, **153**, 1214, 1911; Soret and Couespiel, *Comptes Rendus*, **169**, 431, 1919; Ludewig, *Phys. Zeits.*, **21**, 305, 1920.

[3] Knudsen, *Phys. Rev.*, **21**, 84, 1923.

When a diaphragm is bent by pressure the frequencies of all the partial tones are altered. King [1] has turned this fact to account in order to tune a diaphragm to respond to a signal. The dimensions of a plate cannot be conveniently varied in order to alter its frequency, so that diaphragms to resound to a signal of given frequency were formerly constructed on the " cut and try " method ; but by covering the diaphragm on one side with a reservoir into which air is pumped, a diaphragm at a distance—under the sea, for example—may have its frequency lowered until this is equal to that of the signal.

Valve-maintained Vibrations. In connection with acoustic experiments in general, and with sound-signalling in particular, a simple maintained source of sound is often required. Formerly an organ-pipe or siren was used, but when a source of moderate power taking up a small space is required, it is often convenient to use a telephone receiver of the simple cell type and maintain the diaphragm in oscillation by an alternating or intermittent current in the coil. Such a system may be compressed into a cubic inch, and forms the nearest approach to a " point source " for sound, as the apparatus supplying the current can be placed at any distance from the diaphragm. The latter is too small to permit of an intermittent contact like that applied to the tuning fork, but the source of energy may be an alternator, running at the appropriate speed to produce forced vibrations of the diaphragm at the desired frequency ; if this is one of those natural to the diaphragm the sound will be loud. The " triode valve " offers to the modern experimenter a more convenient means of maintaining the diaphragm in oscillation. A circuit for this purpose is shown diagrammatically in Fig. 49.

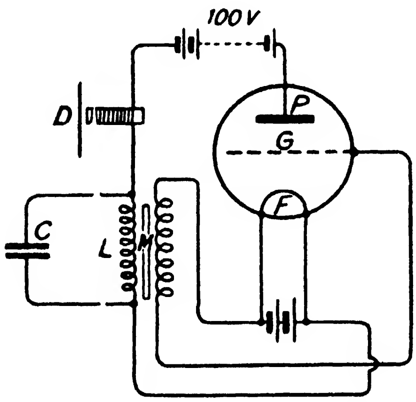


FIG. 49.—Valve-Maintained Oscillations.

The branch between plate P and filament F contains self-inductance L and capacitance C in parallel. It is well known that such a circuit has a natural period of oscillation for alternating

[1] *Journ. Frank. Inst.*, **187**, 611, 1919, and *Roy. Soc. Proc.*, **99**, 163, 1921, and *Journ. Sci. Inst.*, **3**, 241, 1926.

current of $2\pi\sqrt{LC}$ (cf. p. 228). Then, with a given value of L and C, oscillations of the frequency $\frac{1}{2\pi\sqrt{LC}}$ are produced by the valve, which react by means of the mutual inductance M in the grid circuit, on the diaphragm D with its accompanying electro-magnet in the plate circuit. Thus by fittingly adjusting L or C, vibrations of any frequency can be impressed on the diaphragm. The mutual inductance M must be "negative." In practice this means the connections to the terminals of one of the coils of M may need interchanging. The same apparatus will furnish alternating current of a desired frequency to the coil of a maintained electro-magnet.

The Condenser Microphone. Most of the above types of microphone suffer to a greater or less extent from the disability that their response is non-uniform over the musical scale. Recently

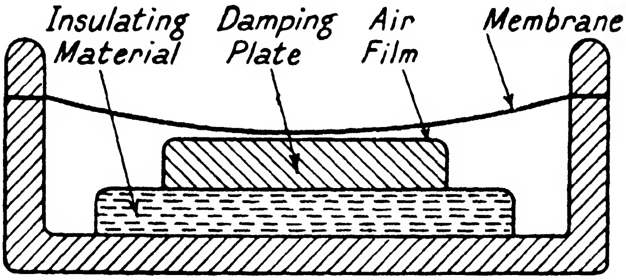


FIG. 50.—Condenser Microphone.

an electrostatic form of microphone, with transmitters and loud-speakers on the same principle, has been developed out of the original form due to Wente. In this microphone a layer of air is sandwiched between a thin metal diaphragm clamped at its edges and another fixed metal electrode embedded in an insulator such as ebonite [1] (Fig. 50). A steady potential difference is applied to the electrodes causing the diaphragm to be attracted inwards into the shape of a paraboloid, leaving an air space only about 10^{-3} cm. thick between. Any motion of the membrane will cause a ripple in the potential difference between the plates due to the change in capacity of the electrostatic condenser, which can produce a current in a string galvanometer or valve amplifier in whose circuit it is connected. The reason for the distortionless response

[1] Riegger, *Zeits. f. tech. Phys.*, 8, 510, 1927; West, *J.I.E.E.*, 67, 1137, 1929; Oliver, *J. Sci. Inst.*, 7, 113, 1930.

of this microphone is to be sought in the action of the air film. At low frequencies this provides viscous damping, as the air surges to and fro from centre to circumference with the motion of the diaphragm. At high frequencies the motion is too rapid for the air to follow so that it simply acts as a buffer. At low frequencies then, the system provides extra *resistance* above that natural to the diaphragm, at high frequencies comes extra *stiffness*, which will be a function of the frequency (cf. p. 228). Finally, these two factors are found to act in such a way as to give uniform response over a considerable region of frequency (300 to 5,000, as usually manufactured).

Both diaphragm and air film must be extremely thin. Jakowleff [1] notes that resonance in the air cavity is innocuous if its natural frequency is two or three times that of the highest note to be recorded and if the damping factor is 1·35 times this natural frequency.

In an attempt to produce a small acoustic measuring instrument which should not unduly disturb the field in which it may be placed, Hall [2] has succeeded in constructing a condenser microphone, the diaphragm of which consists of a sheet of aluminium foil 0·0004 in. thick and 2 cm. in diameter, put under tension by screwing down a brass ring over the edge of a hard rubber bushing. The brass shell into which the system was screwed served as the other plate of the condenser. Used as a transmitter the system has an output of about one millivolt per bar and is practically independent of frequency up to 6,000 cycles/sec. Such an apparatus —even smaller ones might be constructed—offers great possibilities for the plotting of sound fields where larger microphones would produce distortion of the field.

Bells. Plates are used as instruments of percussion in music under a number of different forms, with or without air cavities, as cymbals, gongs, etc. That shape which possesses most interest from a physical point of view is the bell. The theory of this instrument has been attacked mathematically only by making several reservations. On the one hand the bell may be regarded as a development of the hollow rod, on the other as a bent plate of, generally, non-uniform thickness. The use of hollow cylinders in imitation bells has already been referred to (*glockenspiel*). The

[1] *Jahrb. d. drahtl. Tel.*, **30**, 151, 1927; *see also* Schweikert, *Zeits. f. Phys.*, **41**, 775, 1927.

[2] *Acoust. Soc. J.*, **4**, 83, 1932; *see also* Sacerdote, *Alta. Freq.*, **2**, 516, 1933.

form of the church bell is more nearly approached by the half-shell, a form which has been considered by Chladni [1] and Rayleigh.[2] The latter made experiments on church bells proper, of various sizes, determining by ear the inharmonic tones which make up the sound. The commercially made bell is of special metal (bell metal, about 80 parts of copper to 20 of tin); a common form is shown in Fig. 51*a*. The clapper hangs loosely inside, and smites the bell on the "sound-bow" *SB* near the open end, where the inside and outside sections have opposite curvature. Only the smallest bells are rung by hand, a handle being placed at the centre of the upper surface for the purpose. Large bells are "swung" by means of a wheel and axle, passing through the same point.

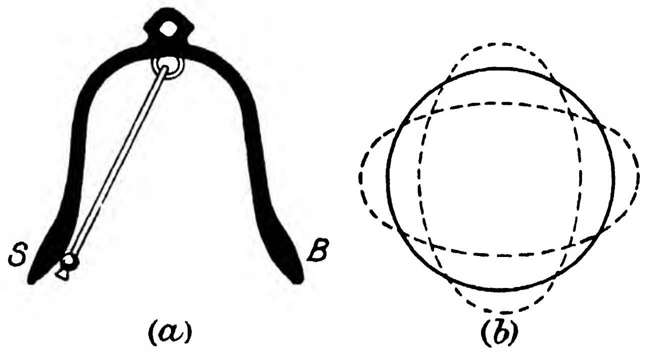


FIG. 51.—The Church Bell.

Bells may also be kept still while they are struck, usually on the outside, by a hammer.

Lord Rayleigh,[3] and recently Biehle,[4] examined a large number of bells, and found that the various partial tones formed by division into a number of segments, formed a series approximately as follows :—

Approx. Relative Pitch.	*Nodal Lines.*
1. Fundamental	4 sectors
2. Octave	4 sectors and ring
3. Octave and minor third	6 sectors
4. Twelfth	6 sectors and ring
5. Double octave	8 sectors etc.

It is to be understood that these relations are approximate only, as the tones of a bent plate are inharmonic, but the bell-founder,

[1] *Akustik*, 192, 1802. [2] *Sound*, 1, 420, 1897.

[3] *Phil. Mag.*, 29, 1, 1890.

[4] *Phys. Zeits.*, 20, 429, 1919, and 22, 337, 1921, and 23, 80, 1922. *See also* Blessing, *Phys. Zeits.*, 12, 597, 1911; Sizes, *Comptes Rendus*, 154, 340 and 504, 1912.

by adjusting the thickness at various sections, strives to make the lower tones as nearly harmonic as possible. In practice only one nodal ring is formed on a bell; this eliminates some of the discordant overtones pertaining to a plate. Beside these tones which can be approximately predicted by applying the theory of plate vibrations, there is another tone which, immediately after striking, overpowers these but decays more rapidly. This is known as the "striking note" and its pitch, by which the founder names the bell, seems to lie near that of the octave or second in the series of partial tones; in a good bell the striking note is made coincident with or harmonic to this second overtone, even if the other overtones have to be left inharmonic to each other. The occurrence of this striking tone is very curious, and has so far baffled explanation. The partial tones can be elicited by resonance with a tuning fork, but not so the striking note; nor can the latter be picked up by a resonator, or made to produce beats with a neighbouring tone. Its origin may be subjective—formed in the ear itself—but its abnormal intensity is against this idea; however Biehle found that it was most prominent when the lower tones of the bell were loudly produced, and as it dies out rapidly compared with the normal tones of the bell, it may be formed in the ear by the large "forcing" of the initial stroke. On the other hand Taber Jones [1] thinks that the phenomenon is an aural illusion; that the striking note is really the fifth partial (double octave) but that its tone location is masked by the lower overtones, making it seem to be in the lower octave.

Further work on bells has been done by Jones and Alderman,[2] also by Curtiss and Giannini.[3] The former have determined the acoustic spectrum of a bell at various intervals after striking. Since the bell is theoretically a struck anisotropic plate we find analogies with the struck string in the rate of decay of partials. The variation in this respect is one of the characteristics of bell tone. The latter workers have succeeded in examining the nodal lines by the method of Tyzzer,[4] viz., excitation of each partial separately by means of a valve oscillator and loud speaker, and

[1] *Phys. Rev.*, **16**, 247, 1920, and **31**, 1092, 1928; *Acoust. Soc. J.*, **1**, 373 and 382, 1930.

[2] *Acoust. Soc. J.*, **3**, 297, 1931, and **4**, 331, 1933.

[3] *Acoust. Soc. J.*, **4**, 245, 1933, and **5**, 159 and 211, 1933.

[4] *Frank. Inst. J.*, **210**, 55, 1930; *see also* Obata and Tesima, *Acad. Tokyo Proc.*, **10**, 211, 1934.

exploration of the vibrating surface with a sort of stethoscope. Whereas, immediately after striking, the fifth partial is the strongest, after 1 sec. it has practically disappeared leaving the third partial master of the field. In 12 secs. this too has nearly fallen below audibility, leaving the fundamental which shows the least attenuation of all.

RELATIVE AMPLITUDES OF BELL OVERTONES AT VARIOUS TIME INTERVALS AFTER STRIKING. (*Jones and Alderman*)

Order of Partial.	1	2	3	4	5	7
At strike	1	·7	2·4	·7	4	2·8
After ½ sec.	1	·6	1·8	—	·8	·75
„ 1 sec.	1	·6	1·8	—	·1	·4
„ 2 sec.	1	·5	1·1	—	·1	—
„ 3 sec..	1	·4	1	—	—	—
„ 7 sec.	1	·4	·8	—	—	—
„ 12 sec.	1	·2	·4	—	—	—

This is for a tenor bell of 345 cycles/sec. In the treble the bells have a thicker waist (zone of minimum thickness of metal) and the attenuation of the partials is more nearly uniform. Giannini thinks, however, that the large bells with finer waists which have the acoustic properties outlined above have a better and more characteristic bell tone and that this pattern (thick sound bow and thin waist) should be adopted throughout the peal.

The nodal lines are positions of no motion normal to the surface, but a considerable circumferential movement may take place at points on these lines. This is shown by the accompanying figure of a section of the bell when sounding its fundamental (Fig. 51*b*).

Another point of interest noted by Rayleigh is that these nodal "lines of longitude" tend to move slowly round the bell causing a waxing and waning of intensity to a stationary observer as the antinodes and nodes are directed towards his ear. The nodes may be traced out by touching a suspended pith-ball to various points on the surface; the normal motion will throw the ball off, but at the nodes it will remain undisturbed. For such experiments a hollow hemispherical glass bowl suffices. The note may be excited either by striking, bowing the edge, or rubbing a wet finger along the surface, or by simple resonance with an appropriate fork. With the first two methods the point of excitation naturally becomes

an antinode; with tangential rubbing the point is usually, at first, a node.

Attempts to imitate bell-tone by gongs made of steel wires bent into a spiral and struck at the free end with a hammer have been made by clockmakers with a certain amount of success. Investigations have been made by Pomp and Zapp [1] to determine what is the best type of steel for this purpose. Carbon steel (0·63 per cent.) tempered for 15 mins. at 300° C. gave the maximum duration of note.

Musical Sand. An interesting phenomenon, which at first sight seems to involve the elastic vibrations of spheres, is the production of sound from certain grains of sand when struck by a rod. This was discovered by Miller at Eigg in 1850. In England there is, or was until recently, a narrow patch of sand running along the shore of Studland Bay between Sandbanks and Swanage which would emit sounds of definite pitch. The grains must be more or less of the same size, and can be excited by blows with a stick. The objection to the theory that the tone is due to elastic vibrations of a granule in a number of sectors as in a solid bell, is that the fundamental of such small spheres would be of very high pitch. Carus-Wilson,[2] who has made a detailed study of the Studland sand, ascribes the note to simple friction between highly polished grains of quartz, but it is not clear why the pitch of the emitted note is then so definite.

Musical Bubbles. The sounds of air bubbles breaking in water, to which the purling of a brook is due, cannot, for the same reason that the pitch would lie outside the audible limit, be ascribed to compressional vibrations of the air pocket. Minnaert [3] has, however, shown that the frequencies of possible *radial* elongations and contractions of the liquid envelope agree closely with those heard. Moreover, the tones are practically independent of temperature, which would not be the case if the vibrations of an air resonator were in question.

[1] *Engineer*, 105 and 120, 1934.

[2] *Nature*, **81**, 69 and 109 1909. *See also* Irving, p. 99, and Gray, p. 126, in same volume.

[3] *Phil. Mag.*, **16**, 285, 1933; Smith, *ibid.*, **19**, 1147, 1935; Gorbatschew and Severny, *Koll. Zeits.*, **73**, 146, 1935.

FURTHER REFERENCES: (drum) Raman, *Ind. Acad. Sci. Proc.*, **1**, 179, 1934; Obata and Tesima, *Acoust. Soc. J.*, **6**, 267, 1935; Rao, *Ind. Acad. Sci. Proc.*, **7**, 75, 1938; (plate) Wood and Smith, *Phys. Soc. Proc.*, **47**, 149, and 794, 1935; Pavlik, *Ann. d. Physik*, **28**, 353, and 634, 1937; Hayes, *Acoust. Soc. J.*, **8**, 220, 1937; Mary Waller, *Phys. Soc. Proc.*, **50**, 70, 1938.

CHAPTER SIX

VORTEX FORMATION AND ÆOLIAN TONES

Before the production of tone in a column of air or in a solid body by aerial vibrations can be properly understood, it is necessary to make a digression, and to discuss the growth of vortices in a fluid, as far as it has been clarified by recent physical research.

Potential Streaming past an Obstacle. Consider a solid cylinder placed in a moving fluid, like that shown in section in Fig. 52. The ideal motion most amenable to mathematical treatment is that in which the particles are supposed to exercise no dragging effect on each other. The path of particles in an infinite channel in which uniform flow is taking place, will then be represented by parallel straight lines; the so-called "stream lines." On dipping the cylinder into such a fluid we introduce the "boundary condition" that at the surface between the fluid and the cylinder there can be no normal velocity. This leads to the requirement that the surface must itself be a stream line. The paths of particles which pass near the cylinder are shown in the figure. The pattern is identical with that of the equi-potential lines of an electrostatic field containing a conducting cylinder, and can be mapped out practically as such; for this reason this type of ideal flow is called "Potential Streaming." The assumption of no friction in the fluid requires that the kinetic energy gained by particles passing from the front to the mid-point of the side of the cylinder shall be just lost when they reach the stern, so that they leave the cylinder with the same velocity with which they struck it. The fluid which actually passes in contact with the body therefore "slips" past the surface without any retardation. This leads to the paradox that there is no force on the cylinder due to potential streaming. This is in obvious contradiction to fact, and yet, except very close to the cylinder, the lines shown in Fig. 52 do closely resemble those observed experimentally by the method of Hele-Shaw[1] pro-

[1] *Brit. Ass. Rep.*, p. 136, 1898.

vided that the velocity of flow is not too great. This method consists in allowing coloured filaments of liquid to enter a small observation tank in which the flow of clear liquid is taking place. The mistake arises evidently in neglecting to take into account the friction, not only between the solid boundary and the adjacent fluid which prevents slipping to the extent postulated, but also between neighbouring layers. The friction approximates the velocity, if any, at the boundary to that in the body of the fluid, so as to produce a steep velocity gradient across the stream lines near the surface of the body. There is thus a shearing force on the fluid tending to retard it, or, if we think of the contrary but dynamically identical case, of the cylinder dragged through the still fluid, there is a resistance offered to the motion of the cylinder,

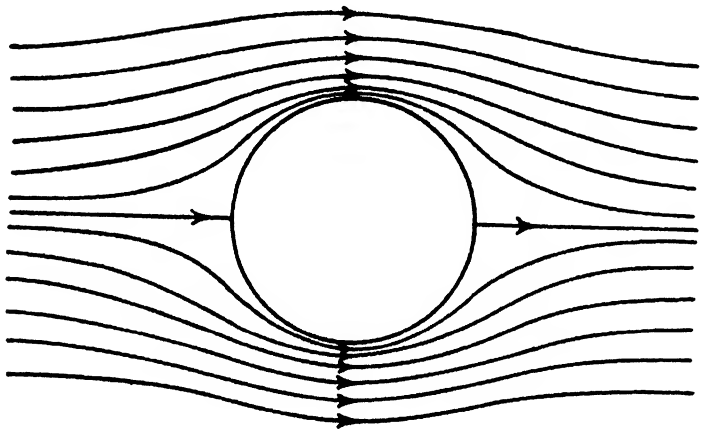


Fig. 52.—Potential Streaming Past a Cylinder.

as there would be if it were dragged over a solid surface. The part of the resistance due to shearing effects between the body and neighbouring strata of fluid is known in this country as "skin-friction" and must be distinguished from the resistance due to eddy formation in the rear of the obstacle. This will be considered later. When the body is without sharp changes of contour, and moves at a low velocity through the fluid, skin-friction produces the major part of the drag-resistance.

Boundary Layer. A large number of attempts have been made to introduce viscosity into the mathematical equations of fluid motion, but an exact solution has so far defied all efforts. An idea developed by Prandtl,[1] though admittedly an approximation, has stood the test of twenty years' application to the prediction of skin-resistance of profiles, and deserves elaboration.

[1] *Proc. Math. Congress*, Heidelberg, 1904.

The shearing force per unit area between neighbouring layers of fluid is given by the expression

$$F = \mu \frac{dV_x}{dy}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

where μ is the coefficient of viscosity, and $\frac{dV_x}{dy}$ is the gradient of velocity (tangential to the boundary where $y = 0$) measured across the layers. Now if there were any slipping between fluid and solid, a shearing force would be brought into play, which would be infinitely great compared with that between fluid layers. We believe, therefore, that slip is impossible at the boundary, and take as the boundary conditions $V_x = V_y = V_z = 0$, these being the component velocities in the three rectangular directions. Prandtl remarked that the viscosity of air is a small quantity, and its effect noteworthy only—in accordance with (54)—where the change of velocity from layer to layer is very great. He therefore proposed to neglect viscosity except in a thin layer at the liquid-solid surface. Within this " boundary layer " the tangential velocity rises in a very small distance from 0 at the boundary to the mean velocity of the body of the fluid. Within this layer " stream-line " motion takes place even when the motion outside is unsteady.

For example, we may imagine the motion outside the boundary layer to be simple harmonic with respect to time, and due to aerial waves ; or to vary from time to time in an incoherent way about an average value in the fashion denoted " turbulent." In order to calculate the thickness of this layer, Prandtl takes a new coordinate ζ, normal to the surface in place of y, such that $\zeta = 0$ at the surface, and $\zeta = \infty$ at the outer edge of the boundary layer. He writes down the equations of motion introducing the viscous term $\mu_1 \frac{dV_x}{d\zeta}$ representing the shearing force between adjacent strata of the layer where μ_1 is a greatly increased viscosity coefficient consequent on the greatly reduced scale of ζ. In the steady state wherein $\frac{dV_x}{dt} = 0$, it appears that the thickness of the boundary layer depends on $\sqrt{\frac{\mu l}{\rho \overline{V}_x}}$ where $\overline{V}_x$ is the average velocity in the main fluid, ρ is the density, and l is the length, in the direction of the motion, of the surface on which the fluid has rubbed. On

inserting the dimensions it will be seen that the above expression is of the dimension of a length, and that the thickness of the boundary layer increases progressively with the square root of the length of the boundary. It is therefore cumulative. At the end of a wooden board 2 metres long the thickness has been estimated at 1 or 2 cms.

The quantity $\frac{\mu}{\rho}$ is called the "kinematic coefficient of viscosity" (ν), and plays an important part in fluid motion. For the application of this theory to the skin-friction of various profiles textbooks on Aerodynamics must be consulted.

Formation of Eddies at the Rear of a Cylinder. The ideal motion depicted in Fig. 52 differs from reality in a further respect. Save at indefinitely slow velocities the liquid would not hug the stern of the cylinder in the manner shown by the stream lines, but would cut off part of the "corner" in the manner shown above

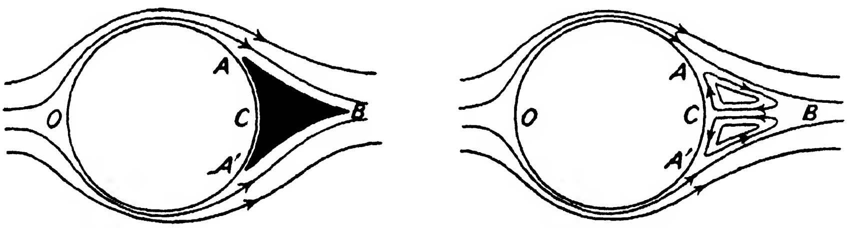


FIG. 53.—Formation of Eddies Behind a Cylinder.

(Fig. 53*a*); leaving the cylinder at A, A', and resuming parallel motion at B. The fluid of the shaded area ABA' is "dead water," since it is not carried along by the stream, and the surfaces represented in section by the lines, AB, $A'B$ are "surfaces of discontinuity" to which Helmholtz [1] ascribed the drag resistance of the cylinder. Owing to the shearing effect of the stream on this dead wake, the fluid in it is set in rotation in the form of two eddies (Fig. 53*b*). That the fluid leaves the solid at A, A', may be ascribed to the intense rates of shear in the boundary layer. A, A', are probably the points at which turbulence is produced in the boundary layer itself, so that Prandtl's original theory breaks down. If the surface of the cylinder from O to A and C (Fig. 53*b*) is made to move in the same direction as the fluid, this reduces the shearing forces in the boundary layer all over the surface, and postpones the separation of the fluid from the body. Thus, if the whole

[1] *Berlin Akad. Sci. Ber.*, 1868.

cylinder be given a rotation in the clockwise direction, the point A is moved towards C, but A' is pushed back towards O. Between C and A—we are speaking of the stationary cylinder again—the fluid in the boundary layer is moving towards A, i.e., opposed to the motion from O to A. At A and A' the tangential velocity in this layer is zero. These points, however, represent regions of considerable instability, and when a sufficient velocity in the body of the stream is reached, vortices formed in the wake no longer remain attached to the cylinder, but on reaching a sufficient size are carried down the stream as if they were solid bodies. It is these travelling vortices with which we are more particularly concerned in sound; but we may point out here their importance in hydrodynamics as they represent energy being dissipated as heat by friction. This energy is carried away from the body, setting up a resistance component far exceeding the skin frictional component. If the points A, A', on a surface where the boundary layer is disrupted, can be pushed farther astern there is less dead water, consequently less eddy-formation and less resistance. That is why bodies which have to move rapidly through a fluid are "stream-lined," sudden changes of contour in the stern being avoided.

Strength and Influence of a Vortex. The ideal vortex consists of a core of fluid rotating with constant angular velocity, surrounded by fluid in which there is no rotational velocity. In practice there is no discontinuity in linear velocity at the edge of the vortex, but the motion tails off rather sharply and continuously to the average velocity of the stream. The "strength" of the vortex is defined as twice the product of the angular velocity ω and the cross section. Consider first a single isolated vortex of radius r_0 and of strength K. Then

$$2\pi r_0^2\omega = K, \text{ from } r = 0 \text{ to } r = r_0,$$

and

$$\omega = 0, \text{ from } r = r_0 \text{ to } r = \infty.$$

Now the linear velocity at a point in the vortex, say at $r = l$, will be given by $l\omega = \dfrac{Kl}{2\pi r_0^2}$; thus at the circumference the linear velocity is $\dfrac{K}{2\pi}.\dfrac{1}{r_0}$ normal to the radius. In order that the linear velocity may be continuous at the circumference we must have also at a point outside distant b from the centre, the velocity normal to the radius due to the vortex equal to $\dfrac{r_0^2\omega}{b} = \dfrac{K}{2\pi}\dfrac{1}{b}$. Now when a number of vortices exist in a fluid, it is permissible, in order to find the resultant motion

at a point due to the system, to superpose vectorially the velocities due to the separate vortices. In particular, if the point in question is the centre of one of the vortices, we can obtain the motion of this one vortex due to all the others. Thus if there are two equal single vortices, one clockwise and one anti-clockwise, distant b apart, each imparts to the other a velocity $\frac{K}{2\pi b}$ in the direction at right angles to the line joining them, and they both move with this velocity, preserving the same relative orientation.

Vortices in Parallel Rows. From the acoustic point of view the case of prime interest is that in which we have an "avenue" of vortices at equal distances apart. Let the vortices in each row be distant l apart, and the distance between the two rows be h, and the amount of "stagger" of one row upon the other be a (Fig. 54).

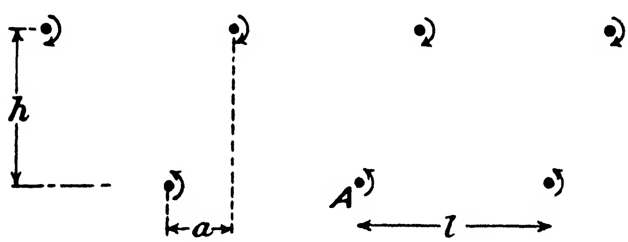


FIG. 54.—Avenue of Alternate Vortices.

The velocity imparted to the vortex A in a direction across the avenue by any other vortex in the upper row is

$$\frac{K}{2\pi}\frac{\cos\phi}{\sqrt{(a+jl)^2+h^2}}=\frac{K(a+jl)}{2\pi[(a+jl)^2+h^2]},$$

j being any integer and ϕ the angle between the line joining the vortices and "l." Now the induced velocities due to those in the same row mutually cancel, but those due to the upper row, resolved in the same direction, add up to :—

$$V=\frac{K}{2\pi}\left\{\frac{a}{[a^2+h^2]}+\frac{(a+l)}{[(a+l)^2+h^2]}-\frac{(l-a)}{[(l-a)^2+h^2]}+\right.$$
$$\left.\frac{(a+2l)}{[(a+2l)^2+h^2]}-\frac{(2l-a)}{[(l-2a)^2+h^2]}\right\}+\ldots\text{ to }\infty$$

This will be equal to zero only in two cases :—

(1) $a=0$. The first term of the series is zero, and the alternate positive and negative terms become equal in pairs, and so cancel.

(2) $a=\frac{l}{2}$. Then the first term cancels with the third, the second with the fourth, etc.

If these two cases be imagined pictorially, the first corresponds to vortices opposite to each other in each stream. As far as the corresponding motion produced on any vortex by the rest is concerned, the effects of all those on either side cancel in pairs, leaving only that

due to the opposite vortex, and the component of this across the stream is nil, since the velocity produced by this one is entirely in the down-stream direction. In the second case, any single vortex is equidistant from the two jth vortices in the other row, counting from the vortex in question. In order then that the two series of vortices may remain in parallel rows, every vortex in one row must be oriented symmetrically with respect to the series in the opposite row. Note that although there is then no motion of the vortices across the rows, each and every is actuated with the same velocity along the rows, given by :—

$$V = \frac{K}{2\pi} \sum_{j=-\infty}^{j=+\infty} \frac{\sin \phi}{\sqrt{h^2 + (j + \frac{1}{2})^2 l^2}} = \frac{Kh}{2\pi} \sum_{j=-\infty}^{j=+\infty} \frac{1}{[h^2 + (j + \frac{1}{2})^2 l^2]}$$

in the " alternate " arrangement, and of :—

$$V = \frac{Kh}{2\pi} \sum_{j=-\infty}^{j=+\infty} \frac{1}{(h^2 + j^2 l^2)}$$

in the " opposite " arrangement. The system therefore remains in equilibrium, moving with velocity V in the direction of the rows, and with no cross component.

Experiment shows that the main production of vortices occurs behind the body, as described on page 144, each vortex being formed and then detached to move down the stream after the others in procession. As long as all have the same initial strength (as we should expect, if the velocity of the stream, and the position of the cylinder are unchanged), this procession of vortex pairs can remain in equilibrium and move down the stream only if the vortices occupy one or other of the two orientations discussed above; therefore they must be detached periodically from the rear of the cylinder, either in pairs (case 1), or alternately (case 2), from each side. Now although these two possible systems are both equilibrium positions, they are not both stable arrangements.

The calculation of the relative stability of these two equilibrium systems proceeds on the usual lines. One supposes a small displacement ξ_0 given to one of the vortices in the row due to some accidental disturbance, and calculates the net effect on the velocity $\frac{d\xi}{dt}$ of this vortex due to all the others; the solution of the equation for $\frac{d\xi}{dt}$ may be written $\xi = \xi_0 e^{\alpha t}$. The calculation is too abstruse to be given here, but the result shows for the opposed position, α always positive, any accidental displacement growing with time,

system unstable; for the alternate position, α negative, accidental displacements damped out, system stable. Furthermore, in the second case, maximum stability occurs when α has its greatest (negative) value; this occurs when $\frac{h}{l} = 0{\cdot}28$.

Experimental Confirmation. This procession of alternate vortices behind an obstacle in a stream was first noted by Mallock [1] and investigated (independently) by Bénard in 1908.[2] Bénard allowed the body to dip into a stream of water, and photographed the vortices formed in the stream behind the body. The positions of the vortices were made visible by the light which they scattered as dimples on the surface. The experiments established the fact that the vortices left the obstacle alternately on either side, forming rows with separation depending on the velocity of the stream and the diameter of the obstacle. Kármán [3] investigated mathematically the stability of this vortex system on the lines indicated in the latter part of the last paragraph, and using some further photographs by a collaborator, confirmed the predicted value $\frac{h}{l} = 0{\cdot}28$ for the system formed behind a cylinder: it makes no difference experimentally whether the fluid streams past the obstacle, or the obstacle is dragged through the fluid.

When it is desired to follow the formation of these vortices at frequencies too great for simple counting, Relf and Simmons [4] have found that an electrically heated hot-wire placed in the wake of, and parallel to the cylinder, showed by periodic cooling the passage of the vortices past it. These perturbations were made apparent by a vibration galvanometer tuned to the variations of current through the hot-wire, caused by the periodic change of resistance, which was in turn caused by the cooling. Thus the tuning of the galvanometer to get the best response, with certain precautions, gave the frequency of production of the vortices. Kármán also calculated the effect of this formation on the drag-resistance of the cylinder (cf. p. 144) in a paper [5] of fundamental

[1] *Proc. Roy. Soc.*, **9**, 262, 1907.

[2] *Comptes Rendus*, **147**, 839 and 970, 1908, and **156**, 1003 and 1225, 1913, and **182**, 1375 and 1823, 1926, and **183**, 20 and 184, 1926.

[3] *Göttinger Nach.*, 547, 1912.

[4] *Phil. Mag.*, **49**, 509, 1925. *See also* Tyler, *Journ. Sci. Instruments*, **3**, 398, 1926.

[5] Kármán and Rubach, *Phys. Zeits.*, **13**, 49, 1912.

importance in hydrodynamics, but which does not concern us here.

We are now in a position to discuss these vortex-rows of Bénard in the light of certain important acoustical phenomena.

Æolian Tones. The periodic detachment of vortices from alternate sides of the obstacle in a stream, imposes periodic cross forces, alternating in direction, on the obstacle. If the obstacle is free to move in a direction at right angles to the stream, it will execute transverse vibrations when the frequency of detachment of a pair of alternate vortices corresponds to one of its natural tones. These are the sounds which Rayleigh called Æolian tones, most noticeable when the wind strikes a system of wires or cords. The phenomenon has been known from Biblical times, harps and even violins being constructed to work on this principle, but the Æolian Harp is not a serious musical instrument, as no proper

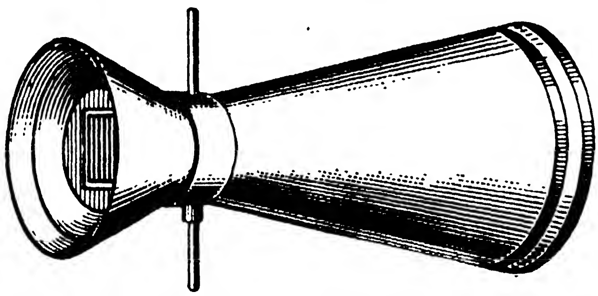


Fig. 55.—Æolian Harp.

control can be exercised over the source of the sound. They are still made as toys, however, and one such is shown in Fig. 55. This is arranged to pivot on a post, and to turn always to face the wind. The wires are usually tuned to a common pitch, but are of different thicknesses, in order to increase the probability of the resonance of the vortex system with one of the wires at any given wind speed.

The first scientific investigation of the phenomenon was made by Strouhal.[1] He stretched a wire between the ends of two rods, and an axle parallel to the wire passed through the mid-points of these rods, so that the rotation of the axle whirled the wire through the air round the circumference of a circle, and in a direction at right angles to its length; i.e., the wire described the curved surface of a cylinder. Not only the fundamental of the wire, but harmonics also would respond at appropriate speeds.

[1] *Ann. d. Physik*, 5, 216, 1878.

The connection between the diameter D of the wire, frequency of the tone n, and velocity of the wire through the air V, was :—

$$\frac{V}{nD} = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (55)$$

Later Rayleigh pointed out that the vibration was always across the stream, as it must be if it is due to the Bénard phenomenon, and showed that the above formula could be predicted by dimensional analysis. About the same time, Krüger and Lauth,[1] by applying the result found by Kármán and Rubach, showed that the frequency of production of the vortices corresponded to that of the vibrating solid, that $\frac{h}{l}$ was a constant, independent of D and of V; where h is the distance between the two rows of vortices, and l the length between two successive ones in the same row. Also l was approximately proportional to, and greater than, D, so that $l = bD$ say; the velocity U of the vortex system relative to the (stationary) fluid was less than, but proportional to, V—i.e., $U = aV$, where a and b are constants. These facts led Krüger and Lauth to a theoretical basis for Strouhal's formula. The frequency of the vibration represents the number of vortices formed on one side of the wire in one second. If the body swings with the vortices, $\frac{l}{V - U}$ is the time between the disengagement of two successive vortices from the same side of the body—i.e., the period of swing :—

$$\frac{1}{n} = \frac{l}{V - U}$$

$$\frac{V}{nD} = \frac{V}{D}\left(\frac{l}{V - U}\right) = \frac{l}{D}\left(\frac{V}{V - U}\right) = \frac{b}{1 - a}$$

which is a constant. From Kármán and Rubach's results for a and b, Krüger and Lauth obtained the value 5·0 for this constant, which is in fair agreement with Strouhal's numbers, which lie between 6·3 and 4·9.

The methods for investigating the laws of Æolian tones are (1) the whirling machine method (Strouhal)—this is open to error if recourse is made to comparison by ear for the pitch; (2) a pendulum dipping into a revolving tank of liquid; the pendulum is free to oscillate across the stream, i.e., radially to the tank, and

[1] *Ann. d. Physik*, **44**, 801, 1914.

its movements are slow enough to be counted (Riabouchinsky)[1]; (3) a wire or cord stretched in a frame and held upright in a wind or water channel, the frequency being found by stroboscopic means (Richardson).[2] By allowing light to be reflected from the surface of the water into a camera a photograph can be obtained, which shows the alternate vortices behind the obstacle.

The complete apparatus for the wind-channel method is shown in Fig. 56. The channel had a long glass window in one side, facing a smaller window in the opposite side. The wire was so placed in the channel that it could be observed in a direction inclined about 20 degrees to the axis, through a telescope of a short focus near one end of the long window, the wire being illuminated by an opal-glass lantern, placed behind the small window.

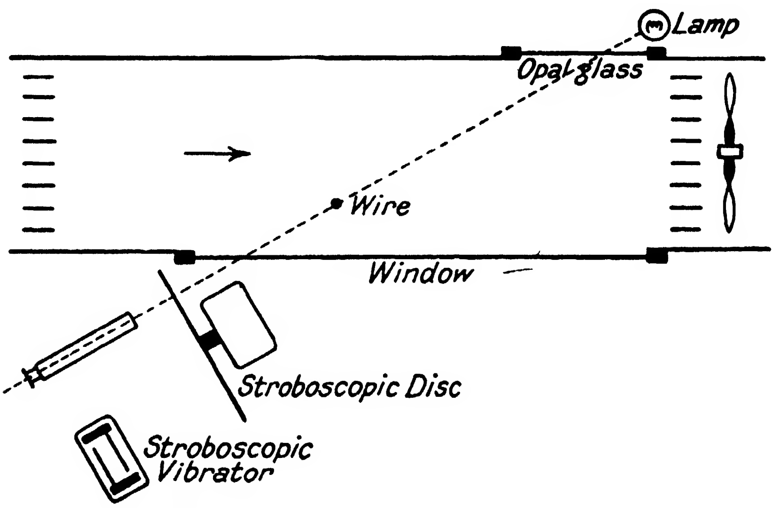


FIG. 56.—Wind Channel Apparatus for Æolian Tones.

As the noise of the fan and motor prevented all but the louder tones from being distinguished, the vibration of the wire was observed through the telescope, the frequency being found by a stroboscopic method. A motor, carrying a stroboscopic disc, having twelve slits equally spaced round its circumference, revolved directly in front of the object glass of the telescope, so as to cut off the light twelve times in each revolution. The speed of this motor was adjusted by means of a rheostat, until the fastest speed of the disc, at which the vibrating wire appeared stationary, was

[1] *Aérophile*, **19**, 15, 1911. *See also* Rayleigh, *Phil. Mag.*, **29**, 433, 1915; Krüger, *Ann. d. Physik*, **60**, 279, 1919; Relf, *Phil. Mag.*, **42**, 173, 1921; Sellerio, *N. Cimento*, **4**, 60, 1927; Thom, *Roy. Soc. Proc.*, **141**, 651, 1933.

[2] *Phys. Soc. Proc.*, **36**, 153, 1924, and **37**, 178, 1925; *Roy. Aero. Soc.*, **1927**.

obtained. This speed in revolutions per second was known by looking through a stroboscopic vibrator, just beside the telescope, and the speed multiplied by twelve, gave the frequency of the wire, independently of the ear, or of any comparison of pitch. The velocity of the air current was found from a manometer. The wires were fixed vertically in small clamps, with adjusting screws for altering the fundamental frequency.

The modern results establish the constancy of $\frac{V}{nD}$ at a value of 5 for a cylindrical body, except for small values of V or of D. This is noticeable when thin wires (of less than 0·02 cm.) are used; then the "constant" soars up to 8 or more. The lateral extent of the wake depends on the shape of the body, and therefore h alters with the shape, and this affects the value of this constant for bodies of different shape.

Effect of Viscosity on Fluid Vibrations. Experiments made in connection with oscillating wires, or with pendulums in liquids of (kinematic) viscosities varying from 0·01 to 0·5 c.g.s. units, showed little or no effect of viscosity on the quantity $\frac{V}{nD}$, the tones of a wire being produced at practically the same stream velocity in every case. Apparently then viscosity does not appreciably change the rate of formation of the Bénard vortices, though, in recent papers, their original investigator thinks that a change of $\frac{V}{nD}$ is detectable in his photographs. Viscosity has, however, a very important influence on the initiation of the vortex system. There is a minimum velocity ("critical velocity") below which no vortices, and therefore no tones are produced. If the ratio of velocity to viscosity is small, and if the obstacle is sufficiently narrow and tapered in the stern, the two parts into which the stream is divided at the front of the obstacle, can re-unite exactly at the centre of the stern of the obstacle. There is no dead-water region, no discontinuity, and no vorticity. We have in fact stream-line motion corresponding closely to the "potential streaming" of hydrodynamical theory. When the critical velocity is exceeded, the stream lines can no longer hug the stern, stagnation points appear on the sides, and vorticity in the wake.

This critical velocity then depends on the viscosity and density of the fluid, and the width and form of the body, especially of the stern. The two former effects we unite under the coefficient

of kinematic viscosity $\nu = \frac{\mu}{\rho}$, the latter pair under a single linear dimension l.

Principle of Dynamical Similarity. The exact form of the dependence of Æolian tone phenomena on viscosity may be deduced from the method of dimensions. To give a simple illustration of the application of the method ; first we will suppose that, as experiment shows, the frequency of the tone depends only on the velocity of the stream and the diameter of the body, so we write $n = f(V, D)$. Now n represents vibrations per unit time, and is of the dimensions $\frac{1}{\text{time}}$, $\left(\frac{1}{T}\right)$.

V represents distance travelled, and its dimensions are $\left(\frac{L}{T}\right)$.

D represents a length (L).

It is necessary that the right-hand side of the equation should have the same dimensions as n, otherwise the equation would depend on the system of units used, e g., if it were true for metric units it would cease to be valid on changing to English units, therefore $f(V, D)$ must be of dimension $\left(\frac{1}{T}\right)$. If we assume that this function can be written out in powers of V and D, say $V^x D^y$, its dimensions are $\frac{L^x}{T^x} . L^y$. In order that this may equal $\frac{1}{T}$ we must have $x = 1$, $y = -1$. Therefore the relation connecting V, n, and D is $n = C\frac{V}{D}$, or $\frac{V}{nD} = \text{const.}$ (cf. 55), C being a non-dimensional factor.

It is to be noted that this formula does not necessarily apply when some quantity other than V, n, or D is changed—the viscosity of the fluid for example, or the shape of the tail of the body, a thing we have not taken into account—these things are latent with others in our non-dimensional factor C.

Now let us introduce the viscosity in its common form μ, density ρ, and write $n = V^a l^b \mu^c \rho^d$, (the dimensions of μ are $\frac{M}{LT}$, of ρ are $\left.\frac{M}{L^3}\right)$; proceeding as before, and equating powers

$$\begin{aligned} &\text{of } M\,; \quad c = -d \\ &\text{of } L\,; \quad 0 = a + b + 2c \\ &\text{of } T\,; \quad -1 = -a - c \end{aligned}$$

whence $a = 1 - c$, $b = -c - 1$, so that

$n = V^{1-c}D^{-c-1}\mu^c\rho^{-c} = \dfrac{V}{D}\left(\dfrac{\nu}{VD}\right)^c$, putting $\nu = \dfrac{\mu}{\rho}$. There are not enough equations to determine c and this may be given any value. In general we should expand it and write :—

$$n = \frac{V}{D}\left[C_0\left(\frac{\nu}{VD}\right)^0 + C_1\left(\frac{\nu}{VD}\right)^1 \quad . \quad . \quad . \quad . \right]$$

or we may write this in the general form :—

$$\frac{V}{nD} = C'f\left(\frac{VD}{\nu}\right)$$

where C' now is a non-dimensional constant not involving viscosity. The form of the function $f\left(\dfrac{VD}{\nu}\right)$ could be found from results in liquids of different viscosity. Our results indicate that it is $\left(\dfrac{VD}{\nu}\right)^0$, or independent of viscosity over practically the whole range investigated, but that the periodic shedding of vortices to which n is due, starts at a definite value of $\dfrac{VD}{\nu}$.

The evidence for the last statement is as follows. With a string tuned to a definite pitch n, it is generally possible to get an Æolian tone when $\dfrac{V}{nD}$ is 5. As the tension in the string is released, n and V for this tone fall, until at a definite value of n, no tone is produced at the appropriate value of V. The inference is, that the motion has become steady—vortices have ceased to be produced. The values of $\dfrac{VD}{\nu}$ when the tones failed to be heard were collected by the author and found to cluster round the value 30 for a cylinder, and 60 for a rubber cord of stream-lined section. The latter is designed to encourage steady motion, and we should expect, *ceteris paribus*, to require higher velocities before vortex motion with the consequent vibration of the body would set in.

The principle of dynamical similarity teaches us in this case that for equal values of $\dfrac{VD}{\nu}$ the motion in the stream will be similar round two bodies of the same shape but of different size. The series of transformations will be identical, though in a different scale, but will take place at different rates. This is what is implied

by the statement $\frac{V}{nD}$ is constant. If then the flow is steady (in stream lines) round one wire at a given value of $\frac{VD}{\nu}$, it will be steady round another wire of different diameter, but at the same value of $\frac{VD}{\nu}$. If vortices begin to form behind one wire at a critical value of $\frac{VD}{\nu}$, then vorticity will appear behind the other wire at the same critical value. That, after the initiation of this oscillatory motion, the similarity does not seem to depend on $\frac{VD}{\nu}$, is a consequence of the experimental fact that at all values of $\frac{VD}{\nu}$ above the critical, the vortices form at the same rate, depending on V only.

To sum up, viscosity in the guise of the expression $\frac{VD}{\nu}$ determines when vortical motion shall commence, but, once initiated, has no effect on the period.

The importance of this quantity was first pointed out by Osborne Reynolds,[1] who found that when fluid was in motion through a tube, turbulent motion set in at a definite value of $\frac{VD}{\nu}$. This case has no acoustical importance, but other periodic motions whose initiation depends on this quantity will appear in the following pages. The critical value of $\frac{VD}{\nu}$ is often known as the critical "Reynolds' number" of the particular type of flow.

Tones of Jets. We have considered above the tones which arise when a stream of fluid passes a linear obstacle; we must now consider the conjugate system of tones produced when a stream issues as a jet from a linear slit in an infinite plate into a stationary fluid. Surfaces of discontinuity arise between the issuing fluid and the stagnant fluid surrounding the orifice; the former tends to curl outwards into the stagnant fluid, and resolve into alternate vortices on each side of the jet, with spacing given by the Kármán formula $\frac{h}{l} = 0{\cdot}28$. As a matter of experimental fact, this system seems to have less stability when, as it does here,

[1] *Roy. Soc. Phil. Trans.*, 174, 935, 1883.

the fast moving fluid lies between the rows of vortices, and there is no certain evidence that such a system is actually produced in the absence of the resonant forcing discussed under organ pipes (p. 169).

The surfaces of discontinuity tend to break up into general vorticity; every vortex pattern seems equally unstable here, and it requires some *deus ex machina* to guide the vortices into a periodically repeated pattern; there is no solid resonator to take up the vibrations corresponding to the wire in Æolian tones.

As a result the tones produced by the issuing fluid are weak, uncertain and fluctuating. They partake of the characteristics of a "hiss," i.e., they correspond to a vortex formation at a high and unsteady frequency. For want of a recognized term we shall call them "Jet Tones": in Germany they are known indifferently as Ausflusstöne or Spalttöne. It is to be understood, unless mention is made to the contrary, that we are dealing with homogeneous jets. The issuing fluid has the same properties as that into which it issues, e.g., air into air, or water into water. Jet tones may also be produced when the orifice is circular, and as such were first observed by Cagniard de Latour. This type of motion may be observed at the orifice of a smoke-stack. Such jet tones formed the subject of an extensive investigation by Kohlrausch,[1] who also found that the general frequency of the tone rose proportionately with the velocity of efflux; the relation between n and D (diameter of orifice) however was not a simple one. Similar investigations have been made on the linear slit, the frequency being estimated by comparison with a sonometer, or in the case of a coloured liquid, by endeavouring to count the issuing vortices.[2] Both methods are fraught with so much difficulty that the results must of necessity be inconclusive.

A case of some interest, as it unites the jet and the Æolian tones, is that of the annular orifice formed between a tube and a concentric disc which nearly fills the end, leaving a ring-shaped opening. If the slit is wide so that the wall of the tube can exert little influence on the flow round the disc, it has been found that the motion corresponds to the Æolian tones for a disc in which the vortex filaments of the cylinder are replaced by embryo vortex-

[1] *Ann. d. Physik,* **13**, 545, 1881.

[2] Krüger and Schmidtke, *Ann. d. Physik,* **60**, 701, 1919. *See also* Fiorentino, *N. Cimento,* **15**, 177, 1908; Skinner and Entwhistle, *Roy. Soc. Proc.,* **91**, 481, 1915; Carrière, *Journ. de Physique,* **5**, 338, 1924.

rings, the liquid curling alternately out from and in towards the system. As the diameter of the tube is reduced, and the wall of the tube approaches the circumference of the disc, it exerts a modifying effect on the frequency of the formation of vortices, expressible by the formula : $n = \frac{V}{d}f\left(\frac{d}{D}\right)$, when D and d are the diameters of the disc and tube (inside measurement) respectively.[1]

Sensitive Jets and Flames. If the progress of a jet be made visible, either by allowing coloured water to emerge into clear water, or by mixing steam with the air which is issuing into the atmosphere, in general a cylindrical filament of issuing fluid will be observed, and this at a definite point breaks up into turbulent motion with radial spreading into the stagnant fluid. The precise cause of this sudden breakdown is obscure, but it is evidently connected with the viscous drag of the stationary on the rapidly moving fluid, which, after the stream has travelled a certain distance, causes such differences of velocity across the section of the jet, that the latter is not able to sustain the large shearing forces brought into play as a consequence, and breaks down into unsteady or turbulent motion similar to that which is believed to be set up in the boundary layer at points of rapidly changing contour. Using very slow speeds of efflux from a nozzle a millimetre or so in bore, and with the nozzle well shielded from accidental disturbances, it is possible to maintain such a jet for a foot or more without breaking up; but while it is in this condition a very slight increase of velocity applied to the issuing gas at the nozzle, even the act of clapping the hands, or sounding an instrument in a distant part of the room is generally sufficient to make the turbulent point jump back nearly to the nozzle; a periodic disturbance causes periodic changes in length.

Such a sensitive jet forms a satisfactory detector of sound, and is used in the form of a narrow ignited jet of inflammable gas, wherein the turbulent point can be distinguished by eye. The phenomenon is not altered in principle by igniting the gas, and the striking-back of the turbulent point to the nozzle is made visible by the sudden flare of the flame. Some light is thrown on the mechanism of the phenomenon by measurements of the length of such jets.[2] Coloured water was allowed to stream from a capillary nozzle (bore ·29 mm.) into a tank. Corresponding

[1] Tyler and Richardson, *Phil. Mag.*, **2**, 436, 1926.

[2] Richardson, *Nature*, **116**, 171, 1925.

measurements of the length of the jet to the turbulent point l, and the velocity of efflux V_0 were taken. The relation between V_0 and l is approximately a hyperbola.

The shape of these curves can be accounted for on principles of similarity. The point at which the jet breaks up is taken to be that at which the Reynolds' criterion $\frac{VD}{\nu}$ (D = diameter of jet and V = mean velocity at this point, ν = kinematic viscosity) has reached that value at which the motion becomes turbulent. If the initial velocity at the nozzle be increased, the critical velocity V will be reached at a point nearer the nozzle; l will therefore become smaller as the velocity at the nozzle increases. The effect of altering D or ν may likewise be predicted.

In a jet issuing from a circular nozzle vortices are formed periodically with frequency n, given by $\frac{V}{nD}$ = a constant, D being a linear dimension dependent on the bore of the nozzle [1] The jet should respond most readily to tones of this frequency, or sub-multiples thereof, and a high-velocity jet to sounds of high frequency. When a jet of gas is ignited the combustion complicates matters, as the visible part of the jet lengthens at first as the velocity increases; but Rayleigh showed that, over the range for which they are sensitive, such flames behave in most particulars like unignited jets, the progress of which is made visible by smoke. Experiment showed that a sudden small increase in the velocity of the gas feeding a sensitive flame brought the turbulent point—visible as a " flare "—nearer to the nozzle.

It is not so much the ignition which divides these from sensitive jets as the absence more or less complete of surface tension in the gas—gas interface. Both Brown [2] and Zickendraht [3] used a valve oscillator for the generation of the tones to which flames were subjected. Photographs of the flame under long exposure (to show visual appearance) and instantaneously are given in their respective papers. The former detected a characteristic frequency for each jet (*vide supra*) by the interesting phenomenon of beats between the jet and the oscillator. This characteristic frequency could readily be found by tuning out the beat, in the same way

[1] Tyler, *Phil. Mag.*, **16**, 504, 1933.

[2] *Phil. Mag.*, **13**, 161, 1932.

[3] *Helv. Phys. Acta*, **5**, 317, 1932; *see also* Humby, *Phys. Soc. Proc.*, **39**, 435, 1927.

that one tunes a wireless receiver. Symmetrical varicosities (to use the late Lord Rayleigh's expressive term) were observed to travel up such jets in the same way that they travel down water jets. More often, particularly with smoke jets, the vagarious motion is not symmetrical, being at its maximum in a plane passing through the jet and the source of sound. Seen from a perpendicular direction, the jet would often appear undisturbed. The sinuosities appear to start from a point a little above the orifice and fork outwards from this point with regularly increasing amplitude, very much as in edge tones (cf. p. 173).

The other common experience with sensitive jets and flames, that they are sensitive only over a small range of gas pressure, and, therefore, of efflux velocity, is shown by the curves and by theory. It is a consequence of the hyperbolic relation between l and V_0, that at a certain value of the latter a small increase in velocity due to aerial disturbances causes a large change in length, so that in using a sensitive flame it is necessary to work on the steep part of the curve (Fig. 57).

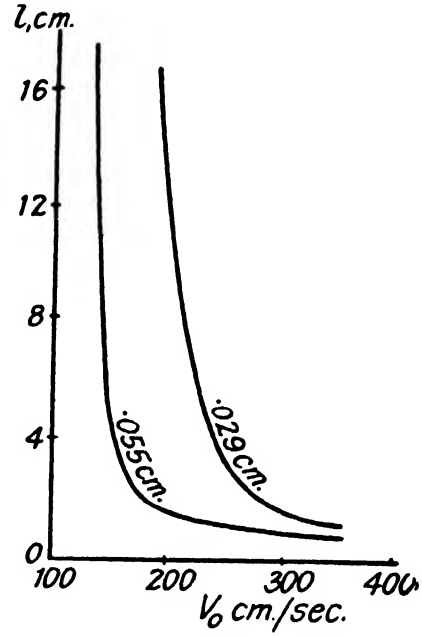


FIG. 57.—Variation in Length of a Sensitive Jet.

Various types of sensitive flame have been devised from time to time, depending on the range of pitch to which they are intended to respond. Tyndall[1] used pin-hole burners, with gas at high pressure for tones high up in the musical scale, coal-gas from the normal supply serving for lower notes. Tyndall recognized that the flame was merely an indicator of the evolutions of the jet of gas. These sensitive jets were formerly much employed as detectors, but have been largely superseded by the more precise instruments discussed in Chapter IX. Rayleigh[2] found that the nodes and antinodes in the stationary waves formed in the air between a source of sound and a wall, could be detected by this method; the flame responded best when placed in an

[1] *Phil. Mag.*, **13**, 473, 1857, and **33**, 375, 1867.
[2] *Phil. Mag.*, **7**, 149, 1879.

antinode, where the fluctuations in velocity were greatest, whereas the ear detected pressure alterations and maximum sound was heard at a node. A simple sensitive flame can be made by unscrewing the tube of a bunsen burner, and lighting the gas issuing from the nozzle which will be found in the base of the burner. This will respond to tones of moderate frequency. Better still, the gas may be lit above the tube in the usual place, and the air-holes below covered with a thin membrane of tissue paper, to which the sound is guided by a small conical trumpet attached to the bunsen.[1]

G. B. Brown[2] has carried out a very extensive and painstaking research on the sensitive jet, enriched with some beautiful photographs. His work is principally concerned with the vortices, the passage of which up the smoky column is the ultimate cause of the breakdown from the steady form, and *ipso facto* of the sensitivity to vibration. The observations were easier to interpret since the slit from which the jet emerged was linear instead of circular, as in the earlier work. Cinematograph films of the motion enabled the progress of the vortices up the jet to be followed. It is interesting that although the author finds approximate agreement between the experimental values of the ratio: velocity of vortices/velocity of jet, and that calculated by Rayleigh (i.e. 1/2) when the slit is wide, yet narrow jets exhibited much lower ratios. This discrepancy between practice and theory is commonly found in related phenomena (æolian tones, edge tones, Kundt's tube, etc.) when the orifice or obstacle falls below $\frac{1}{2}$ mm. in width. A small increase of vortex velocity with frequency was also found, but an important feature of all the films was that the rate (ω) of rotation of the vortex filaments was directly proportional to the frequency (f) of the sensitising sound and independent of jet size or configuration. The author therefore concludes that it is this specified rate of angular rotation which characterises the sensitivity of the jet, i.e., that for which $\omega = \pi f$ radians per second. He has also shown that the origin of the disturbance is at the orifice itself, since, if this part is shielded from the sound, the jet becomes insensitive.

[1] Mache, *Phys. Zeits.*, **20**, 467, 1919. *See also* Jordan and McClung, *Roy. Soc. Canada Trans.*, **18**, 197, 1924; Roberts and Meigh, *Phil. Mag.*, **23**, 368, 1912; Colwell, *Journ. Sci. Inst.*, **1**, 347, 1930; Meier, *Phys. Zeits.*, **35**, 524, 1934.

[2] *Phys. Soc. Proc.*, **47**, 703, 1935; Zickendraht, *Helv. Phys. Acta*, **7**, 773, 1934; Harding, *ibid.*, **7**, 655, and 804, 1935; Schiller, *Akust. Z.*, **3**, 36, 1938; (æolian tones) Holle, *Akust. Zeits.*, **3**, 321, 1938.

CHAPTER SEVEN

COLUMNS OF AIR

Vibrations of Air in Wide Tubes. We have already discussed in Chapter I the plane longitudinal vibrations of air in a wide tube, and have deduced the formula $c^2 = \frac{\gamma p}{\rho}$ for the velocity of sound in such a tube. It will be as well to enumerate the assumptions which are required by the classical theory in order that this formula and those set forth below shall apply. These are :—

(1) That the motion is uniform over the cross-section. This implies that the viscosity effect can be neglected, or the thickness of the "boundary layer" can be neglected in comparison with the width of the tube.

(2) That the oscillations are so rapid that there is not time for heat to be communicated between adjacent layers to equalize the temperature, and consequently the adiabatic formula may be applied.

(3) That vortices or rotatory motion are not set up in the tube.

(4) That the fractional changes of velocity and pressure in the waves are so small that their squares may be neglected.

Under these conditions, the longitudinal vibrations of the air correspond to those of particles of metal forming a rod, and the same equation ((2), p. 3) for the displacement will apply, with the value of the velocity given above :—

$$\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

A quantity frequently introduced in connection with vibrating columns of gas is called the "condensation" s, expressing the fractional change of density of the gas with respect to its original density ρ_0 :—

$$s = \frac{\delta \rho}{\rho_0} = -\frac{\delta v}{v} = -\frac{\partial \xi}{\partial x} . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

This is connected with the pressure change on the adiabatic assumption, as follows :—

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^\gamma = \left(1 + \frac{\rho - \rho_0}{\rho_0}\right)^\gamma = (1 + s)^\gamma = 1 + \gamma s \quad . \quad (58)$$

approximately when s is small.

The solution of (56), representing waves travelling up or down the pipe is :—

$$\xi = f(ct - x) \pm f(ct + x), \quad . \quad . \quad . \quad . \quad . \quad (59)$$

the simplest case being that in which the functions represent S.H.M. At an "open end," as for the "free end" of a rod, $\frac{\partial \xi}{\partial x} = 0$. At a "closed end," as for the "fixed end" of a rod, $\xi = 0$

These conditions with (59) determine the equation for the resulting stationary vibrations. When the origin is an open end :—

$$\xi = f(ct - x) + f(ct + x)$$

to satisfy these conditions ; when it is a fixed end

$$\xi = f(ct - x) - f(ct + x).$$

The tubes with which we meet in practice have one end open, and the other end either open or closed. When both ends are open, we always have antinodes at the ends, and a node in the middle ; the possible modes of vibration are in fact the reverse of those which obtain for the string.

Having regard to the end conditions $\frac{\partial \xi}{\partial x} = 0$, at $x = 0$, and at $x = l$, the possible stationary vibrations of the gas in such a tube are given by (cf. eq. (39), p. 86)

$$\xi = a_0 \cos \frac{j\pi x}{l} \sin \frac{j\pi ct}{l} \; ; \quad . \quad . \quad . \quad . \quad . \quad (60)$$

so that the displacement amplitude is a_0 at $x = 0$ or $x = l$. The factor j can have any integral value, so that the complete harmonic series is possible.

When the tube is open at $x = l \left(\frac{\partial \xi}{\partial x} = 0\right)$ and closed at $x = 0$ $(\xi = 0)$ the corresponding equation is :—

$$\xi = a_0 \sin \frac{j\pi x}{2l} \sin \frac{j\pi ct}{2l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

a_0 being the amplitude at $x = l$, and $j = 1, 3, 5$, etc. Partials

of even order are thus absent. This is a consequence of the fact that we must always have a node at $x = 0$ and an antinode at $x = l$.

The fundamental has thus a wave-length of $4l$, whereas the open tube has a fundamental wave-length of $2l$. The tube with one end closed produces its overtones by unequal subdivision in order to preserve a node and an antinode at its respective ends.

The wave-lengths of partials are shown in the following table :—

	j	1	2	3	4	5	6	7	
Open tube . . .	λ	$2l$	l	$\frac{2}{3}l$	$\frac{l}{2}$	$\frac{2}{5}l$	$\frac{l}{3}$	$\frac{2}{7}l$	etc.
Open-closed tube .	λ	$4l$	—	$\frac{4}{3}l$	—	$\frac{4}{5}l$	—	$\frac{4}{7}l$	

If, instead of quoting the displacement amplitude at any point, we prefer to give the pressure or density changes, we combine the relation $-\frac{\partial\xi}{\partial x} = s = \frac{\delta\rho}{\rho_0} = \frac{1}{\gamma}\frac{\delta p}{p_0}$ with (60) or (61) above; for example, in the open tube the pressure is given by :—

$$\frac{j\pi\gamma}{l} a_0 \sin\frac{j\pi x}{l} \sin\frac{j\pi ct}{l}$$

in terms of the normal pressure as unity. The amplitude of the pressure variation (when $p_0 = 1$) at the node in a tube containing air is therefore $1{\cdot}4\frac{j\pi}{l}$ times the displacement amplitude at an antinode.

Open-end Correction. The above theory supposes that the nodes and antinodes are formed exactly at the end of the tube. The assumption for the closed end cannot be gainsaid, for the unyielding material at the end fulfils our condition that $\xi = 0$ at this point. On the contrary the condensation is not zero $(s = \frac{\partial\xi}{\partial x} = 0)$ at the point where the tube debouches upon the atmosphere, for if $s = 0$, no density changes are possible at the end of the tube. But we know as a matter of fact that the waves are propagated in spherical type outside in the free air, otherwise the sound would be inaudible, so that the inertia of the air

in the neighbourhood of the mouth permits a certain amount of density variation there, although less than that possible within the confining walls of the tube. The point where $s = 0$ corresponds to a small negative value of x (if $x = 0$ at the end of the tube) equal to x_0 say, and the wave-length becomes $2(l + 2x_0)$ for a tube open at both ends.

Another way of considering the problem is in terms of the change of type of wave at the open end. If the antinode were strictly at the end, the waves would have to change from plane to spherical (round the middle point of the end cross-section as centre) with a sudden discontinuity. As this cannot be, the wave-front must gradually curve from the plane-front in the tube to a spherical one having a centre at $x = x_0$. As a consequence of this loss of energy in waves outside, the reflected wave in the pipe has diminished amplitude, causing, apart from viscosity, a damping and ultimate extinction of the sound in the tube, unless the energy is maintained from an external source.

The calculation of the addition necessary to the theoretical length of the pipe, due to the open end, has been made (first by Helmholtz,[1] then by Rayleigh [2]) only by considering an infinite flange flush with the end of the tube; the result shows that to the length of a tube of radius r must be added: $x_0 = \frac{\pi}{4}r = 0{\cdot}786r$ (Helmholtz), $x_0 = 0{\cdot}824r$ (Rayleigh).

The experimental estimation of the end correction is usually done by means of a resonance tube. This consists of a cylindrical tube whose distant end is closed by means of a movable piston, or else by a water surface whose level in the tube can be varied. This is used as a resonator to be tuned to a fork. Starting with the "working portion" of the tube as short as possible, the position of the "stop" in the tube is gradually lowered until the air in it resounds to the fork, and the length l_1 is noted. On further lengthening the distance between the open end and the stop, a second resonant length l_2 will be found. In the absence of end correction l_2 would be $3l_1$, according to the table on page 163. Accurately we have

$$l_2 + x_0 = 3(l_1 + x_0),$$

whence x_0 may be determined. Recent experiments give $0{\cdot}58r$

[1] *Wiss. Abhand.*, **1**, 303.

[2] *Roy. Soc. Trans.*, **161**, 77, 1870; *Phil. Mag.*, **3**, 456, 1877.

for a flangeless tube.[1] By widening out the open end into the familiar "bell" shape of wind-instruments, Helmholtz showed that a tube requiring no end correction could be made if the "bell" formed a hyperboloid (cf. also pp. 240–242).

Conical Tube. If the cross-section of the tube tapers but slightly the theory for the cylindrical pipe may still be applied. But when the tube forms a cone of large angle open at the wide end, sounds proceeding from the vertex will be propagated along the tube as spherical, rather than as plane waves, and be reflected from an end as such. At a point a little outside the end there will still be an antinode, while the vertex or any intermediate barrier will be a node. But when such a tube emits overtones, the intermediate nodes and antinodes will not be found at the same relative positions as in the cylindrical tube.

The extent of this deviation from the nodes of the corresponding cylindrical tube depends on the distance of the node in question from the vertex of the cone; accordingly the overtones in a wide-angled cone are inharmonic apart from any question of end correction.

The position of the nodes corresponding to different frequencies may be obtained by sinking the cone in water until resonance is shown with a tuning fork held over the open end, as Zamminer[2] did. As we have to deal with spherical waves diverging from the tip of the cone we should adopt the form of the general equation of wave propagation which suits this case, and the required form in polar co-ordinates is :—

$$\frac{\partial^2(rs)}{\partial t^2} = c^2\frac{\partial^2(rs)}{\partial r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

r being the radius of a wave, measured from the vertex, and s the condensation. In the actual pipe stationary vibrations will be set up as in the cylindrical pipe, and if we restrict ourselves to restoring forces proportional to rs, so that rs is proportional to $\sin(2\pi nt + \varepsilon)$:—

$$\frac{\partial^2(rs)}{\partial t^2} = -\omega^2(rs) = c^2\frac{\partial^2(rs)}{\partial r^2},$$

[1] Stücker, *Akad. Wiss. Wien. Ber.*, **116**, 1231, 1907; Borini, *Arch. des Sci.*, **1**, 541, 1919; Higgs and Tyte, *Phil. Mag.*, **4**, 1099, 1927; Cermak, *Phys. Zeit.*, **27**, 702, 1927; Anderson and Ostensen, *Phys. Rev.*, **31**, 267, 1928; Bate, *Phil. Mag.*, **9**, 23, 1930, and **10**, 617, 1930; Leopold, *Zeits. f. tech. Phys.*, **13**, 222, 1932.

[2] *Ann. d. Physik*, **117**, 173, 1855.

and the complete solution may be written :—

$$rs = a_0 \sin\left(\frac{2\pi r}{\lambda} + \delta\right) \sin(2\pi nt + \varepsilon) \quad . \quad . \quad . \quad (63)$$

When the cone is continued to the vertex ($r = 0$) we must have at this point s finite, whether the vertex is open or closed. Therefore from (63) $rs = a_0 \sin \delta = 0$.

1. *Open Cone.* First consider a conical tube continued to the vertex, and having the base, usually uppermost, open.

At the open end ($r = l$); $s = 0$ so that

$$a_0\left(\sin\frac{2\pi l}{\lambda}\cos\delta + \cos\frac{2\pi l}{\lambda}\sin\delta\right) = 0$$

This, with the vertex condition, combined with the fact that a_0 is not zero, gives :—

$$\delta = 0, \frac{2\pi l}{\lambda} = j\pi, \text{ or } \lambda = \frac{2l}{j},$$

where j is any integer. The harmonics are the same as for a cylindrical pipe of the same length open at both ends, and this, whether the vertex of the cone is open or closed.

2. *Closed Cone.* When the base of the cone is closed the requisite end condition is $\dfrac{\partial s}{\partial r} = 0$.

$$\frac{\partial s}{\partial r} = \frac{1}{r}\left(\frac{\partial(rs)}{\partial r} - s\right)$$

$$= \frac{a_0}{r}\left[\frac{2\pi}{\lambda}\cos\left(\frac{2\pi l}{\lambda} + \delta\right) - \frac{1}{r}\sin\left(\frac{2\pi l}{\lambda} + \delta\right)\right]\sin(2\pi nt + \varepsilon).$$

Equating this to zero, and adding the vertex condition $\delta = 0$ we get

$$\frac{2\pi}{\lambda}\cos\frac{2\pi l}{\lambda} = \frac{1}{l}\sin\frac{2\pi l}{\lambda}$$

$$\frac{2\pi l}{\lambda} = \tan\frac{2\pi l}{\lambda}.$$

If we put the solution of this, $\dfrac{2\pi l}{\lambda} = j\pi$, j is no longer integral but may have the following values, 0, 1·43, 2·46, 3·47, 4·47, 5·48, etc., so that the overtones of a cone closed at the base are inharmonic, and the nodes of a harmonic are not equidistant along the pipe.

The pressure variation down a conical pipe has been investigated by Webster,[1] one of whose curves is shown in Fig. 58.

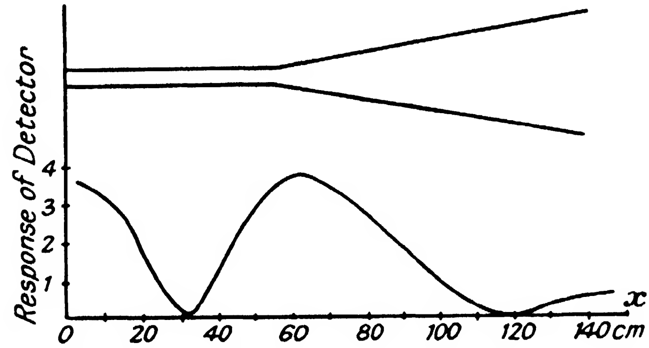


FIG. 58.—Variation of Pressure-Amplitude along Conical Pipe.

The Open Cone as Director and Amplifier. It is customary to put the source of sound at the vertex of a conical horn when it is desired to direct the sound, as in the " megaphone." Rayleigh pointed out that this object can be attained only if the mouth, i.e., the base of the cone, is large compared with the wave-length of the tone emitted, otherwise diffraction will ensue at the mouth, tending to produce equal intensity in all directions. To propagate fog-signals over the sea without unnecessary spreading in a vertical direction, he suggested a rectangular aperture for the horn, prolonged in a vertical direction but narrow horizontally. Recent research suggests that a cone's directive or collective properties are small compared with its amplifying properties. Work by Stewart [2] and by Foley [3] has shown that the amplification due to a conical receiver is much less than theory would predict, if the cone " collected " all the energy falling upon its wide mouth. A considerable proportion of the incident energy is reflected back the way it came without penetrating the cone (cf. also pp. 241, 321).

Scale of Tube or Pipe. One effect of the width of a tube has already been discussed in the first chapter, i.e. that on the velo-

[1] *Nat. Acad. Sci. Proc.*, **6**, 316, 1920.

[2] *Phys. Rev.*, **16**, 313, 1920.

[3] *Phys. Rev.*, **20**, 505, 1922. *See also* Rayleigh, *Advis. Comm. Aeronautics*, No. 1618, 1915; Watson, *Phys. Rev.*, **11**, 244, 1918; Cloud, *Phys. Rev.*, **22**, 73, 1923; Hoersch, *Phys. Rev.*, **25**, 218, 1925; Kennelly and Kurokawa. *Am. Acad. Proc.*, **56**, 3, 1921; Kennelly, *Frank. Inst. J.*, **200**, 467, 1925; Sato, *Rep. Tokyo Aero. Res. Inst.*, 42, 1928; 56, 1930; and **7**, 339, 1933; Trendelenberg, *Zeits. f. tech. Phys.*, **10**, 558, 1929; Larmor, *Camb. Phil. Soc. Proc.*, **30**, 342, 1934; Goldman, *Acoust. Soc. J.*, **5**, 181, 1934.

city of the sound due to the dragging of the walls. The tubes used as sound sources of the type we are discussing in this chapter are wide enough for this drag to be neglected, but there is another important question linked up with the relation of the width to the length, or "scale of pipe" as it is termed. This concerns the ease of production of overtones.

Tubes of very small scale are employed sparingly as connections between the reeds and the main vibrating column in certain bass instruments. When the diameter is a few millimetres only, the whole of the air in the tube may lie within the "boundary layer," so that its movements are, as a whole, subject to viscous drag. In this case not only is there rapid attenuation of the amplitude as the sound is propagated along the tube, but its slow speed reduces the wave-length, and therefore the length of the tube which can "sound" to a given frequency. It was pointed out on p. 143 that the thickness of the boundary layer was of the order $\sqrt{\frac{\nu l}{V}}$, where l is the length of the boundary over which the air rubs. When the air is executing S.H.M. this length is represented by the amplitude a, and the mean velocity by $2\pi na$, therefore the thickness of the layer resisting the S.H.M. is proportional to $\sqrt{\frac{\nu a}{na}}$ i.e., to $\sqrt{\frac{\nu}{n}}$. It can in fact be shown [1] that the velocity of sound in such a tube of radius r, should be :—

$$c\left(1 - \sqrt{\frac{2\nu}{n}} \cdot \frac{1}{2r}\right), \qquad . \quad . \quad . \quad . \quad . \quad . \quad (63a)$$

a formula first deduced by Helmholtz,[2] but modified by Kirchhoff to include the effects of heat conduction which virtually amounts to an increase in ν.

A considerable amount of experimental work has been done to test that, in accordance with the formula, the diminution of velocity below the open-air value is inversely proportional to the radius of the tube and the square root frequency. In particular Kaye and Sherratt [3] have experimented with tubes of various diameters from 2·9 down to 0·9 cm. made of various materials and filled with four different gases in turn. They are satisfied that the Helmholtz-Kirchhoff formula sufficiently represents the facts. This, of course, is of great importance to know since so many measurements of the velocity of sound

[1] Richardson, *Roy. Soc. Proc.*, **112**, 534, 1926. *See also* Foley, *Phys. Rev.*, **14**, 143, 1919; Simmons and Johansen, *Phil. Mag.*, **50**, 553, 1925; Cornish and Eastman, *Phys. Rev.*, **33**, 90 and 258, 1929; Wold and Stibitz, *Science*, **66**, 381, 1927; Henry, *Phys. Soc. Proc.*, **43**, 340, 1931; Vance. *Phys. Rev.*, **39**, 737, 1932.

[2] *See also* Rayleigh's *Sound*, **2**, 324, 1877.

[3] *Roy. Soc. Proc.*, **141**, 123, 1933.

involve a "tube correction" based on (63*a*). For very narrow tubes (less than 3 mm. diameter) the correction is not so certain and further work on them is needed.

Edge Tones. In order to maintain the stationary vibrations in a column of air, it is customary either to direct a blast of air on to a sharp edge at one end of the tube, or to direct air through a channel periodically closed by a "beating reed" leading to the tube. The first device is employed in the "flute organ pipe," and whistle, the second is applied to all wind instruments, either with an *ad hoc* reed, or pair of reeds, or with the player's lips to act as reeds. It was first noticed by Masson[1] and by Sondhauss[2] that when a blast of air is directed against an isolated sharp edge, tones can be produced in the absence of the column of air. Commonly the air issues from a linear slit a few millimetres wide and several centimetres long, and strikes a sharply bevelled edge of wood or metal placed in the plane of the issuing lamina of air. In the last chapter we saw that such a lamina has a tendency to form vortex filaments as it emerges into the undisturbed air on either side of the stream, and that, if a stable system of vortices is formed, these vortex filaments will be alternately spaced, one on each side of the issuing air stream.

Now it appears that when the air strikes an edge, the space between slit and edge acts as a form of resonator, so that the length f from slit to edge, becomes equal to, or a multiple of, the "wave-length of the vortex system" (applying this term to the distance between successive vortices in the same row). Taking the simplest case, a vortex A (Fig. 59) leaves the outer wall of the orifice as the preceding one on the same side B strikes the edge. This obscure action of the edge has been explained in several ways by the authors in the references cited, but it is probably connected in some way with the secondary vortices formed in the boundary layer alongside the edge itself, and which have to "fall into line" with those coming from the slit. There is a minimum distance f_0 for any given velocity of efflux V at which a tone can be produced. When the separation between edge and slit is a little greater than f_0, the state of affairs is as shown in Fig. 59*a*. The vortex B has just struck the edge as the next one on the same side is emerging from the slit. The frequency of the "edge tone" is given by the frequency with which the

[1] *Comptes Rendus*, **36**, 257 and 1004, 1853.

[2] *Ann. d. Physik*, **91**, 214, 1854.

eddies A, B, etc., strike the edge. If they move towards it with velocity U and l = distance between the vortices in the same row, as before (p. 150), then

$$n = U/l,$$

or since $l = f_0$, and $U = aV$, where a is constant (cf. p. 150)

$$V/nf_0 = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad (64)$$

When V is kept constant, i.e., when constant pressure is maintained behind the slit, as f is increased beyond the minimum f_0, the pitch of the "edge tone" falls in accordance with (64) until at a value f_1 approximately double of f_0, the system becomes unstable, and the tone which is now the sub-octave of the original tends to jump up an octave to what it was at f_0. This jump of an octave we may attribute—and experiment verifies the deduc-

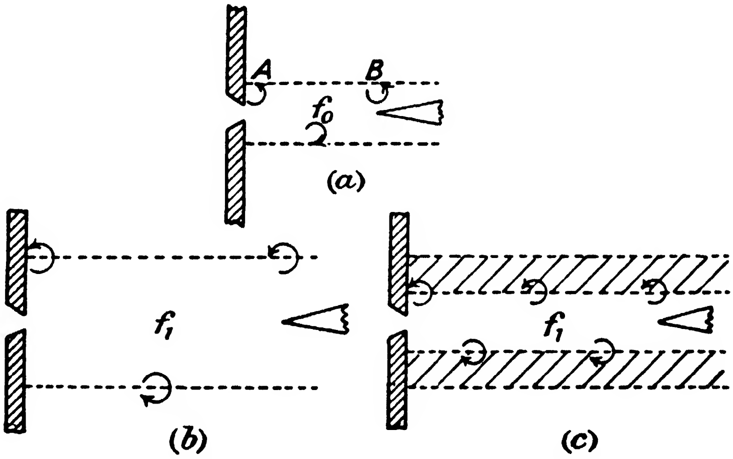


FIG. 59.—Edge-Tones.

tion—to a return to the original spacing of the vortices, but with twice as many between slit and edge. Fig. 59*b* shows their relative positions just before the transition. l has doubled itself with f, and therefore the width of the avenue in accordance with the Kármán law (p. 148) is also double what it was in Fig. 59*a*. After the jump (Fig. 59*c*) l resumes its original value so that $f = 2l$, and, still in agreement with the rule, h is halved, and the original narrow avenue is recovered. The edge must bisect the two rows of vortices if the tone is to be elicited; consequently, in the shaded area of Fig. 59*c*, representing the space between the wider and the narrow avenue at the transition, only the deeper tone can be elicited. In fact after the jump has taken place, the pre-transition tone can be brought back either (1) by moving the edge into

the shaded region, or (2) by pushing an obstacle from the side partly into the path of the blast in Fig. 59*c*, and so deflecting the shaded portion of the stream on to the edge.

For experimental purposes the slit may be formed of stream-lined brass plates, let into an otherwise airtight wooden box of about 20 litres capacity, fed with compressed air from a cylinder. The edge is of steel, tapering from about 1 cm. to a razor edge in 10 cm.; it is firmly fixed to a micrometer traverse as fine adjustments across the stream may be necessary. The slit should lie between ½ and 1 mm. Both slit and edge are 5 cm. deep.

If f is kept constant and V gradually increased, n increases in accordance with (64), and jumps to the octave above when the vortices rearrange themselves, the original tone being recoverable by the same means. One or other of these devices is used on certain open organ pipes, when the blowing-pressure is increased until the edge-tone and with it the pipe-tone jumps to the octave, and is brought back by deflecting the edge or deflecting the air by a wooden " bridge " pushed into its path from the outside. Owing to the greater blast speed employed, more energy is available for sound production with this artificially lowered octave, and a brighter quality of tone is imparted to the pipe than by that which we may call the natural edge-tone corresponding to the smaller f or V.

The existence of a series of these breaks was disclosed by Wachsmuth,[1] who compared the tones with a sonometer. On the basis of some further work by Göller, König[2] proposed a more general formula in place of (64), which with slight alteration we may write :—

$$\frac{Vj}{nf} = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (65)$$

where $j = 1$ from f_0 to f_1, $j = 2$ from f_1 to f_2, etc., and the values for f at the breaks are connected with the minimum f_0 by $f_0 = \frac{f_1}{2} = \frac{f_2}{3} = \frac{f_3}{4}$, etc. Under favourable conditions, half a dozen breaks may be noticed.

This equation has been verified for various gases by Rieth,[3] measuring the average value of V by the fall of the resistance produced in a thin platinum wire heated electrically, placed in

[1] *Ann. d. Physik*, **14**, 469, 1904.
[2] *Phys. Zeits.*, **13**, 1053, 1912.
[3] *Diss. Gieszen*, 1917.

the path of the blast; and for water by Schmidtke,[1] who was able to observe the formation and path of the vortices at the surface of the water. Most careful measurements in air have recently been made by Benton,[2] who has been able to measure h by finding the limiting position of the edge where the tone still continues—presumably this is when the edge lies on the dotted lines of Fig. 59, and no longer parts the two rows of vortices. He finds the value $h/l = h/f = 0{\cdot}27$, approximately the value given by Kármán's calculation. In order to get agreement with Kármán's theory in Æolian and edge-tone phenomena, simplicity in the apparatus is essential. Thus Carrière finds himself in disagreement with the applicability of the Kármán formula to the phenomena at the mouth of an organ pipe. Benton points out that it is possible to demonstrate the application of the theory, if the apparatus is sufficiently "bed-rock" in conception.

Besides the minimum f for tone production, there is a minimum V. These facts are connected with the critical value of $\frac{VL}{\nu}$ discussed in the last chapter, below which vorticity is not present. The linear dimension L is a complex function of f, and of the diameter of the slit, and of the form of the channel through which the fluid has to flow before emerging from the slit. Therefore L is an uncertain quantity, but in confirmation of what was found for Æolian tones, viscosity plays little or no part in the motion once initiated. $\frac{Vj}{nf}$ is practically a "universal" constant for all gases (value 2·0).

An alternative explanation of the edge tone formula may be derived from a consideration of the secondary vortices which may be seen in smoke photographs to form in the boundary layer of the edge itself as the fluid passes along it. These have to fall into their proper position along the two vortex avenues thus formed on each side of the edge. We have the vortices on one side of the "street" starting from the slit (S, Fig. 60) while their fellows on the other side start from the tip of the edge (E) with the same velocity, and the only way in which the procession can

[1] *Ann. d. Physik*, **60**, 715, 1919.

[2] *Phys. Soc. Proc.*, **38**, 109, 1926. *See also* Krüger, *Ann. d. Physik*, **62**, 623, 1920; Richardson, *Phys. Soc. Proc.*, **43**, 394, 1931; Klug, *Ann. d. Physik*, **11**, 53, 1931; Zahradnicek, *Phys. Zeits.*, **34**, 602, 1933; Carrière, *Rev. d'Acoust.*, **2**, 333, 1933.

marshal itself into a Bénard-Kármán avenue of alternate vortices is for one to roll from the edge at the same time as another rolls up from the slit itself. That the vortices do not spring fully grown from these points does not affect the argument if we assume, as visual observation shows, that they all roll up or grow at the same rate.

Edge tones may also be produced by allowing fluid from an annular slit to strike an annular edge. If the latter forms the end of a cylindrical metal tube, an organ pipe may be produced. An organ pipe was so fashioned by Gripon,[1] and the edge-tone formula has been subsequently verified for this case by Krüger and Marschner[2] (but cf. p. 156). Examination of the motion shows that the fluid curls alternately inwards and outwards form-

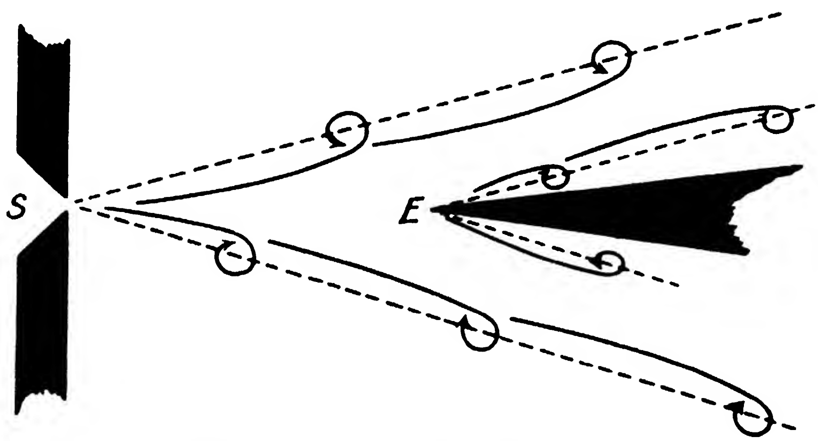


FIG. 60.—Primary and Secondary Edge-tone Vortices.

ing embryo vortices alternately larger and smaller in diameter than the slit; the difference of the radii measures the quantity h. Owing to the difficulty of fixing the pipe to the mouth-piece, organ pipes on this principle are not used, but the "bell-whistle" on locomotives is of this type, as it consists of a short bell-shaped resonator, rigidly fixed by a central pillar to the centre of the annular orifice from which it is blown.

Yet a third system of edge tones is employed in the bird-call. The orifice is a circular hole in a thin plate, while the edge is formed by the circumference of an equal and concentric hole in another plate. As f is made very small, the apparatus forms a compact instrument for obtaining high-pitched tones. Another source of sound related to these edge-tone producers was the metal hose-pipe used by the Germans during the European War

[1] *Ann. d. Physik,* 8, 384, 1874. [2] *Ann. d. Physik,* 67, 581, 1922.

as a substitute for rubber tubing. This consisted of metal tube about 1 cm. diameter, having spirally-bored holes of several hundred windings per metre in the wall of the tube. On blowing through a length of such tubing with the mouth a tone is heard. Cermak[1] ascribes this tone to the wind striking against alternate edges of the spiral bore, so that each convolution may be regarded as the pseudo wave-length of the disturbance. If the mean velocity of the air down its enforced spiral path is V, and if f is the length of one convolution, then $V/f = n$, the frequency of the tone given out. As the whole length of the bore, just like a straight tube, has a fundamental and harmonic series of overtones, the tone is audible only when resonant vibration is excited in the column of air as a whole, that is when n coincides with one of these overtones. These statements were experimentally verified by Cermak.

Hartmann's Jet Oscillator.[2] This may be regarded as an apparatus for producing supersonic edge tones, for it consists essentially of a jet of gas emerging from a nozzle at a speed exceeding the velocity of sound and impinging on a coaxal ring-shaped edge, which may be the mouth of a small bottle-resonator (Fig. 61) somewhat like the Galton whistle (p. 277). The mechanism

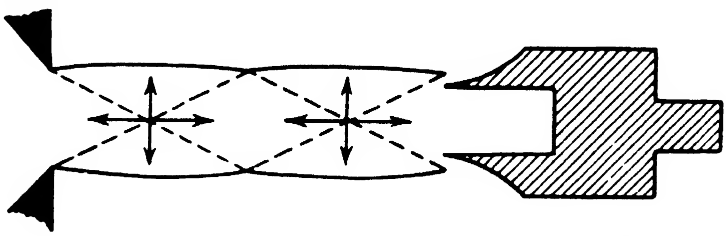


Fig. 61.—Jet Oscillator (*Hartmann and Trolle*).

is not however quite the same, for Schlieren photographs of the jet show waves of compression reflected from the confines of the jet in criss-cross fashion (dotted lines). If the mouth of the resonator is placed in one of these zones of reflection as shown in the figure, its natural frequency is excited with considerable intensity owing to the large energy in the jet. Evidently these zones are regions of instability as in other places no excitation of the resonator is produced. The frequency—usually above the audible limit—of the jet itself apart from the resonator is deter-

[1] *Phys. Zeits.*, **23**, 349, 1922, and **25**, 121, 1924. *See also* Burstyn, *Zeits. tech. Phys.*, **3**, 179, 1922.

[2] Hartmann and Trolle, *J. Sci. Inst.*, **4**, 101, 1927.

mined by the distance between these zones as wave-lengths, and the velocity of the jet. Besides this there is a slow pulsation of the period of several seconds as the resonator fills with gas up to a high pressure and then exhausts back into the jet. This latter is probably a relaxation oscillation.

Flue Organ Pipe. Having shown how the vorticity in the jet from a linear, circular or annular orifice may be stabilized by making it strike a suitably placed edge, we are now in a position to discuss the maintenance of aerial vibrations in a tube by the edge tones from such an orifice, generally a linear slit. Fig. 62 shows a section of an organ pipe called "stopped diapason" which is of wood, and of rectangular section. The wind at a constant pressure of several inches of water enters the mouthpiece M, then emerges from the slit O and strikes the edge E, formed by bevelling the wall of the pipe. An adjustable stop S closes the pipe, so that the "speaking length" is from the neighbourhood of O to S. Wachsmuth[1] was probably the first to recognize the organ pipe as a coupled system, i.e., the edge tones at the mouth coupled to the natural frequencies of the column of air in the tube. He says (1904) "the tone of a flue organ pipe is always one of the possible edge tones determined by f, by the blowing pressure, and by the length of the resonance tube." The f of the edge-tone formula obviously corresponds to OE. At a given blowing pressure, and therefore at a given V, a value of f can be found at which the pipe "speaks" most readily, but the pipe gives the same tone at neighbouring values of f. This is the generally accepted view to-day; before the present century it was thought that the issuing lamina of air vibrated transversely like a reed, at a frequency governed entirely by the column of air, but the notion has been superseded. To Wachsmuth and to Hensen[2] we are indebted for the modern conception of the functioning of flue organ pipes. By mixing smoke with the issuing air and examining it stroboscopically, or by taking instantaneous

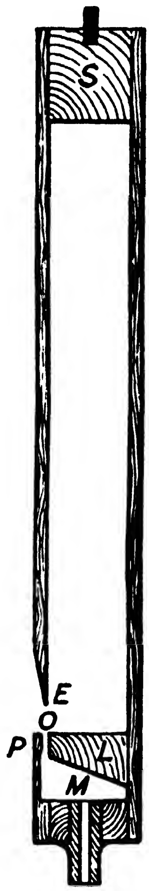


Fig. 62.—Flue Organ Pipe.

[1] *Ann. d. Physik*, **14**, 469, 1904.

[2] *Ann. d. Physik*, **2**, 719, 1900, and **4**, 41, 1901, and **21**, 781, 1906.

photographs, van Schaik[1] and also Weerth[2] was able to show the existence of these movements in the issuing air stream, though as Bénard's work was still to be done, they understood the mechanism of the edge tones imperfectly. Carrière[3] has recently drawn the appearance of the smoke under the stroboscope, showing the analogy with the edge-tone photographs of Schmidtke. The organ pipe should be designed so that at normal blowing pressure, the edge tone $n \propto \frac{Vj}{f}$ is equal to the fundamental of the pipe, so that pipe and edge tone are coupled at resonance. In accordance with this principle (though not in recognition of it) organ-builders make f continually decrease from bass to treble on each "stop" of pipes. Such a coupled system as that with which we are here concerned is governed in the main by the less strongly damped component, in this case the column of air. A certain amount of mutual accommodation takes place between pipe and edge tone, but whereas the edge tone may be considerably pulled out of its natural period of vibration in order to secure equality of period, that of the column of air is alterable to a very small extent. Analogies may be found with the coupled vibrations of tuning fork and string which form the basis of Melde's experiment (p. 118). To exemplify the properties of the coupling, we may suppose the blowing pressure to be continuously increased beyond the normal. As V is increased, the natural frequency of the edge tone rises beyond the fundamental of the pipe, but the latter succeeds in forcing its own period upon the edge tone until the natural frequency of the edge tone, if isolated, would be nearer to the first overtone of the pipe than to the fundamental. Up to this moment the frequency of the coupled system has remained in the neighbourhood of the fundamental of the pipe, but now a jump occurs to the overtone, both edge tone and pipe tone rapidly picking up the new frequency which they retain with slight alteration until a jump to the next overtone takes place. Actually the behaviour is more complex in that the overtone may appear before the fundamental has ceased, producing a complex note. The procedure is shown graphically in Fig. 63, where the actual pipe tones of an organ pipe are shown by thick lines, and the natural frequency of the edge tones in the absence of the pipe, by a dotted line.

[1] *Archiv. Néerl.*, **25**, 308, 1892. [2] *Ann. d. Physik*, **11**, 1036, 1903.
[3] *J. de Physique*, **6**, 52, 1925; **8**, 215, 1927; Bate, *Phil. Mag.*, **8**, 750, 1929.

This increase of V resulting in the formation of overtones is known as "overblowing" the pipe.[1]

The converse phenomenon of the "underblown" pipe presents several items of interest. Suppose V to be continuously decreased below normal. Very soon the edge tones will be so far below the pipe tone that the latter ceases to sound. But now, the two systems being still coupled, the pipe is still endeavouring to impose one or other of its own tones upon the vortex production at the mouth, and in an endeavour to conform, the edge tone reduces its pseudo "wave-length" by a transition like that described on p. 171 to $\frac{1}{2}f$, $\frac{1}{3}f$, etc., and the same tone is again, but feebly, elicited. Thus if the fundamental is normally produced at V_1, so that $\frac{V_1}{n_1 f}$ = constant, it may also be produced under conditions given by:—

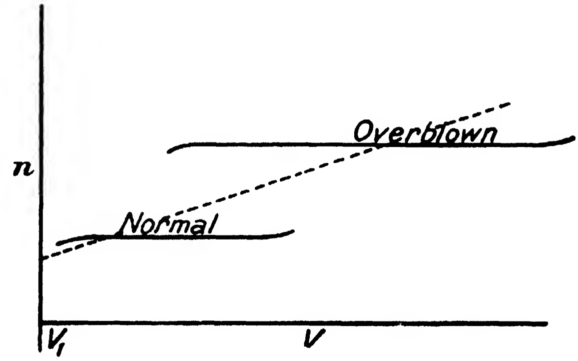


FIG. 63.—Tones of Normal and "Overblown" open Organ Pipe.

$$\frac{\frac{1}{2}V_1}{n_1\frac{1}{2}f} = \frac{\frac{1}{3}V_1}{n_1\frac{1}{3}f} = \text{a constant,}$$

or the octave (in the case of the open pipe) given by:—

$$\frac{\frac{1}{2}V_1}{2n_1\frac{1}{4}f} = \frac{\frac{1}{3}V_1}{2n_1\frac{1}{6}f} = \text{a constant.}$$

These transitions (shown in Fig. 64) can the more easily take place if f is large—if the mouth is "cut up high"—as the organ-builder phrases it. Thus we are faced with the curious fact, that overtones can be formed both by extra and reduced pressure in blowing the pipe. Of course the

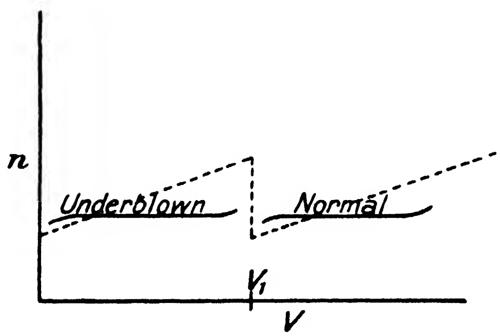


FIG. 64.—Tones of "Underblown" open Organ Pipe.

[1] *See* Lough, *Phil. Mag.*, **43**, 72, 1922; Bhargava and Ghosh, *Phys. Rev.*, **20**, 452, 1922; Carrière, *J. de Physique*, **3**, 7, 1926.

intensity of the sound is very different under the two conditions. The input of energy to the pipe is measured by the product pressure of wind × volume delivered per second, for this is the work which the blower has to do per second. A certain percentage of this—which we may call the "efficiency" of the pipe—is converted into sound. Now V^2 is proportional to the blowing pressure, and the volume delivered per second is V times the area of the slit, so that the intensity of the sound, *ceteris paribus*, varies as V^3. Therefore the tones of overblown pipes are louder than normal, whereas those of under-blown pipes are very feeble. They may be heard when the air is first driven into the pipe—this is technically known as "coughing" or "murmuring"—but may be extinguished by so designing the mouth, that the jet is not directed on to the edge until normal pressure is reached (cf. p. 170).

The stopped pipe is tuned by adjusting the position of the stop; the open pipe may be sharpened by cutting off part of the end of the tube. What is known as "voicing" is an adjustment of the coupled system to give the note required, and is mainly accomplished by operations on the mouth.

Some of the considerations which determine the quality of the note are enumerated here.[1]

1. The "scale" of the pipe, i.e., ratio of diameter to length.
2. Height of the mouth.
3. Shape of upper lip or "edge"; whether thick or thin, convex or concave.
4. Shape and position of languid (see Fig. 62; *L*).
5. Nicking of lower lip, *P*.
6. Material of the tube.

The explanation of some of these points has been given in our text, others are at present beyond scientific explanation. One is reluctantly compelled to admit that the organ-builder with his empirical methods is years in front of the physicist.

Reed Pipes. In this class of sound-producing systems a column of air is closed at the mouth by a "beating reed" (cf. p. 117). Normally the reed stands clear of the orifice leading to the pipe, but when wind under pressure is introduced to the chamber surrounding the reed, it is deflected by the rush of air into the pipe, until it shuts up the pipe orifice; then, being under tension, it springs back and lets the air into the pipe again, and the cycle

[1] Bonavia Hunt, *Church Organ*, 68, 1920.

is repeated. This coupled system consists of "bar" in transverse vibration, and "air column" in longitudinal vibration. Various possibilities may occur, depending on the form of the "coupling."

Firstly, let us consider the reed organ pipe. The air tube or pipe proper is generally conical. The reed, generally of brass, is at the vertex, and is contained at the end of a narrow tube, to which one end of it is clamped. The reed and this narrow tube are enclosed in a short supply pipe termed the "boot." The conical pipe and the reed are tuned to the same frequency, the tuning being accomplished by a spring which presses on the reed near its clamped end, shortening it or lengthening it as desired (Fig. 65). The resulting note is due to the fundamental of the reed with the coincident fundamental of the pipe, together with some overtones of the latter. As the overtones of the reed are inharmonic, these are not produced because they do not form part of the "note" of the column of air. The vibrations of the reed are always simple harmonic although the air vibrations involve the overtones proper to the form of the air column. Trouton[1] first observed that the length of the supply-tube exerted a control on the tone of the complete pipe, which is really a tripartite system. If the length of the supply tube is of the order of the length of the pipe, the best arrangement is to have the former about a quarter of the wave-length, or an odd multiple of a quarter. The reed apparently vibrates most freely when placed near a node of the column of air in supply-tube and pipe proper.

FIG. 65.—Reed Organ Pipe.

Examination of Phenomena inside the Pipe. We have already dealt with the behaviour of the column of air in a pipe as given by theory; and it now remains to describe the methods employed to test what is happening to the air in the pipe. The first methods will be qualitative, later some methods claiming to give quantitative results will be described. The methods may be classified in accordance with the particular property of the air

[1] *Roy. Dublin Soc. Proc.*, 132, 1888. *See also* Hoppe, *Ann. d. Physik*, **39**, 677, 1912; Vogel, *Ann. d. Physik*, **62**, 247, 1920; Carrière, *J. de Physique*, **5**, 338, 1924; Vogel, and Max Wein, *Ann. d. Physik*, **62**, 649, 1920; Auger, *Comptes Rendus*, **195**, 516, 1932.

which they measure, as displacement, pressure, density or temperature. Most of these instruments were originally used on organ pipes but can be adapted to columns of air maintained in vibration in other ways.

One of the earliest methods of obtaining an idea of the relative displacement in different parts of the pipe was to lower a little paper membrane formed on a ring and covered with sand into the pipe held vertically. Maximum agitation of the sand particles is shown at an antinode, minimum agitation at a node. A corresponding pressure indicator is the manometric capsule devised by König.[1] This consists of a membrane of parchment or thin rubber gripped between two rings, dividing the capsule into two halves. One half can be sealed by glue or wax to any point of the pipe

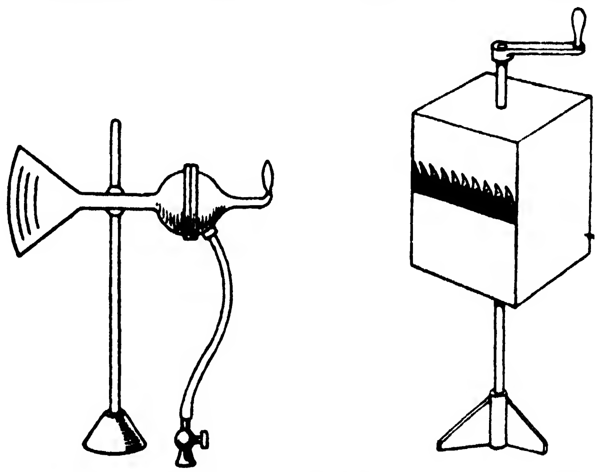


FIG. 66.—Manometric Flame (König) and Revolving Mirror.

at which a hole is made, through which the pressure variations at the point in question are communicated to the membrane. The other side of the membrane faces a little box, closed save for an inlet by which coal-gas impinges on the membrane from a supply, and a gas outlet ending in a pin-hole burner. The gas lit at this burner acts as a sensitive indicator of the motion of the membrane by its movement up and down; to this end both inlet and outlet should point directly at the membrane. The flame is also able to show the presence and relative intensity and phase of any harmonics which may be present in the pipe, but owing to its inconstancy the instrument is unsuited to absolute measurements of the pressure amplitude. The motion of the flame can be examined stroboscopically, or in the revolving mirror invented by Wheatstone (Fig. 66).

[1] *Ann. d. Physik.* **146.** 161. 1872.

When it is desired to record the note of a pipe, a record of the flame's movement can be obtained, either by passing a strip of paper over the flame, when its soot will form a periodic record on the paper (Brown),[1] or by mixing as much acetylene with the gas as the flame will take without becoming smoky, and photographing the motion on a rapidly moving plate. By this means Merritt[2] obtained records of speech and musical notes directed on to the membrane of the capsule, in the open atmosphere.

Kundt's dust tube method has already been described and may be used for an organ pipe if one side at least be of glass, indeed blowing such a pipe with another gas forms an additional method for finding the velocity of sound in the gas.

The position of the nodes and antinodes may be found by a thin listening tube pushed into the pipe from one end, and connected to the ear. A minimum sound will be heard at an antinode. As the search tube is liable to interfere with the motion in the pipe, König[3] replaced one side by a water surface, and pushed a tube bent twice at right angles through the water into different parts of the pipe.

Measurement of the Pressure Amplitude in the Pipe. A liquid manometer of the bent tube type possesses too much inertia to record the small but rapidly alternating changes of pressure in the pipe. To obviate this, Kundt[4] conceived the idea of admitting the compressions only to the manometer by means of a valve which opened only outwards. This was placed at a node of the pipe, and the manometer showed a steady increment over the atmospheric pressure, by which Kundt claimed to measure the pressure amplitude at the node. Raps[5] endeavoured to improve this valve manometer by a mechanically operated valve which opened at the phase of maximum compression, at a frequency which was made to coincide with that of the pipe.

The valve itself exercises a considerable influence, both on the readings of the gauge, and the performance of the pipe, so that

[1] *Phys. Rev.*, **33**, 442, 1911.

[2] *Phys. Rev.*, **1**, 166, 1893. *See also* Marbe, *Phys. Zeits.*, **7**, 543, 1906; and **8**, 92, 1907; Seddig, *Phys. Zeits.*, **8**, 449, 1907; Athanasiades, *Comptes Rendus*, **145**, 1148, 1907, and **146**, 533, 1908; Marbe and Seddig, *Ann. d. Physik*, **30**, 579, 1909.

[3] *Ann. d. Physik*, **42**, 549, 1891; Marty, *Ann. de Physique*, **1**, 622, 1934.

[4] *Ann. d. Physik*, **128**, 337, 1866. *See also* Dvorak, *Ann. d. Physik*, **150**, 410, 1873; Rilbentrop, *Zeits. f. tech. Phys.*, **13**, 396, 1932.

[5] *Ann. d. Physik*, **36**, 273, 1882.

it is not surprising to find that the values by these stroboscopic manometers do not agree with later measurements, being in fact much too large. A convenient and simple membrane manometer may be made on the lines of the manometric capsule, but with the flame replaced by a mirror as indicator.

Fig. 67 shows a full-size drawing of the instrument convenient for application to a wooden pipe. The capsule widens out conically from a short cylindrical piece to a flange between which and a corresponding ring there is fixed by screws a membrane of the thinnest sheet rubber tightly stretched in order that its natural frequency may be high. The mirror has to be given an angular motion in order that it may deflect a spot of light on a scale. To accomplish this, the mirror may be placed on a little lever which presses on the membrane. It must be remembered, however, that a membrane has its natural frequency considerably lowered by a load of this kind, and one wants to keep this frequency above the range of possible harmonics of the pipe. Because of this it is better to cement the mirror by rubber solution in an eccentric position. On the instrument used by the writer the mirror was made by silvering a tiny fragment, about 1 mm. square, cut from a microscope slip. This was fixed at a distance equal to half the radius from the centre of the membrane. With the lamp and scale a metre away, the instrument had a sensitivity of 1 cm. for a change of pressure equal to a thousandth of an atmosphere. The calibration is carried out statically by applying various small pressures (measured on a water manometer) and noting the corresponding deflection of the image of a narrow slit of light on the scale. When the apparatus is on the sounding pipe, a hole being made into which the cylindrical portion of the capsule is sealed by soft wax, the pressure amplitude can be read off from the calibration line, when the amplitude of the deflection has been observed. For exact work a correction for the lag of the membrane may be necessary. Instead of attaining the corresponding deflection θ_0 instantaneously, the light approaches it on an exponential curve. As the instrument is lightly damped the approach

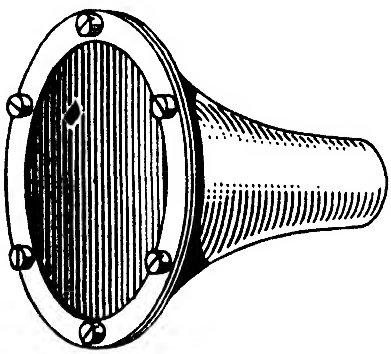


FIG. 67.—Manometric Membrane with Attached Mirror.

to the final deflection is rapid. A curve of the pressure amplitude down a stopped pipe obtained by means of the manometer is shown (Fig. 68).

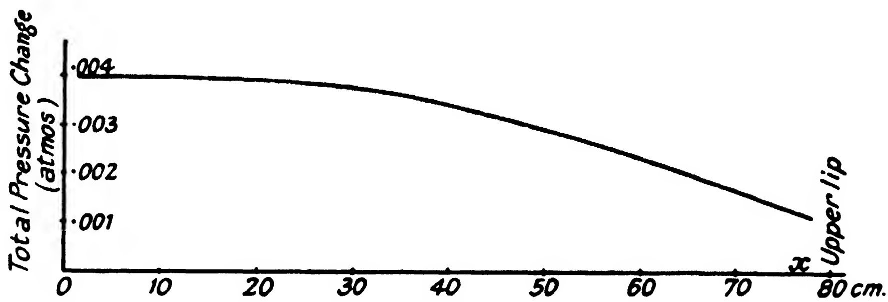


FIG. 68.—Pressure Amplitude along Stopped Organ Pipe.

Density Amplitude by Optical Interference Method. Töpler and Boltzmann[1] introduced a method for examining the extent of the density changes at the node at the end of a stopped pipe, based on the Jamin optical interferometer. Light fell on an inclined glass slab *A* (Fig. 69), so that part entered the glass and was totally reflected by the back surface, passed through a glass window at the end of the pipe, and out again by an opposite win-

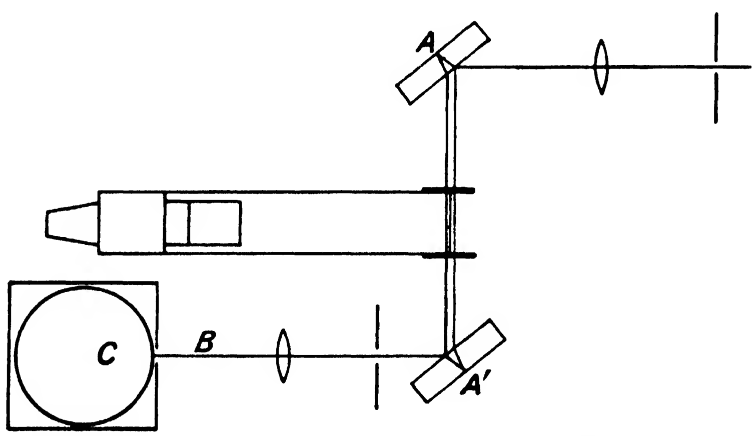


FIG. 69.—Optical Interference Method for Organ-Pipe (Töpler and Boltzmann).

dow. The rest of the light entering the glass slab was reflected at the front surface, and emerged parallel to the other ray but passed outside the pipe. The two beams were recombined by a second glass slab *A′*, but owing to the somewhat different paths they had traversed, coloured interference bands were observed in an eye-piece placed at *B* (Fig. 69). The position of these bands depends on the optical distance traversed, and if the air in the

[1] *Ann. d. Physik*, **141**, 321, 1870.

pipe increases in density owing to compression the velocity of light in the air is decreased, so that a greater relative retardation of the two beams occurs which results in a shift of the interference bands. Now when the organ pipe sounds, rapid changes of density take place at the node, so that on looking through the eye-piece the bands seem to be broadened, as the eye cannot follow the rapid movements of the bands following the rapid changes of density. Examined in a stroboscope, the amplitude of the movements of the bands is disclosed, and knowing the dependence of the velocity of light on the density of air, the density amplitude could be calculated. This is connected with the pressure amplitude by the formula $\delta p = \gamma p \frac{\delta v}{v} = \gamma p s$ (cf. 58).

Raps,[1] who repeated the method, found it better to calibrate the interferometer by compressing statically the air in the pipe, and noting the shift of the bands produced. He also photographed the movement of the bands by a revolving-plate camera C, employing both a reed and a flute organ pipe.

The method has the advantage that there is no question of lag, at any rate there is no lag of the order of the period of the tone, but it is extremely necessary that the walls and windows of the pipe shall be quite rigid, a slight vibration of the latter being sufficient to cause shifts of the bands greater than that due to the density change sought.

Measurement of the Temperature Change at a Node. The compressions and rarefactions taking place at a node cause temperature changes connected therewith by the adiabatic law. The amplitude of the change is very small, about 0·1 degree Centigrade, and requires a resistance thermometer of very fine wire to measure it. This difficult feat has been attempted by Neuscheler.[2] He used a Wollaston wire of 0·001 in. diameter, the oscillatory change in resistance being observed. The maximum estimated temperature change was 0·13° C., corresponding to a pressure variation in the node of 0·0155 atmosphere, the pipe being blown at a pressure of 5 inches of water. Pressure and temperature θ are connected by the relation $\delta\theta = \frac{(\gamma - 1)}{\gamma}\frac{\theta}{p}\delta p$. Taking the temperature coefficient of resistance of platinum as

[1] *Ann. d. Physik*, **50**, 193, 1891.

[2] *Ann. d. Physik*, **34**, 131, 1910. *See also* Heindlehofer, *Ann. d. Physik*, **37**, 247, 1912, and **45**, 259, 1914; Johnson, *Phys. Rev.*, **45**, 641, 1934.

40×10^{-4}, this would produce a resistance change of 4×10^{-4} ohms. It is doubtful whether the indications of a resistance thermometer can be relied on in such small but rapid fluctuations, in spite of the careful technique developed by Neuscheler. In a recent paper, Friese and Waetzmann [1] claim that such a "thermometer registers a fraction (depending on the fineness of the wire) only of the temperature changes in an oscillation of 100 periods per second."

Displacement Measurement. The cooling of a hot wire by the current of air in which it is placed forms a more convenient method of measuring the amplitude in a pipe, since the changes of resistance involved are considerable, and a string galvanometer is unnecessary. It was discovered by Richards [2] that when a hot wire is placed in an alternating draught—actually the wire was placed on the prong of a tuning fork—it assumed a steady resistance corresponding to the maximum velocity in the alternation, i.e., to $2\pi na$, when a is the maximum displacement, and n is the frequency (see p. 35). In other words, the resistance of the vibrating wire as measured on a Wheatstone Bridge with a dead-beat galvanometer, is the same as that which it would have if placed in a steady wind of this velocity. By measuring the steady drop of resistance of a nearly red-hot wire (0·001 in.) placed at different positions in the pipe, we are able to find the variation of a along the pipe. By this method the writer [3] obtained the curve shown in Fig. 70. Notice how the curve deviates from theory as the mouth is approached; a deviation due to the vorticity caused in this neighbourhood by the edge phenomena. This method provides a convenient and apparently accurate way of investigating the amplitude changes down an organ pipe, wind instrument or resonator—possible errors are discussed in the paper cited—with less interference with the motion than in any except the optical interferometer methods.

Recently the possibility of a direct measurement of the amplitude of the gas molecules by intermixing with visible particles has been explored. Lewis and Farris [4] allowed dust particles to

[1] *Ann. d. Physik*, **76**, 39, 1925.

[2] *Phil. Mag.*, **45**, 925, 1923.

[3] *Roy. Soc. Proc.*, **112**, 522, 1926; *see also* Goldbaum, Müller, Waetzmann, *Zeits. f. Phys.*, **54**, 179, 1929, and **62**, 167, 1930; Müller, *Phys. Zeits.*, **31**, 350, 1930; Kröncke, *ibid.*, p. 30.

[4] *Phys. Rev.*, **6**, 491, 1915.

fall through a sounding tube held horizontally, measuring their sideways displacement in a microscope. Gehlkopf[1] using oil drops and Carrière[2] and Andrade[3] using smoke have been able to hold particles in suspension long enough for their amplitude in vibration—they appear as streaks under the microscope—in a sound-

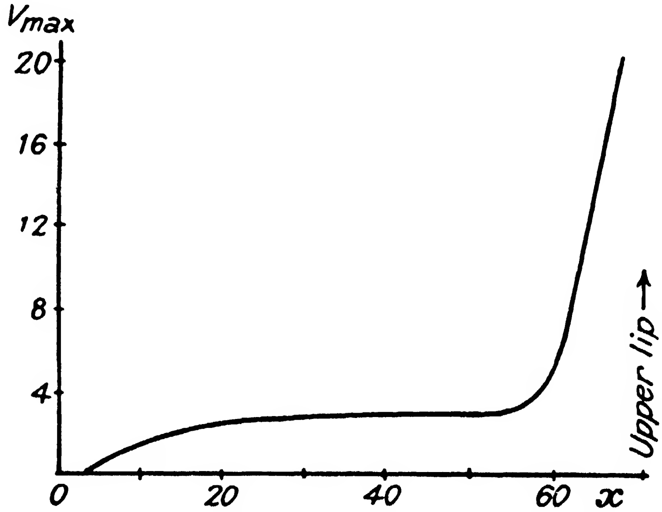


FIG. 70.—Displacement Amplitude along a Stopped Organ Pipe.

ing tube to be measured. Naturally the smaller the particles introduced the greater the fraction of the total (molecular) motion which they assume. Both W. König[4] and Andrade have gone into the theory of this method and the latter has shown how to calculate the total amplitude by extrapolation.

Wood-wind Instruments. The *flute* and *piccolo* are types of organ pipes of variable length. Fig. 71*a* shows a simple flute in section without keys or levers. In the modern form (*cross-flute*) edge tones are produced by the player directing his breath across a hole *M* to its opposite edge. A number of side holes are provided and when all these are covered by the finger-tips or by levers and keys the speaking length is from the mouth-hole to the open end, and the instrument sounds "middle C." As each hole is opened in turn antinodes are formed which terminate the effective length of the pipe at a shorter distance from the mouthpiece and raise the tone. As in the organ pipe this reacts upon the edge tones; but the player assists the adjustment by blowing across the mouth

[1] *Zeits. f. Phys.*, **3**, 330, 1920.
[2] *J. de Physique*, **9**, 187, 1928; **2**, 165, 1931.
[3] *Phys. Soc.* "*Audition*," 79, 1931. [4] *Ann. d. Physik*, **49**, 648, 1916.

with a greater velocity as the frequency goes up; f being an invariable quantity, roughly equal to the diameter of the hole.

Steinhausen [1] found that the vibrating length of the air column did not terminate at the open hole nearest to the mouth hole. Thus, in the figure, hole No. 1 is supposed to be closed while 2 to 6 are all open. If these holes were wide compared to the diameter of the tube, the vibrating column would stretch from one antinode at M to another at 2, but owing to the small diameter of the holes, the vibration is extended a little beyond 2. The upper line exhibits the relative pressure amplitude along the flute. By overblowing, the octave of the whole series of notes is obtained, intermediate semi-tones being formed by keys opening additional

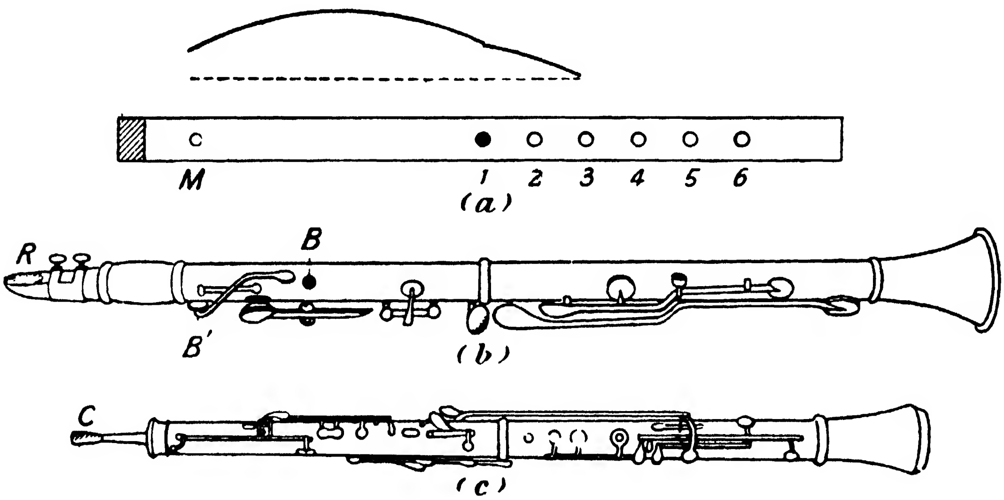


FIG. 71.—Wood-Wind Instruments.

holes. The flute is characterized by its pureness of tone and absence of harmonics.

The *clarinet* (Fig. 71*b*) has a beating reed R of cane secured to the mouthpiece by clamps, but the frequency of the reed is at the mercy of that of the cylindrical column of air, which the player varies by opening side-holes. The reed forms a node (cf. p. 179), while the open end of the cylindrical tube, or some intermediate point if the side-holes are open, is an antinode, so that the series of tones is the same as that in a column of air stopped at one end

[1] *Ann. d. Physik*, **48**, 693, 1915. *See also* Dunstan, *Phys. Soc. Proc.*, **31**, 229, 1919; Walker, *Ind. Assoc. Proc.*, **6**, 113, 1921; Ratz, *Ann. d. Physik*, **77**, 195, 1925; Cermak, *Ann. d. Physik*, **53**, 49, 1917; Richardson, *Roy. Soc. Arts. J.*, **78**, 203 and 224 and 258, 1930; Obata and Ozawa, *Phys. Math. Soc. Japan Proc.*, **12**, 285, 1930, and **15**, 125, 1933; Das, *Ind. J. Phys.*, **6**, 225, 1931; Redfield, *Acoust. Soc. J.*, **6**, 34, 1934.

(p. 163); and overblowing, which is assisted by opening a hole B' near the closed end of the tube, produces the twelfth instead of the octave of the fundamental series, and the quality of the note is one rich in the odd partials. The difficulty in playing the instrument is, that unless the mouthpiece is carefully constructed, the reed may escape from its bondage to the column of air and give out that distressing " quack " which characterizes its natural note. The player assists in adjusting the length and pressure on the reed, by his lips, with the flat part of which he grips the reed (see Fig. 71*b*).

The *oboe* (Fig. 71*c*) and *bassoon* have a *conical* tube, opening out from the vertex, where the mouthpiece is situated. As explained on p. 166 the column of air in such a tube corresponds to that in a doubly-open cylindrical pipe, so that overblowing gives the octave of the fundamental notes formed by successively shortening the effective length of the air column. The reed differs from the clarinet reed, as on these instruments it consists of two thin pieces of cane C with their free ends almost touching, and projecting into the player's mouth. Such a double reed is more flexible than the thick clarinet reed, so that it is not necessary to adjust the pressure on the reed to a nicety; literal overblowing will produce the higher notes of the instrument, as in the flute. The " note " is very penetrating and rich in high harmonics.

The *saxophone* is a mongrel instrument, invented by Sax in 1840, but not much used till recent years. It has a conical tube like the oboe, but a clarinet reed and mouthpiece. In consequence of its conical shape, overblowing elicits the octave of the air column, which is enclosed in a metal tube. Here we have a proof that it is the shape of the tube which determines what partial is produced by overblowing, and not the type of reed. Although the quality of the saxophone is not unpleasant, the combination produces a rather distressing slowness and uncertainty of " speech."

Brass Instruments. The instruments which consist merely of a simple conical tube like the *post horn* and the *bugle* can of course give only the harmonic series of tones, of which it is not generally possible to sound the fundamental. The mouthpiece consists of a cup or cone-shaped orifice, which fits over the player's lips which form the reeds of the instrument. In order to elicit the overtones of the instrument the player must compress his lips together more tightly and blow harder in order that the " reeds " may vibrate with the higher frequency. ' Music for these " natural horns " must

be confined to the series of notes formed by the partials of the instrument—the " harmonic scale " as it is called. In order that the full range of semi-tones may be obtained, orchestral instruments of this family are provided with three or four branch tubes, normally closed by " pistons." By depressing one or other of these valves the player can lengthen the tube and obtain a new harmonic series based on a fundamental several semi-tones lower than the normal. With three or four of these harmonic series available, the gaps in the scale of the natural horn can be bridged.

The fundamental of the horn can also be lowered by the player partially shading the bell-shaped open end of the instrument by means of his hand, thus increasing slightly the effective length of the vibrating column. It is said to be possible to produce two notes at once if the player blows to elicit one note, and at the same time hums another note into the horn, but it is doubtful whether this has been achieved when the two notes do not form part of the harmonic series, that is to say, the humming probably serves to emphasize one partial tone in the series which is already present in the vibrations of the column of air in the instrument.[1]

The *trombone* exceptionally employs a " slide " which lengthens the speaking portion on the principle of the device figured in Quincke's interference tube (Fig. 20). The player can thus vary the fundamental tone of his instrument continuously, but in practice the slide is employed in eight positions, with each corresponding series of harmonics. The relation between the slide trombone and an instrument " à pistons " is analogous to that between a violin and a banjo with its fixed frets for the fingers. The advantage lies with the former instruments, inasmuch as the player can " make his own notes "; a skilful player unconsciously adjusts the sounding length of string or tube to suit the pitch of other instruments in the orchestra, or to allow for temperature variations. On wind instruments without slides one can make small increments of the length by pulling out the mouthpiece joint before playing, but such an adjustment will be correct for one side hole and therefore for one note only. Thus, if the uncorrected lengths corresponding to a note and its octave are l and $\frac{1}{2}l$, the lengths corrected by an additional l_0 will be $l + l_0$, $\frac{1}{2}l + l_0$, and

[1] Kirby, *Musical Times*, 66, 815, 1925. *See also* Barton and Helen Browning, *Phil. Mag.*, 50, 951, 1925.

the ratio of these no longer being 2 to 1, the octave relation will be imperfect.

A somewhat similar fault occurs when more than one piston on a brass instrument is pressed so as to lengthen the tube by adding more than one branch tube; thus, if the first piston depresses the fundamental a semi-tone, and the second a whole tone, the first increases the speaking length in the ratio 15 : 16, the second in the ratio 8 : 9, and hence the two together lengthen it by $\frac{1}{15} + \frac{1}{8} = \frac{23}{120}$ which is less than the increase necessary for three semi-tones. Hence the two branches together are insufficient to obtain the required lowering of a tone and a half. To overcome this fault short compensating tubes are added which come into action only when the appropriate combination of pistons is depressed, and which bring the total added length to the amount necessary. Besides these transitory adjustments of the speaking length, more permanent alterations to suit the composer's requirements may be made by inserting "crooks," U-shaped bends of tubing added to the length of the tube which change the whole series of notes produced by the horn with or without the employment of pistons. The crooks give quite the same effect as the pistons except that they are in use all the time, having no valves to cut them off.

The quality of wind instruments is determined by many factors. Apart from the shape and scale of the tube and the form of the mouthpiece, the material and thickness of the walls of the tube play an important part. The damping is not a constant quantity for the harmonics of a brass instrument, but changes as the pitch goes up, so that the relations deduced on p. 51 for the sharpness of resonance do not apply. Analysis of the wave-form of the notes from wind instruments shows that not only the relative number of harmonics but also their absolute position in the scale determines the quality. Thus the trombone tends to bring out with emphasis all overtones lying between the notes 485 and 580, whatever the fundamental pitch.[1]

Barton and Laws[2] by placing a little manometer in a corner of the player's mouth, found the blowing pressures necessary for various notes of the scale on a brass instrument. It is rather difficult to co-ordinate their results with theory, as the latter has not advanced far enough to take all the human factors into account. But the experiments show: (1) that the blowing pressure rises proportionately

[1] Hermann-Goldap, *Ann. d. Physik*, **23**, 979, 1907. *See also* Marage, *Comptes Rendus*, **148**, 709, 1909; Sizes and Massol, *Comptes Rendus*, **151**, 303, 1910.

[2] *Phil. Mag.*, **3**, 385, 1902. *See also* Foord, *Phil. Mag.*, **27**, 271, 1914.

to the logarithm of the pitch, the intensity being kept as constant as possible; (2) that at constant frequency the pressure rises with the intensity. In view of what was said with regard to overblowing an organ pipe and the energy used therein, these variations are in the direction we should expect.

Recent Work on Edge Tones. Kruger and Caspar[1] have pointed out the affinity between Brown's photographs of sensitive flames and those of their own and other workers of smoke-jets producing edge tones. Brown[2] has himself extended his work in this direction. In the edge-tone formula (65, p. 171) he denies that f is an exact multiple of l, but proposes instead the empirical formula $f = 0{\cdot}0466j(V - 40)(1/l - 0{\cdot}07)$, where $j = 1$, $2{\cdot}3$, $3{\cdot}8$, $5{\cdot}4$ for the first four stages of the sub-division of the slit-edge space. He then examines critically the various edge-tone theories and favours a modified form of "escapement" theory, whereby any casual deviation of the jet from the forthright direction sets up a pressure increase in the air on one side of the wedge, forcing the jet back towards the other side, so making it pendulate. It will be noted that this does not completely account for the period of the pendulation. Two other investigations, which are germane to this difficulty, are to be found in papers by Carrière[3] and Coop.[4] Both these authors, acting independently, placed a wire in the path of a jet parallel to the slit from which it was emerging, though they used an ignited gas-jet. Thus, the new wind tone is a sort of combined æolian and edge tone. In fact, Carrière found that an edge-tone formula satisfied the observations, while Coop considered the sound as an æolian tone. The subject is being pursued further in the writer's laboratory, since the experiment seems very suggestive in respect of the escapement theory which Brown endorses. For if the wedge against which the pendulation is set up can be replaced by a thin wire without upsetting the mechanism of the edge tone, it seems difficult to see how the postulated differences of pressure can be maintained when there is free communication between the two sides of the jet by way of the far side of the wire.

[1] *Phys. Zeits.*, **37**, 842, 1936.

[2] *Phys. Soc. Proc.*, **49**, 493, and 508, 1937; *Nature*, **141**, 11, 1938.

[3] *Rev. d'Acoustique*, **5**, 112, 1936. [4] *Acoust. Soc. J.*, **9**, 321, 1938.

FURTHER REFERENCES: (wind instruments) Seiberth, *Hochfrequenz.*, **45**, 148, 1935; Lottermoser, *Akust. Zeits.*, **2**, 129, 1937; Redfield, *Acoust. Soc. J.*, **6**, 34, 1934; Aschoff, *Akust. Zeits.*, **1**, 77, 1936; Jarnak, *Fysisk. Tids.*, **34**, 137, 1936; Trendelenburg, *Zeits. tech. Phys.*, **17**, 578, 1936, and **18**, 477, 1937; Young and Longridge, *Acoust. Soc. J.*, **7**, 178, 1936; Ghosh, *ibid.*, **9**, 255, 1938; Mokhtar, *Durham Phil. Soc. Trans.*, **9**, 352, 1938; *Phil. Mag.*, **27**, 195, 1939; Bonar, *Acoust. Soc. J.*, **10**, 32, 1938; Carrière, *Rev. d'Acoust.*, **5**, 1, 1936; (end correction) Bate, *Phys. Soc. Proc.*, **48**, 100, 1936; *Phil. Mag.*, **24**, 453, 1937; King, *ibid.*, **21**, 128, 1936; Dongen, *Rev. d'Acoust.*, **7**, 1, 1938; (absorption in tubes) Lehmann, *Ann. d. Phys.*, **21**, 533, 1934; Waetzmann and Keibs, *ibid.*, **22**, 247, 1935; Norton, *Acoust. Soc. J.*, **7**, 16, 1935; Oberst, *Zeits. tech. Phys.*, **17**, 580, 1936; Tyler, *Phil. Mag.*, **24**, 665, and 908, 1937; Salceanu, *Comptes rendus*, **205**, 1219, 1937, and **206**, 329, and 502, 1938; Bürk and Lichte, *Akust. Zeits.*, **3**, 259, 1938.

CHAPTER EIGHT

HEAT-MAINTAINED SOUNDS

The body whose vibrations are to be maintained by expenditure of thermal energy may be a solid or a gas, and in general it is necessary that the heat should be applied intermittently. In a solid, the prime cause of the vibration is a rapid expansion and contraction of part of the system, first noticed in Trevelyan's experiment, but latterly come to the fore with the development of the alternating electric current. In a gas a periodic heating may produce either (1) a periodic convection current in a certain direction, or (2) a periodic compression. Under (1) we classify varieties of singing flames and tubes; under (2) we include the singing electric arc, though the actual compression is probably due rather to the disruptive action on the air particles, than to actual heating of the air. Besides the phenomena described in this chapter, there are others where the effect is more obscure and difficult of study, but doubtless due to the same cause. Such are the "singing" of a kettle before the water boils, the sounds heard occasionally when a hot glass bulb is blown on the end of a tube,[1] and finally the sounds produced when the sun shines on certain large cavities of air, e.g., Fingal's cave, and the storied Colossus of Memnon.

Trevelyan Rocker. When Trevelyan[2] in 1831 accidentally discovered that a hot iron fork laid on a block of lead gave rise to musical sounds, he set to work to devise a model which should best exhibit the effect, and produced a prismatic block of copper having a groove on its under surface so that it would rock about the ridges on the lead block, the other point of support being a knob at the end of a thin round handle, generally known as Trevelyan's rocker. Leslie suggested a theory of the action, which

[1] Obtainable in suitable form under the name of "Singing Tubes" from the Thermal Syndicate, Wallsend, England.

[2] *Phil. Mag.*, 3, 321, 1833.

was accepted by Trevelyan. Trevelyan found it necessary to have the ridges very smooth and clean, but the lead was best roughened. Heat being communicated to the lead by the copper, the rugosities of its surface were supposed to expand, to push up each ridge in turn, and then to contract as the heat diffused through the lead, the rocking being due to inequality of inertia of the portions of the rocker on opposite sides of the ridge, causing a lateral movement. Faraday, as a result of experiments with various pairs of metals, went further and said that the success of the experiment depended on the difference of conductivity of the two metals; the hot one must readily transfer its heat to the cold one, but the heat must not be able to diffuse rapidly into the latter, but remain near the point of contact causing local expansion; it was immaterial which one formed the rocker, cold lead would vibrate on a hot copper block.

In further support of the theory that the phenomenon was maintained by heat, Page[1] made a light rocker vibrate on two rails connected to the terminals of an electric battery, the heat being produced by the thermo-electric effects at the points of contact.

This theory was put to test by the writer,[2] and also by Bhargava and Ghosh,[3] using different methods. In both researches, corresponding values of frequency and amplitude were found, resulting in satisfactory confirmation of Leslie's theory. The writer's method of measurement was as follows.

A block of lead with rounded top is screwed down to the bench. A small hole is bored through the rocker, through which a thick steel knitting needle is thrust and held in position by a small screw, so that the needle lies horizontally across the rocker, when the latter rests with its two ridges across the lead block. The end of the needle or indicator is observed through a microscope. The light by which the field of the microscope is illuminated comes from a little electric lamp placed behind the slits of a stroboscopic disc, driven by an electric motor. When the disc is rotating, therefore, the illumination is intermittent, and its period may be made to coincide with that of the rocker by finding the fastest speed of the motor which makes the end of the needle appear stationary in the microscope. This speed was measured by a stroboscopic

[1] *Sill. Journ.*, **105**, 1856. [2] *Phil. Mag.*, **45**, 946, 1923.
[3] *Phys. Rev.*, **22**, 517, 1923. *See also* Eccles, *Phys. Soc. Proc.*, **23**, 204, 1910.

vibrator in conjunction with the usual print of differently spaced circles of dots pasted on the disc. By using a micrometer eyepiece, and setting the disc slightly out of step with the rocker, the amplitude at the end of the indicator could be readily observed as the needle appeared to move slowly up and down. No resonant vibration of the indicator was observed, so that the amplitude at the ridge could be directly calculated from the observed motion of the end of the needle (Fig. 72).

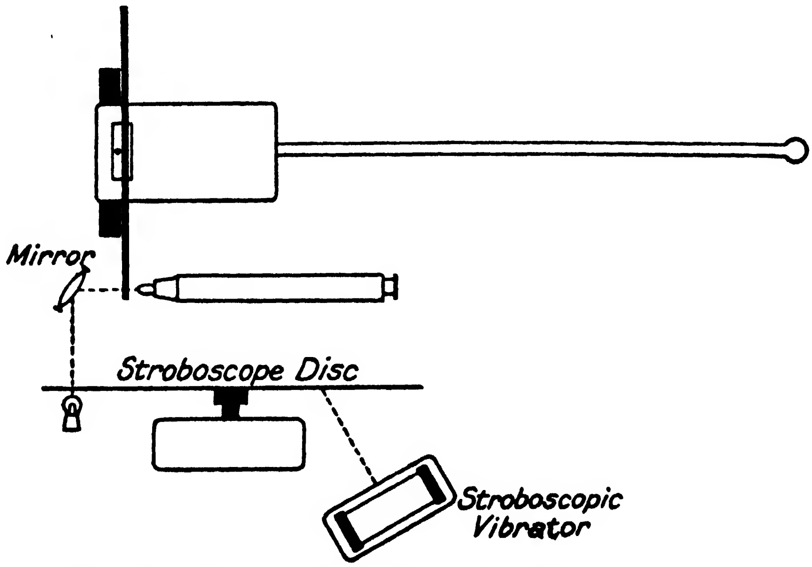


FIG. 72.—Apparatus for Tones of a Trevelyan Rocker.

On the assumption that the motion takes place about some point between the ridges, and corresponds to that which the instrument would have if lifted by one ridge and let go, we can obtain an equation for the motion [1] thus: If M = mass of rocker, K its spin-radius about a ridge, ϕ the angular deviation from the vertical, $2l$ = distance between ridges, the equation of the motion is :—

$$MK^2\frac{d^2\phi}{dt^2} = -Mgl.$$

An integration for $\frac{1}{4}$-time-period gives :—

$$K^2\omega = lg\frac{T}{4},$$

if a is the amplitude at a ridge, and ω = angular velocity,

$$\tfrac{1}{2}\omega = \frac{\phi_{max}}{\frac{1}{4}T}, \text{ or } \phi_{max} = \frac{a}{2l}, \text{ and } \omega = \frac{4a}{lT},$$

[1] Chuckerbutti, *Indian Assoc. Sci.*, 6, 143, 1921.

Therefore
$$T^2 = \frac{16aK^2}{l^2g}.$$

If n stand for the frequency, this equation may be written :—
$$n^2a = \frac{l^2g}{16K^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

We see, then, that the gravity theory requires the amplitude and frequency to be connected by the formula $n^2a =$ constant. The heat merely gives each ridge a little push which maintains the oscillation against gravity.

The results of the experiments can be summarized as follows :—

(1) n^2a is fairly constant, though rather higher than the calculated value.

(2) An alteration of the length of the handle does not affect the average value of n^2a, nor does it affect the range of frequency.

(3) Amplitudes are greater at higher temperatures, though this is probably dependent on the excess temperature of the copper over the lead, not on the actual temperature of the former. Similar results were obtained with a rocker consisting of a cylindrical rod of copper, having a narrow groove cut near one end. No frequencies were observed which would correspond to elastic (e.g., longitudinal or torsional) vibrations of the rod.

Mary Waller[1] has discovered an interesting extension of this method of maintaining a solid in vibration, viz., by contact with solid carbon dioxide. The conditions for success are the same, i.e., a good conductor placed on a colder bad conductor of heat. With the solid carbon dioxide, however, another factor intervenes which makes for greater efficiency and allows of the maintenance by heat transfer of the vibrations of objects of quite high natural frequency which would be difficult to manage with the usual lead block. This factor is that the heat transfer sublimes the carbon dioxide, setting up considerable gas pressures which help to tip the plate over.

Alternating Current in a Wire. If a steady electric current be passed through a wire of considerable resistance, and high coefficient of expansion, such as a thin platinum wire stretched on a monochord base, the wire sags if the current is sufficient to raise its temperature by a considerable amount. If the current were rendered intermittent with a frequency n the wire would sag and recover with the same frequency. In the early days of alternat-

[1] *Phys. Soc. Proc.*, **45**, 101, 1933; **46**, 116, 1934; **49**, 522, 1937.

ing currents it was noticed that the stretched wires sometimes used as rheostats, would sound their fundamental or an overtone when in a condition of resonance with the periodic heating produced by the alternating current passing through them. Now this heating is independent of the direction of the current in the wire, so that the wire is heated and cooled twice in each cycle of the alternating current, or the frequency of the impressed force on the wire is double of that of the current. Yet the frequency of the tone in the wire is found to be half of that of the impressed force (as in the longitudinal form of Melde's experiment), so that finally the frequency of the tone and of the current are the same. Analogous effects can arise in a wire which forms part of a circuit in which an intermittent spark discharge is taking place, e.g., the "secondary" of an induction coil.[1]

Krishnaiyar[2] has examined the amplitude of vibration produced in a sonometer wire whose length and tension could be varied, when currents of fixed periodicity and of constant strength are passed through it, and so has obtained the resonance curve of the system. These vibrations being of the type where the impressed force has a frequency double the frequency of the emitted note, a marked resonance peak is not obtained (cf. p. 56). Instead, there is a gradual increase in the amplitude of response as the frequency of the wire is increased up to, and beyond, that of the forcing vibration. Krishnaiyar found the frequency of the wire by calculation from the tension and length, but apparently no comparison with a standard frequency was made.

Thermophone. The periodic heating of a strip of platinum by an alternating current is the underlying principle of an instrument known as the thermophone. Such an instrument was first described by de Lange,[3] who ascribes the actual discovery to a Russian engineer named Gwodz. It gives a pure but feeble tone and so is usually provided with a resonator to amplify its vibrations. As a source of sound it has been elaborated by Arnold and Crandall[4]

[1] Viol, *Ann. d. Physik*, **4**, 734, 1901; *see also* Gozate and Naik, *Nature*, **125**, 819, 1930.

[2] *Phys. Rev.*, **14**, 490, 1919 and *Phil. Mag.*, **43**, 503, 1922. *See also* Strientz, *Phys. Zeits.*, **16**, 137, 1915; Guillet, *Comptes Rendus*, **161**, 561, 1915; Butterworth, *Phys. Soc. Proc.*, **27**, 410, 1915.

[3] *Roy. Soc. Proc.*, **91**, 293, 1914.

[4] *Phys. Rev.*, **10**, 22, 1917. *See also* Wente, *Phys. Rev.*, **18**, 333, 1922; Geffcken (*with* Keibs), *Ann. d. Physik*, **16**, 404, 1933; (*with* Waetzmann), *Phys. Zeits.*, **34**, 234, 1933; Cellerier, *Comptes Rendus*, **190**, 45, 1930.

using a strip of platinum 0·0007 cm. thick, in which either A.C. or A.C. superposed on D.C. was used. Applying a current of periodicity which was varied over a considerable range, they found for the unassisted thermophone without resonator that the intensity of response, measured by the sound emitted fell regularly as the applied periodicity was raised. This result is similar to Krishnaiyar's, and the instrument is probably to be regarded as a telephone transmitter in which the vibrations, normally produced by electro-magnetic means, are in this instrument produced by rapid expansions and contractions of the diaphragm strip under the action of the alternating current.

The Singing Flame. The fact that a jet of hydrogen, burning in an open tube, would under certain conditions cause a musical note to be emitted, was first observed by Higgens in 1777, and other observers studied various aspects of the phenomenon, without attempting to account for it, until De La Rive advanced his theory, i.e., that the periodic condensation of water vapour by the burning hydrogen caused the emission of the note. That this cannot be the true cause of the " singing " was shown by Faraday, who was able to replace the hydrogen by carbon monoxide, a gas which produces no moisture in its combustion, without detriment to the effect. Faraday himself put forward an alternative theory, that the note was caused by successive explosions of the gas with the oxygen of the air, the flame then dying out until a further supply of air arrived at the jet, when it was re-ignited. For Wheatstone had shown in his revolving mirror that the flame was not steady, but vibrated up and down, so as to appear in the moving mirrors as a succession of images. As a result of his experiments, Sondhauss [1] considered the cause of the singing to be the heating of the air in the neighbourhood of the jet, the subsequent change of density causing a compression to flow away from the jet, thus starting the air in the large outer tube (hereafter referred to as the air tube) to sound its natural tone. He also found that certain combinations of the air tube and gas-supply tube would not sing, and concluded that the length of the gas tube must vary with different gases, in order that the oscillations of gas and air may be in step near the jet. If these oscillations were stopped by a plug of cotton-wool in the air tube near the jet, the singing ceased.

Rayleigh [2] showed theoretically that, unlike ordinary resonance,

[1] *Ann. d. Physik*, **109**, 1 and 426, 1860. [2] *Theory of Sound*, **2**, 227, 1877.

the impulses given to the resonator (i.e., the air in the tube) by the impressing force (the heat) must occur at the phase of maximum displacement or condensation, and not in the neutral position. Also that for the continuance of the vibrations the gas tube must be of such a length and in such a position, that a condensation at the jet will travel down it and back again so as to arrive at the jet in phase with a condensation in the air tube (as Sondhauss had previously stated), in fact, that there should be stationary waves in the gas tube as well as in the air tube.

Finally, Gill,[1] apparently without having seen Rayleigh's paper, has revived the older theory, wherein an accidental change of pressure in the air tube sets the latter faintly sounding, the successive condensations forcing the flame down the jet, and each rarefaction letting it grow again.

Now the effect of an intermittent addition and withdrawal of heat on the air in the neighbourhood of the flame fairly obviously is to impose a periodic increment and decrement of pressure. If this alternating force could be likened to the piston of a reciprocating engine (p. 34), if indeed it could be represented by $F \sin pt$, then the reasoning developed in Chapter II for a system maintained against damping by a periodic force would apply, and in particular the conclusion that when the period of the applied pressure synchronizes with that of the tube, the maximum applied force should occur when the alternations of pressure in the air are passing through their mean position, i.e., the forcing system should lead the forced by $\frac{\pi}{2}$. It will be shown later, however, that this applied force is of a different nature; the heat is given discontinuously in pulses—one had almost written "in quanta"—a pulse in each period, and Rayleigh having instinctively surmised this, asked himself at what phase in the period must the additional compressive pulse be given in order to maintain the sound.

We can best consider this question graphically. Suppose owing to some casual disturbance of the air near the tube—a momentary edge tone for example—pressure oscillations are set up in the pipe. In the absence of maintaining forces these would be rapidly damped (Fig. 73*a*), but consider the effect of suddenly increasing the pressure at the instants *A* or *C* (normal pressure), *B* (maximum compression), or *D* (rarefaction). At *C* a small increase of pressure represented by the short upstroke (Fig. 73*b*) will delay the fall of

[1] *Sill. Journ.*, 4, 177, 1897.

pressure below the normal, and therefore increase the period; but because $\frac{dp}{dt}$ is already considerable, it cannot affect the amplitude appreciably; it merely repeats the course of events between C' and C, otherwise the motion goes on being damped as before. Similarly if the extra compression is given at A, the period is shortened without affecting the amplitude. At B, however, the sudden application of heat increases the pressure amplitude, without

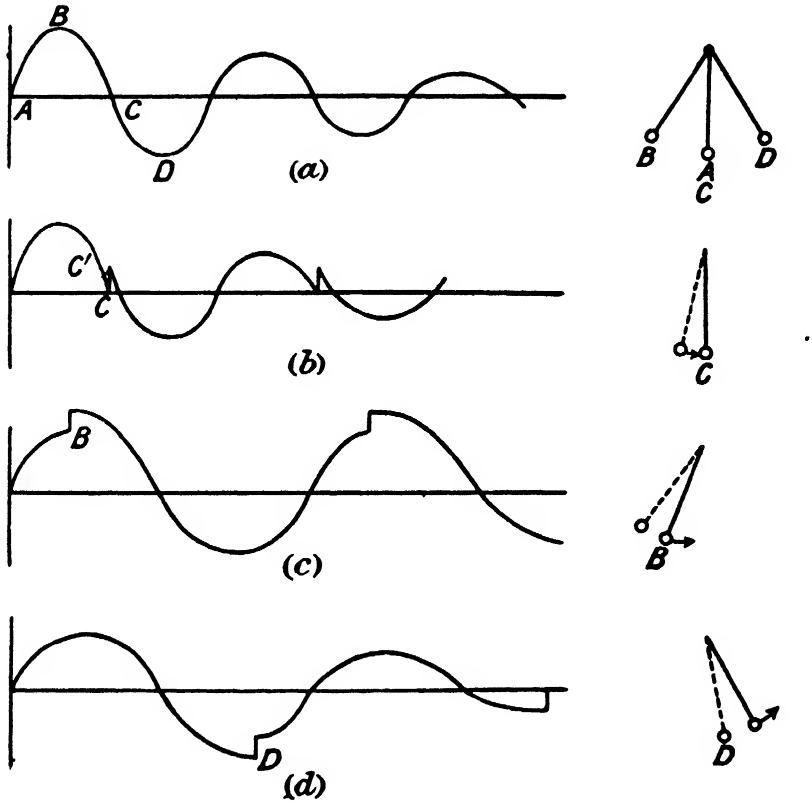


FIG. 73.—Action of Singing Flame.

(so far as the period is independent of the amplitude) altering the period. If this addition of pressure is sufficient to make up for what is lost by damping in each preceding period, the motion will be maintained (Fig. 73*c*). If the compression is given at D, then the motion is rapidly damped (Fig. 73*d*). The pendulum at the side of the figures is intended to illustrate a mechanical analogy. The pendulum is to be given a small displacement from right to left once in each period. The amplitude will be increased (dotted line) if this occurs at the maximum displacement to the left, reduced at the maximum displacement at the right, but unaffected—

provided no acceleration is given to the bob—when this occurs at the mean position.

The writer has made some experiments which support this view.[1] A brass tube, 60 cm. by 4 cm., was taken and a gas tube with a jet placed in it in such a position that the singing started spontaneously and vigorously. A hole, 1 cm. diameter, was bored in the brass tube at this point, also a larger hole, over which a piece of glass was fixed by sealing wax, to serve as a window through which the singing flame could be observed (Fig. 74). Over the small hole a König manometric capsule was placed, the burner of which was brought right round in front of, and close to, the window, so that when lit this manometric flame appeared vertically beneath the singing flame. The jets were placed as close as possible to each other, one inside and one outside the air tube; and all joints were carefully plugged, as a leakage of pressure is inimical to the "singing." Both flames were fed by coal gas, and in order to compare the flames they were looked at directly through a stroboscope. By this means it was seen that they rose and fell almost simultaneously and in the same phase. Fig. 75 shows the appearance of the two flames in the stroboscope. The theory of Rayleigh with regard to the phase of the heat supply receives its justification in this experiment.

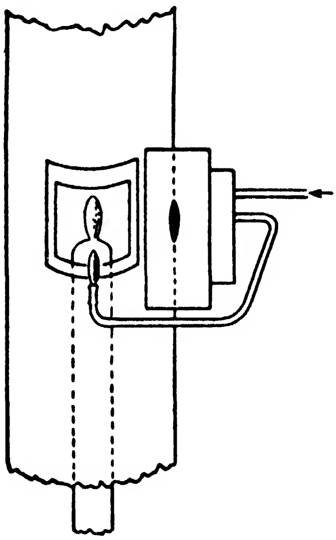


FIG. 74.—Phase Relationship of Singing Flame and Column of Air.

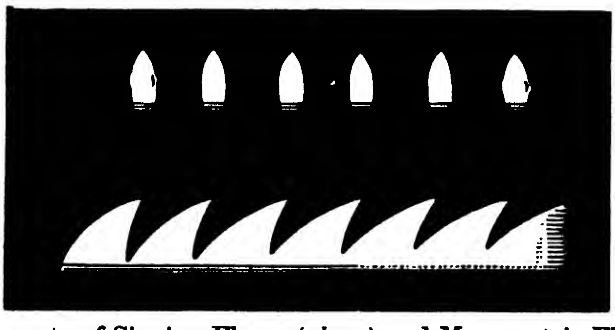


FIG. 75.—Movements of Singing Flame (*above*) and Manometric Flame Compared.

[1] Richardson, *Phys. Soc. Proc.*, 35, 47, 1922. *See also* Athanasiades, *Comptes Rendus*, 146, 533, 1908; Pelligrini, *N. Cimento*, 19, 83, 1920; Banerji, *Indian Assoc. Proc.*, 7, 47, 1921; Foch, *Comptes Rendus*, 188, 697, 1929.

Position of Flame and Length of Supply-tube. Granted that the action of the "singing flame" is to increase the pressure amplitude, it must be placed at a point where the change of pressure is appreciable, i.e. near a node. The ends of an open tube being antinodes for all harmonics, the flame will not sing near these points. On an average this "silent zone" extends over about a fifth of the length of the tube from the mouth. From what has been said with regard to reed-pipes, we should expect to find vibrations in the gas-supply tube, and since the systems are coupled and must have the same frequency, we should expect the maintenance to be most perfect when stationary waves are formed in the gas. By fitting a manometric capsule to the *supply-tube* just below the flame, one found that maximum compression occurred there just after the flame struck back to the jet, as if one had turned off the flame by a tap and the pressure accumulated then in the gas main behind it. When the singing was most powerful, stationary waves could be traced in the gas tube by adjustable manometric flames, a node always being found near the jet and an antinode below at the end of the gas tube in the reservoir. Rayleigh suggested that the gas tube should be of length about $\frac{1}{4}\lambda$, $\frac{3}{4}\lambda$, $\frac{5}{4}\lambda$, etc., for best maintenance, λ being the wave-length of the tone in the *gas*. When this adjustment is made, the violence of the flame's oscillation as it is pushed up the tube is sufficient to extinguish it before the central node is reached. When the adjustment of the length of the supply tube is less perfect (and considerable variations on each side of the "resonant" length are possible) the sound is more feeble, and the singing continues without extinguishing the flame, as it is moved from near the bottom to near the top end. When the supply tube has a length nearly equal to $\frac{1}{2}\lambda$, λ, $\frac{3}{2}\lambda$, etc., the phase of the flame is most inimical to maintenance, and no sound is heard.

The tones of a column of air maintained by the singing flame are particularly pure and free from overtones. Harmonics alone can be excited, especially if encouraged by appropriate side holes. Of course the proper tones of the pipe are all raised by the higher temperature due to the general heating by the flame.

The Rijke and Bosscha "Gauze Tones." In contradistinction to the purity of the above tones, there are other ways of maintaining the sounds of columns of air, in which vibration is so

vigorous and therewith the overtones are so prominent that they have earned the well-merited name of "howling tubes." Besides using thermal means to increase the *pressure* amplitude in a stationary vibration we may employ the source of heat to create a temporary convection current, and so increase the *velocity* amplitude. Curiously enough no external means of rendering the heat-supply intermittent, corresponding to the gas-supply tube of the singing flame, is necessary with these heating effects. The first of these was discovered by Rijke,[1] who placed a piece of metal gauze in the lower half of a vertical tube (best about a quarter of the way up) and heated it red-hot by a flame. On withdrawing the flame the air in the tube sounded until the temperature of the air and the gauze were nearly equalized. By passing an electric current through the gauze to maintain it at red heat he was able to make the sound continuous. The sound could not be excited when the gauze was within one-thirty-third of the length of the tube from the lower end, nor when it was anywhere in the upper half. This phenomenon was attributed by Rayleigh to a combination of loop and node effects. Under the action of the first cause, every upward movement of the air in vibration brings cold air on to the heated gauze, whereas the downward movements bring the already hot air back on to the gauze. Thus the greatest temperature difference occurs in the "upstroke," consequently the greater heat transfer and consequent compression occurs as the air goes up. When the gauze is in the lower half, this upward phase occurs just before the maximum compression (because the air is then moving towards the central node) and the vibration is assisted. When the gauze is in the upper half this phase precedes a rarefaction, and so the motion is damped. This hypothesis, which we have seen verified for the singing flame, explains the complementary phenomenon discovered by Bosscha,[2] that if a cold gauze be placed in the upper half of the tube, and a current of hot air be passed up the tube, a similar sound is produced. To make Bosscha's gauze and tube sound continuously, the gauze can be cooled by water in a circulating pipe on which it is woven, while hot air rises from a candle near the lower end of the tube.

Lissajous' Whistling Flame. It is possible to maintain the sounds of the air column at the antinode formed at the end of the tube itself, by means of a gauze and a gas supply, with the gas lit

[1] *Ann. d. Physik,* **107**, 339, 1859. [2] *Ann. d. Physik,* **127**, 166, 1866.

above the gauze. A Méker burner, being constructed on this principle, will act in the same way. This apparatus was examined by Gill,[1] who calls it a " whistling flame," and ascribes it to Lissajous. Gill gave a theory of the action, which is briefly as follows. When a rarefaction is taking place, a current of air leaves the tube with velocity U; if the upward through draft of air due to combustion have velocity V, the resultant downward current is $U - V$, causing a smaller amount of gas to enter the flame. During a condensation more gas enters the flame, because the upward current is $U + V$. The fluctuations of the gas supply then maintain the vibrations. Recently an examination was made of the phase relation between the vibrations of the flame and the stationary air waves formed in the pipe, when the fundamental tone is sounding.[2] This was accomplished by the method previously applied to the singing flame, in which a König manometric capsule is applied to a hole in the tube near the burner flame, the manometric flame being brought round and in front of the former, which is viewed through a window in the tube. As the gauze is best placed at the mouth of the pipe, i.e., at an antinode, it is necessary to apply the manometric capsule about an inch from the mouth. With such an apparatus it was seen that the whistling flame rose to its maximum during the quarter period preceding a condensation.

The action of the gauze or grid should be to cool the flame rapidly between each of its hot periods. To test this, it was necessary to find the widest mesh that would support the sounding. For iron, the limit appears to be 64 meshes per sq. cm.; gauzes of a better conductor like copper were effective up to 36 per sq. cm. It was found that the gauze and flame could be replaced by an ignited Méker burner, which is of similar construction.

That the through draught of air, postulated by Gill, is not essential in the pipe, is shown by these two observed facts :—

(1) A pipe completely stopped at the upper end will sound with a flame at its mouth, provided that the burnt gases can escape downwards between the grid and the pipe.

(2) The whole apparatus may be turned through an angle of 160° from the upright position, without stopping the sound. In

[1] *Am. J. Sci.*, 4, 187, 1897.

[2] Richardson, *Am. J. Sci.*, 6, 11, 1923.

this position, with the pipe lying wholly below the flame, the combustion draught cannot pass through the pipe.

It appears, then, that the to-and-fro motion of the air in the pipe, due to stationary waves, can cause a change in the quantity of air admitted to the flame sufficient to maintain the phenomenon. This also explains why the most favourable position for the gauze is at an antinode, where the movement of the vibrating particles is a maximum.

The Singing Arc. When an electric arc is struck between two carbons fed by direct current, the incandescent spark produced is peculiarly sensitive to oscillations of the current supply. For example, any inductive effects superposed on the steady current will be repeated by the arc, and owing to the powerful disruptive effect of the spark, become audible, generally as a "spluttering" of the arc. To make the arc "sing" a constant tone it is necessary so to adjust the conditions that the arc is unstable, ready to oscillate, and then to include it as part of a resonant circuit. In this way Duddell[1] was able to make the arc produce powerful sounds of

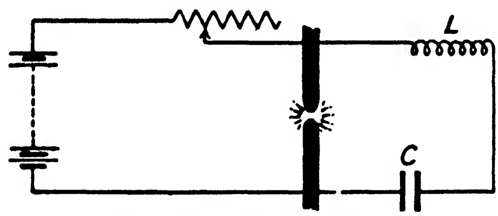


FIG. 76.—Singing Arc.

frequency $n = \dfrac{1}{2\pi\sqrt{LC}}$, where L is the self inductance, and C the capacity of a circuit placed in parallel with the arc (Fig. 76). Of course, an arc fed by alternating current will reproduce the frequency of the supply in the same way. The phenomenon is reversible, sound waves impinging on the arc will cause corresponding variations in the current through the arc. Thus if the single coil L and condenser C be replaced by two insulated but interwound coils, of which the secondary coil remains connected in the arc circuit, while the primary is connected to an ordinary telephone transmitter, any sounds—the notes of tuning forks, or even words—falling upon the latter will be forced upon the arc circuit and emitted as sounds of similar pitch and quality by the

[1] *Electrician*, 46, 269 and 310 and 356, 1900.

arc with increased vigour. The simple formula connecting the frequency of the "singing arc" with the constants of the simple shunt circuit suffices for general purposes but becomes untrue if large intensities and arc-lengths are employed.[1]

[1] *See* La Rosa, *Accad. Lincei. Atti.*, **17**, 112, 1908; Nasmyth, *Phys. Rev.*, **27**, 117, 1908; Taylor Jones and Owen, *Phil. Mag.*, **18**, 713, 1909, and **20**, 660, 1910; Rihl, *Ann. d. Physik*, **36**, 647, 1911; Hoyt, *Phys. Rev.*, **35**, 387, 1912; Lichte, *Ann. d. Physik*, **42**, 843, 1913; Faszbender and Hupke, *Phys. Zeits.*, **14**, 222, 1913; Zenneck, *Ann. d. Physik*, **43**, 481, 1914; Freda and Mortara, *N. Cimento*, **13**, 297, 1917; Braüer, *Phys. Rev.*, **20**, 409, 1919; Mayer, *Zeits. tech. phys.*, **2**, 18 and 40 and 73 and 94, 1921; Blondel, *Comptes Rendus*, **182**, 900, 1926; Seidel, *Phys. Zeits.*, **27**, 64, 1926; Seidel, *Ann. d. Physik*, **84**, 384, 1927; Lichtenecker, *Zeits. Tech. Phys.*, **8**, 161, 1927.

CHAPTER NINE

ANALYSIS OF SOUND IN AIR

In the preceding chapters, grouped under appropriate headings, appear a large number of methods by which the vibrations of particular systems, wires, bars, columns of air, etc., may be studied. In general, the sounds from these bodies are conveyed to our ears by waves in the air. The open air—open, that is, in the sense that there are no partially enclosed spaces which can resonate—gives a faithful reproduction of the vibrations impressed upon it by the source, so that if we can devise an instrument which will faithfully analyse the sonorous air waves, we have a universal instrument for analysing the notes produced by any sound-making body whatever. The construction of such an instrument is by no means simple, and it will be best to consider some of the difficulties in the way, before describing those instruments themselves which analyse sound waves in air.

Requirements of a Sound Analyser. If the air waves are due to a sustained unvarying note, the analysis consists in finding the amplitudes and frequencies of the Fourier components which make up the given note. The instrument will register these by a movement of certain amplitude, e.g., of a spot of light on a scale. This movement is known as the "response" of the instrument. The instrument must then give *adequate and equal response* to tones over the whole audible range, provided the tones are of equal amplitude. It is not essential that the amplitude of the response should be always a constant factor times the amplitude in the air, as long as the law connecting them is known, and is independent of frequency. Some considerations which mitigate against this desideratum are :—

(1) The reproducer has a natural period or periods of its own and gives excessive response to tones of coincident period. This is known technically as "distortion."

(2) The average amplitude of the motion of the air is minute (about 10^{-7} cm.).

(3) On this account a horn is added to many instruments to obtain increased response. The horn adds more resonances.

Laying aside the problem of measuring the vibrations of the source by calculation from those of the air at the point where they are measured, we will now classify the instruments which analyse the sonorous vibrations of the atmosphere.

(A) Instruments of the phonograph type which give a visible record of the air wave to be subsequently submitted to Fourier analysis and correction for resonance.

(B) Recording resonators, which measure the relative amplitudes of a tone *at a particular frequency.* The resonators are often adjustable, but no comparison can be made between the amplitudes of tones, before and after adjustment, as the response will generally be different.

(C) Absolute instruments, free from distortion.

It may be added in parentheses, that claims have been made, not necessarily by the inventors themselves, for several of the instruments in class (B) as giving absolute measurements of sound. This title cannot rightly be given to an instrument in which resonance either designedly or unavoidably plays a part, and where a large and generally uncertain correction has to be made in comparing tones of different frequencies. On the other hand, it must be confessed that the instruments of class (C) are still in the embryo stage.

A. Recorders.

Phonautograph or Phonograph. The first instrument for recording sound waves was Scott's Phonautograph, 1859.[1] The membrane was stretched over the narrow end of an ellipsoidal horn and its movements were recorded through the medium of a lever bent at right angles, one end resting on the membrane, and the other carrying a pencil which drew the wave-form on a drum rotated by hand, the records being obtained in the same way as those from a tuning fork (cf. p. 76). Edison replaced the lever by a stylus attached to the membrane in such a way that the former pressed with greater or less force against the drum as the membrane vibrated. He found that, when the drum was covered with wax or similar soft material the sounds impinging on the membrane could be reproduced after the "record" had been taken, if the style was run a second time over the minute crests and troughs

[1] *Cosmos*, **14**, 314, 1859.

formed in the wax by the original sound waves. Edison's phonograph was at first intended as a scientific instrument for recording and reproducing sounds, but the modern application of the instrument to music and phonetics is familiar to all.

Its development as a reproducer has run along lines leading to increased but more uniform response over the entire musical scale, by horns of peculiar shape, and by multiple resonances in the housing of the diaphragm, which tends to equalize the response over the whole musical range by overlapping resonance peaks (cf. p. 325).[1]

A photograph exhibiting the wave-form of the motion may be obtained from the record by many devices, all of which consist in principle of an optical lever with one leg dipping into the groove moved slowly along, so that the indentations are copied on a moving photographic film by the deflection of a beam of light.[2]

The Phonodeik. The receiver of this instrument by Miller[3] is a diaphragm of thin glass (0·003 in.) at the end of a resonator horn.

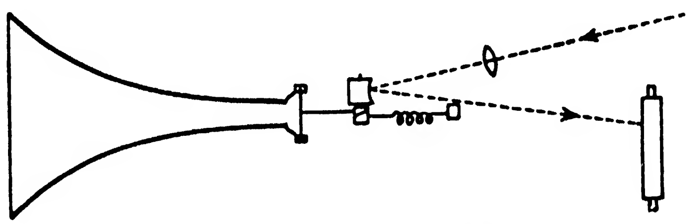


FIG. 77.—The Phonodeik (*Miller*).

A silk fibre or a very thin platinum wire leads from the centre of the diaphragm over a pulley on the end of a minute spindle to a light spring. On the spindle is a mirror, which the movement of the diaphragm causes to rotate. The movement of the mirror deflects a beam of light reflected on to a moving sensitized film (Fig. 77).

The spot of light thus traces a record of the sound wave on the

[1] *See also* Berliner, *Frank. Institute Journ.*, **176**, 189, 1913; Rothwell, *Nature*, **111**, 254, 1923; Porter, *Phys. Soc. Proc.*, **36**, 129, 1924; Rankine, *Phys. Soc. Proc.*, **36**, 115, 1924; Brown, *Phys. Soc. Proc.*, **36**, 149, 1924; and the refs. on pp. 306–310.

[2] McKendrick, *Nature*, **80**, 188 and **81**, 488, 1909; Rosset, *Comptes Rendus*, **149**, 1,511, 1910; Gianfranceschi, *N. Cimento*, **7**, 19, 1914; Hauser, *Akad. Wiss. Wien. Ber.*, **117**, 143, 1908; Bensdorf and Pöch, *Akad. Wiss. Wien.*, **120**, 1,811, 1911.

[3] *Engineering*, **94**, 550, 1912. *See also* Anderson, *Opt. Soc. Am. Proc.*, **11**, 31, 1925.

film, magnifying the motion of the diaphragm 2,500 times, and giving a record $2\frac{1}{2}$ in. wide. A zero line to serve as axis of the curve is also drawn, and time flashes are given from a tuning fork.

An ideal instrument would give the same response to sounds of the same intensity over the whole of the musical scale. As a matter of fact, the diaphragm, and especially the horn, have natural frequencies of their own, and when a sound corresponding to one of these falls upon the instrument, the amplitude of this component is magnified by the resonance of the instrument. It was found impossible to eliminate these; the natural period of the diaphragm can be made high, but not that of the horn, and a horn is indispensable if sounds of the normal intensity in the air are to be recorded. Miller therefore set out to reduce the trace of a sound wave to absolute proportions by first calibrating the instrument with tones of constant loudness over the whole range of frequency. Then when the analysis of the trace has been accomplished, each amplitude is reduced as demanded by this calibration curve. The coefficients of the requisite Fourier series are determined by operating on a magnified trace of the corrected curve by mechanical analysers.

There are available other methods for recording the movement of the diaphragm of such a recording instrument. The diaphragm may form one side of a manometric capsule, and the movements of the flame be photographed (cf. p. 181), or a telephone receiver may be used and the fluctuations of current caused by its vibration measured on a galvanometer sufficiently damped to respond to rapid current changes; such an instrument is the Einthoven string galvanometer or oscillograph.[1] In place of the oscillograph, the current may be rectified by a crystal detector, and an ordinary galvanometer employed.[2]

Garten's[3] registration method employs a soap-film, and is only

[1] Watson, *Phys. Rev.*, **30**, 471, 1910; Boehm, *Phys. Rev.*, **31**, 329, 1910; Barlow and Keene, *Roy. Soc. Phil. Trans.*, **222**, 131, 1921; *see also* Kasansky and Rschevkin, *Zeits. f. Phys.*, **47**, 233, 1928; Hajek, *Akad. Wiss. Wien. Ber.*, **137**, 529, 1928; Meyer, *Zeits. f. tech. Phys.*, **12**, 606, 1931; Sivian, Dunn, White, *Acoust. Soc. J.*, **2**, 330, 1931; Snow, *ibid.*, **3**, 155, 1931; Ballantine, *ibid.*, **5**, 11, 1933; Mallina, *ibid.*, **6**, 37, 1934.

[2] Pierce, *Am. Acad. Proc.*, **43**, 375, 1928. *See also* Goldhammer, *Ann. d. Physik*, **23**, 192, 1910; Tuzman, *Phys. Soc. Proc.*, **34**, 166, 1922. *See Engineering*, **117**, 108, 1924; Karcher, *Bureau of Standards Bulletin*, **19**, 105, 1923; Grützmacher, *Zeits. tech. Phys.*, **8**, 506, 1927; Gerlach, *ibid.*, **8**, 515, 1927.

[3] *Ann. d. Physik*, **48**, 273, 1915.

"absolute" to frequencies removed from the resonant frequency of the "membrane" (about 2,000) though the part played by resonance is unimportant since the damping is large—the free vibrations fall to half amplitude in about four swings. The film used by Garten is in the shape of a four-pointed star 2·5 mm. across, with a minute iron particle weighing 0·0001 mg. held in suspension in the centre of the film between the poles of a magnet. This particle, viewed in a microscope with micrometer eyepiece, serves as indicator of the amplitude of motion of the centre of the film. To protect this film from draught it is backed by a glass cover at 3 mm. distance. The instrument is calibrated by static changes of pressure, and is claimed to indicate a pressure of 0·012 mm. of water.

The Cathode Ray Oscillograph. The cathode ray tube, an illustration of which will be found in most modern books on electricity, consists of a vacuum tube one end of which is cylindrical while the other is broadened out to hold a fluorescent screen. At the narrow end a heated filament and associated shields direct a beam of electrons axially along the tube to fall on the screen and there produce a bright spot. Two pairs of plates to which electrostatic fields may be applied let into the side of the tube allow the beam to be deflected either horizontally or vertically. To use this apparatus as an oscillograph one pair of plates is connected to the alternating field (suitably amplified) produced by a microphone. If this field oscillates the spot in a vertical plane we can pass a sensitized film in a horizontal direction in front of the screen, and get a record of the oscillatory current from the microphone. Alternatively by means of a "sweep circuit" connected to the other pair of plates the spot may at the same time be swept slowly from left to right horizontally with a jump back to the start when it has reached the edge of the screen and again the slow sweep to the right, *ad inf.* This apparently complicated motion is simply achieved by the slow charge and sudden discharge of a condenser connected to a source of e.m.f. and suitable resistance. If the period of this relaxation oscillation coincides with the fundamental period in the sound to be recorded, its wave form will be held stationary on the viewing screen as if drawn on paper.

The cathode ray oscillograph in conjunction with a condenser microphone and a distortionless amplifier is probably the most accurate sound-recording device known. Since the recording is done directly by an electron beam there is no lag or distortion due

to heavy or damped moving parts such as other oscillographs show. It is particularly useful in recording speech sounds where high overtones are important. Curry[1] has been able to photograph the motion of the fluorescent spot on a film moving at 6 ft. per sec., and the upper limit to the frequency which can be recorded seems to be set by the response of the microphone rather than by the oscillograph itself.

Standard Sources. It is important, not only in connection with sound analysis, to have reliable standards of sound. It is not generally necessary, and not even feasible until we have a reliable absolute measurer of sound, to have a complete series of tones covering the musical scale and producing sound of intensity expressed as so many units. Miller's approximation to a series of tones of equal loudness, consisted of a "stop" of sixty-one organ pipes, voiced by the builder so as to give equal loudness, as judged by a skilled ear. He was thus reduced to a subjective standard in calibrating his instrument, and that standard one of uncertain constancy. For most purposes it is sufficient for an experimenter to have a source whose frequency and intensity can be relied upon not to vary from day to day, and on whose constancy he can base his results; but it is a standard only for the experimenter, it is in no sense a universal standard. Such is the tuning fork, driven by a constant current, and maintained at constant amplitude, with a small correction for temperature. The reliability of the tuning fork has led to its constant use as a fixed frequency standard.

The Siren. When a source of greater intensity is required the "siren" may be used. This instrument, designed by Cagniard de la Tour,[2] consists in principle of a wheel rotating at constant speed, having a number of equidistant holes which pass over the orifice of a tube leading from a reservoir containing air under pressure, and which release a number of puffs of air as the orifice is opened and closed (Fig. 78*a*). If these follow each other sufficiently rapidly a tone is produced whose frequency can be calculated if the speed of rotation of the disc is known. The note given by the siren is simple in character, hence its advantage as a standard. The intensity is proportional to the energy in the issuing blast, i.e., to the work done per second on the issuing air, which is given by the product of the volume delivered per second and the pressure

[1] *Sci. Inst. J.*, **11**, 162, 1934.

[2] *Ann. Chim. Phys.*, **12**, 167, 1819. Robison *first conceived the idea*, *Encyc. Brit.*, 1799.

in the reservoir. As the delivery is also proportional to the pressure, it is necessary to keep this constant. It is also necessary, of course, to secure constancy in the speed of rotation of the disc. This may be done by stroboscopic means, or the disc may be attached to the axle of a phonic motor, so that the frequency ultimately depends on a tuning fork. The siren used by Helmholtz [1]

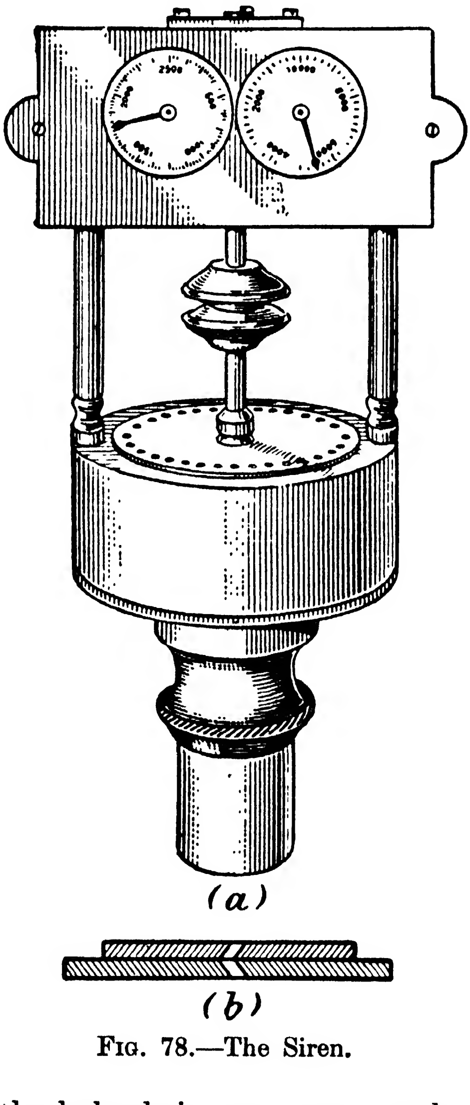


FIG. 78.—The Siren.

was self-rotating, the holes being set at an angle to the orifice (*see* Fig. 78*b*) like the blades of a turbine, so that the pressure-air was able to rotate the disc. This type has the disadvantage that intensity and frequency cannot be independently altered; increasing the pressure of the blast raises both. When the disc is independently rotated it would be possible to secure equal intensity

[1] *Sensations of Tone*, p. 163. *See also* Milne and Fowler, *Roy. Soc. Proc.* **98**, **414**, **1921**; Simeon, *Phys. Soc. Proc.*, **42**, 293, 1930.

in all tones, if one could be certain that the same proportion of kinetic energy in the air was converted into sound—this factor might be called the efficiency of the siren—at all frequencies; but this is doubtful.

Frequency Standardization. In order to find the exact number of vibrations per second made by a standard, it is necessary to compare the standard with a period which can be accurately measured. This is best a pendulum, and in Rayleigh's method [1] a standard fork is made to drive a phonic wheel (p. 116) carrying a stroboscopic disc, through the slits in which the oscillations of the pendulum can be observed. The number of poles on the phonic wheel and the number of slits on the disc are so adjusted that the interval between successive glimpses is about one-eighth of the period of the pendulum. The operation then proceeds by the method of coincidences, i.e. the time is measured which elapses between successive appearances of the pendulum at any given phase of its swing, viewed through the slits. The clock by which this time, and the period of the pendulum have been determined must be compared with a chronometer or other reliable time source. König [2] made the tuning fork control the escapement of a clock, which could be compared with a standard time source. For a more rapid determination of frequency, Scheibler in 1834 constructed a set of tuning forks and Appunn [3] a set of reeds to cover part of the musical scale, with four vibrations separating each of the set. Each thus produced four beats per second with its neighbours, and any source of sound could be located between two of these tones and its exact frequency found by counting the beats with either of the near-lying tones.

B. Resonators

The air columns contained in organ pipes which we have already described can serve as examples of resonators, but in analysis something possessing sharper resonance is required. Helmholtz [4] found that vessels having an internal capacity large compared with the orifice or neck by which communication with the atmosphere was made, were more selective as resonators, the criterion

[1] *Roy. Soc. Phil. Trans.*, **174**, 316, 1883.

[2] *Ann. d. Physik*, **9**, 394, 1880.

[3] *Ann. d. Physik*, **64**, 409, 1898; *see also* Klein and Rouse, *Opt. Soc. Amer. J.*, **14**, 263, 1927; Moon, *Bur. Standards Res. J.*, **4**, 213, 1930.

[4] *Sensations of Tone*, p. 373.

being that the condensation in the main reservoir should be practically uniform, and the to-and-fro motion of the air in vibration should be significant only at the neck. Two forms of Helmholtz resonator A and B are figured.

On the above assumption, we can imagine the air in the neighbourhood of the neck to act as a piston alternately compressing and rarefying the air in the main chamber.

If ρ = density of air, A = area, and l = length of neck, the equivalent mass of the piston is $A\rho l$. Now, if unit displacement of the "piston" has caused a compression equal to δp in the body of the resonator, the force on the "piston" is $A\delta p$. The restoring force per unit displacement $= A\delta p = A\gamma p\frac{\delta\rho}{\rho}$ if the compression is

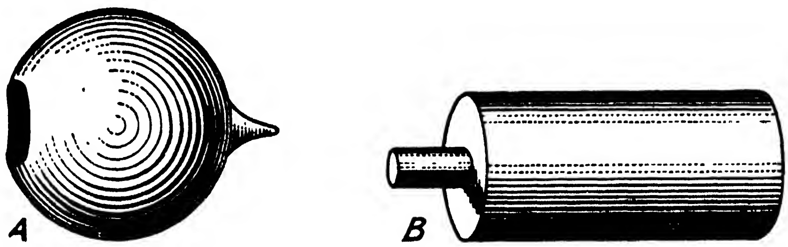


FIG. 79.—Resonators (*Helmholtz*).

adiabatic. Also the volume of air in the body of the resonator when this unit displacement has been produced $= v$, the volume of the resonator, whereas before, this mass of air occupied a volume $v + A$.

Hence
$$v(\rho + \delta\rho) = (v + A)\rho$$
$$\delta\rho = \frac{A\rho}{v}.$$

Whence the restoring force per unit displacement

$$= A\delta p = \gamma p\frac{A^2}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

and the time of oscillation of the system

$$T = 2\pi\sqrt{\frac{A\rho l}{\gamma p\frac{A^2}{v}}} = 2\pi\sqrt{\frac{\rho l v}{\gamma p A}}.$$

Or, putting $\sqrt{\frac{\gamma p}{\rho}} = c$, the velocity of sound in air :—

$$n = \frac{c}{2\pi}\sqrt{\frac{A}{l}\cdot\frac{1}{v}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

The quantity $\frac{A}{l}$ is called the "conductivity" of the neck, and the value of it given by this theory is only approximate. The solution requires that it should have the dimensions of a length. It represents the conductivity of the neck to air-flow through it. For a circular orifice in a thin wall (A, Fig. 79) the conductivity is equal to the diameter.

Helmholtz resonators of the cylindrical type can be adjusted in volume and therefore in frequency by arranging a part to slide over the other on the principle of the trombone. The law of variation of frequency given in (68)—n^2V = a constant—was found experimentally much earlier by Sondhauss.[1]

Before describing the use of resonators, it is necessary to emphasize the fundamental difference between the theoretical "pipe" and the "resonator." The motion in the former is a case of stationary waves, its longest dimension is comparable with the wave-length, and the displacement along the interior rises and falls continuously. The dimensions of the latter are small compared with the wave-length, and the orifice or neck is so narrow that the motion may be considered as confined to its neighbourhood. In practice, the bodies of air we deal with may form a compromise between these two aspects.

In order that a resonator may be sensitive, it must exhibit sharp resonance, and therefore the damping must be small (cf. p. 51). The damping of resonators of different forms has been examined experimentally by Leiberg[2] and Pochettino.[3] Their experiments showed that the mouth is the main region where damping takes place. This we should expect as most of the motion of the air occurs in this part of the vessel. The shape of the orifice had no influence, but an increase in width of the mouth reduced the damping.

Resonant cavities have been found in an unexpected quarter. The sounds of splashes produced by dropping a small sphere, or a drop of the liquid itself, into a vessel containing a liquid, have been shown to be due to the vibrations of the little cavity of air trapped by the falling body as it enters the liquid. The frequency of the note depends

[1] *Ann. d. Physik*, **81**, 235, 1850. *See also* Rayleigh, *Roy. Soc. Proc.*, **92**, 265, 1914, and *Phil. Mag.*, **32**, 188, 1916; Sizes, *Comptes Rendus*, **161**, 634 and 781, 1915, and **162**, 634, 1916; Lifschitz, *Comptes Rendus*, **154**, 1218, 1912; Hahnemann, *Zeits. tech. Phys.*, **3**, 265 and 281, 1922; Hartmann, *Phys. Rev.*, **28**, 719, 1922; Taber Jones, *Phys. Rev.*, **25**, 696, 1925.

[2] *Journ. Russ. phys. chem. Ges.*, **27**, 93, 1896.

[3] *Accad. Lincei. Atti.*, **8**, 260, 1899.

then on the size of this cavity, and the intensity on the height of the fall.[1]

Resonators as Analysers. Sets of resonators have been constructed either with fixed or with variable capacity, to pick out the partials in a musical sound. Several devices have been employed to show the extent of the response of particular resonators. One of the earliest was an adaptation by König of his manometric flame to register the vibrations of the air inside the resonator. He connected the listening orifice at the back of the resonator to the air side of the capsule, by a short tube, and had the usual gas jet on the other side. Sets of ten or more resonators were fixed on a stand, each with its appropriate capsule, the gas jets of which were brought into line, so that their movements could be examined simultaneously in a long revolving mirror. This has now been replaced by an apparatus giving more sensitive indications, by use of either the cooling effect on a hot wire by the current, or the effect which tends to set a suspended disc across the current of air.

Hot-wire Microphone. This instrument devised and developed by Tucker and Paris [2] consists of a Helmholtz resonator of type B

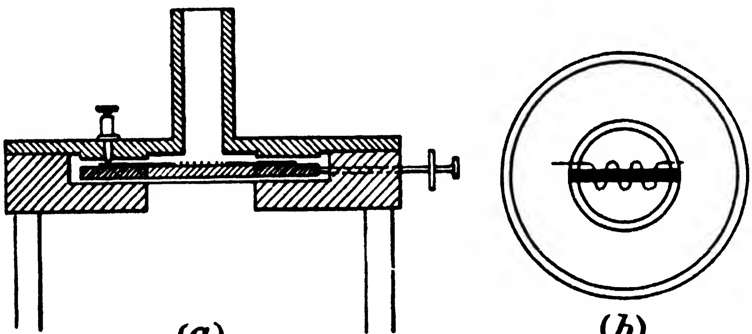


FIG. 80.—Hot-Wire Microphone (*Tucker*).

with a "grid" formed of a 0·0006 cm. diameter platinum wire bent into the shape shown (Fig. 80*b*), and placed at the entry of the neck into the main body of the resonator. Connection is made from the two ends of this coil of wire to two annular discs of silver foil, one above and one below a piece of mica, which forms the frame of the grid.

[1] Mallock, *Roy. Soc. Proc.*, **95**, 138, 1918; Raman and Dey, *Phil. Mag.*, **39**, 145, 1920; Narayan, *Phil. Mag.*, **42**, 773, 1921.

[2] *Roy. Soc. Phil. Trans.*, **221**, 389, 1921. *See also* V. Hippel, *Ann. d. Physik*, **66**, 293, 1921; Friese and Waetzmann, **76**, 39, 1925; Tugman, *J. Opt. Soc. Amer.*, **15**, 110, 1927.

The frame is clamped into the "collar" of the resonator (Fig. 80*a*), and a current passed by the terminals through the platinum wire until it is just below red-heat. Under these conditions a very slight motion of the air through the neck is sufficient to change the resistance by a measurable amount. The change in resistance in the alternating draught which takes place in the neck when the resonator is responding to a tone, partakes both of a steady drop, and of a complex periodic change containing the fundamental and harmonics of the frequency of the air motion. It is possible to measure the amplitude of the response, either by means of a vibration galvanometer tuned to the fundamental of the resonator, or else by measuring the average resistance of the hot wire during the motion, by a simple Wheatstone Bridge method. The resistance so measured is the same as would result from a steady draught of velocity equal to the maximum velocity ($2\pi na$) of the S.H.M. in the neck (cf. p. 185). So that if the resistance-velocity curve of the grid in a steady draught has been obtained, this can be used as a calibration curve to obtain from the measured resistance the amplitude of the motion in the neck, which amplitude indicates the extent of the response of the resonator.

For small velocities it is found that the steady drop in resistance is proportional to the square of the air velocity, and therefore to the energy of vibration in the neck of the resonator, or to the intensity of the sound therein. The grid is more sensitive, i.e., shows the greater resistance change to a given amplitude of air motion, as the heating current is increased. Normally, the grid is kept just below red heat. Apart from its use in analysis, the instrument, as its name indicates, serves as a sensitive detector for sounds of the frequency of the resonator.

Paris [1] has adapted the same principle to the double resonator, consisting of two Helmholtz resonators joined by a neck in which the heated grid is placed, protected from casual draughts. The vibrations of the two component resonators are "coupled," and as the analysis of page 55 showed, the two tones to which the system responds are not those due to each resonator acting alone, but lie outside these frequencies, the smaller vessel having its frequency raised, the larger lowered by the presence of the other.[2]

[1] *Roy. Soc. Proc.*, **101**, 391, 1922, and *Phil. Mag.*, **48**, 719, 1924, and *Science Progress*, **20**, 68, 1925, and *Phil. Mag.*, **2**, 751, 1926.

[2] Rayleigh, *Roy. Soc. Phil. Trans.*, **161**, 77, 1870, and *Phil. Mag.*, **36**, 231, 1918.

Paris placed the grid in a Wheatstone Bridge, the balance of which was disturbed when the double resonator responded to sound, the loss of balance causing a deflection of the galvanometer. Using a siren of an output as far as possible constant, he calibrated the instrument over a range of frequencies, using the galvanometer deflection as an indication of the response of the resonator. A typical curve is shown (Fig. 81).

If the frequencies of the component resonators are equal, the sensitivity of the double resonator can be made much greater than that of the single form, by making the inner resonator of small volume compared with the outer. If the conductivities of the two necks be denoted by l_1, l_2 the volumes of the resonators by v_1, v_2 :—since $n_1 = n_2$, therefore $\frac{l_1}{l_2} = \frac{v_1}{v_2}$. Other things being equal, the rate of flow will be greater through the neck having the

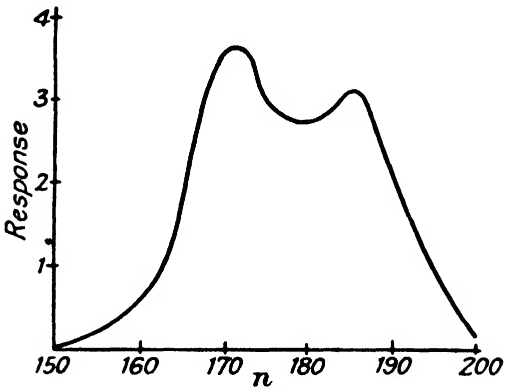


FIG. 81.—Response of Double Resonator (*Paris*).

smaller conductivity, and this, according to the above reasoning, will be at the entrance to the smaller vessel. Thus, by making the inner vessel smaller, we get a greater draught in the neck separating the vessels, than in that opening from the atmosphere, and therefore greater sensitivity of the grid in this position.

Rayleigh Disc and Phonometer. Rayleigh[1] observed that a light disc suspended in a sounding cylindrical resonator tended to set itself across the tube, i.e., at right angles to the direction of the alternating air current, and further that, if the disc was suspended by a torsion thread so as to lie at an angle to the opening when the resonator was unexcited, the extent of the turning towards the square-on position was proportional to the energy in the motion. Rayleigh's original disc was a small galvanometer

[1] *Phil. Mag.*, **14**, 186, 1882.

mirror with attached magnets suspended by a fibre in a magnetic field, so that the deflection could be registered by means of a beam of light reflected from the disc itself through the side of a resonator made of glass. The instrument is very sensitive, and has been used in a number of researches. A common form of the instrument is shown in Fig. 82.

The disc is of mica 1 cm. in radius, suspended by a quartz fibre 10 cm. long, and lies, when undeflected, at an angle of 45° to the axis of the tube resonator (in the form of a glass tube 2·5 cm. diameter) in the centre of which it hangs.[1] It is found best to have the radius of the disc about half that of the tube. To indicate the movement of the disc, a mirror as used on galvanometers is attached to the fibre outside the resonator. The hole by which the fibre enters the latter must, of course, be as small as possible consistent with free movement of the fibre.

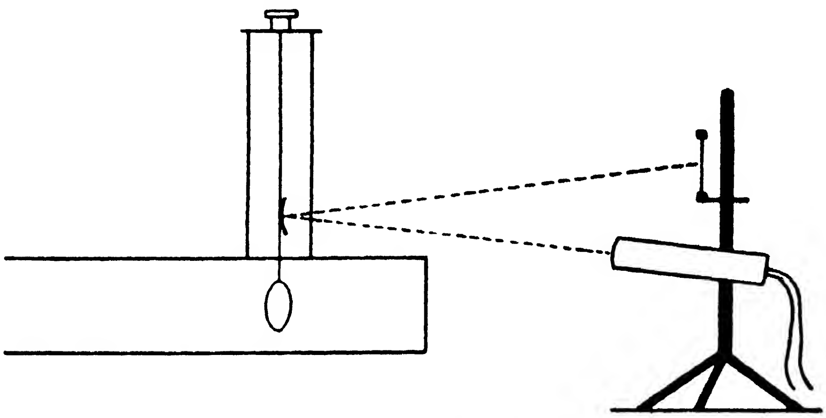


FIG. 82.—Rayleigh Disc.

König[2] showed that the turning moment on the disc, M, is proportional to the mean square of the velocity in the alternating air current, and this again (at constant frequency) is proportional to the intensity at the point in question. If C is the constant of the fibre, i.e., the torque per unit twist, an angular deflection ϕ will be produced such that $C\phi = M$.

When ϕ is small enough for the movement of the spot of light on the scale to be taken as its measure, the deflection on the scale will be proportional to $\overline{V^2}$, i.e., to the intensity. The instrument

[1] Taylor, *Phys. Rev.*, 2, 270, 1913. *See also* Michotte, *Arch. néerl.*, 7, 579, 1922; Skinner, *Phys. Rev.*, 27, 346, 1926; Barnes and West, *J.I.E.E.*, 65, 871, 1927; Sivian, *Phil. Mag.*, 5, 615, 1928; Schmidt, *Zeits. f. Phys.*, 60, 196, 1930.

[2] *Ann. d. Physik*, 43, 43, 1891.

is suited only to the measurement of intensities of sounds of constant frequency and quality, and should be employed at resonance for greater sensitivity. Grimsehl [1] constructed a "phonometer" on this principle for determining frequencies, using a tube resonator terminated at one end by an adjustable stopper, the length of best resonance to the source being found by observing the deflections of a disc near the mouth. Webster's [2] phonometer consists of an adjustable resonator, having a mirror in the neck which tends to set itself across the current of air.

Max Wien's manometer [3] measures the intensity of the response by the alternating pressure exerted upon a thin diaphragm in the rear wall; this is not, of course, the principle of the Rayleigh disc, but the instrument is included here for the sake of completeness.

The formula for the moment on the disc as developed by König is :—

$$M = \frac{4}{3}\rho r^3 \overline{V}^2 \sin 2\theta \quad . \quad . \quad . \quad . \quad . \quad (69)$$

r being the radius, ρ the density of the air, and θ the angle of repose of the disc relative to the stream, and is a maximum when θ equals 45°. The formula has been experimentally verified by Zernov,[4] who moved a box (surrounding the disc) and the air therein on a tuning fork, thus producing known velocity amplitudes. It follows from his work that it is possible to use the disc to measure the absolute velocity or displacement amplitude of the S.H.M. of the air surrounding it, if the constants of the disc and fibre are known.

Striæ in Kundt's Tube. König [5] found that a suspended system consisting of two equal spheres showed a similar tendency to set itself with the line joining the spheres normal to the stream. This behaviour led him to suggest an explanation of the formation of striæ in Kundt's dust figures, wherein the little spheres of dust are found in rows across the tube, i.e., normal to the stationary vibration. He found, for example, that two small spheres of cork suspended by a fibre in such a tube, at an antinode of stationary vibrations adhere together if they

[1] *Ann. d. Physik*, **34**, 1,028, 1888. *See also* Watson and Ham, *Phys. Rev.*, **18**, 178, 1921; Eisenhour and Tyzzer, *Frank. Inst. J.*, **208**, 397, 1929.

[2] *Nat. Acad. Sci. Proc.*, **5**, 173, 1919.

[3] *Ann. d. Physik*, **36**, 835, 1889. *See also* Edwards, *Phys. Rev.*, **32**, 23, 1911, and Hewlett, *Phys. Rev.*, **35**, 359, 1912.

[4] *Ann. d. Physik*, **26**, 79, 1908.

[5] *Ann. d. Physik*, **42**, 343 and 549, 1891.

happen to lie with the line joining their centres across the tube, but repel each other if turned so as to lie along the tube. By similar methods to those employed for the theory of the Rayleigh disc König showed that the force of repulsion along the axis, in such a case was given by :—

$$F = \frac{3}{2}\frac{\pi\rho r^6}{l^4}\overline{V^2}\cos\theta(3 - 5\cos^2\theta) \quad . \quad . \quad . \quad . \quad (70)$$

l being the distance between the spheres making an angle θ with the axis of the tube, and the other symbols having the same meaning as before (cf. 69). The particles ultimately reach equilibrium when the force is zero, i.e., they pile themselves in rows across the tube. The formula has been verified by Thomas,[1] using a device similar to that of Zernov. The force of repulsion which encourages this arrangement depends on $\overline{V}$ which is the mean velocity in an alternating current and therefore decreases in passing from an antinode to a node, as Robinson [2] pointed out, with the consequence that the striation gets less pronounced, and the successive striæ nearer together towards the node. Also the force tending to form the striæ, in accordance with (70) will increase with the intensity of the sound, since this produces an overall increase in $\overline{V}$. It may be noted in passing that an electric spark discharge at the mouth of the tube will fashion dust figures similar to those of the more usual rubbed rod. These are generally ascribed to alternations of definite frequency in the spark discharge.[3]

Recently, Crofutt [4] and others have used as source a telephone diaphragm driven by a valve oscillator (p. 134) in place of the rod, enabling much greater power to be used. Some observations imperfectly noted by Dvorak [5] have been clarified by Irons [6] and by Andrade.[7] There is first a tendency of the dust to form a disc at each antinode which is a distinct feature with the maintained oscillation, but not usually observed under a sporadic rubbing of a rod. Of still greater interest are the general circulations of particles from node to antinode, of which Rayleigh [8] had given a theoretical treatment following Dvorak's observations. Theory indicates currents of the type shown

[1] Described by König, *Phys. Zeits.*, **12**, 991, 1911. *See also Comptes Rendus*, **152**, 1,160, 1911.

[2] *Phys. Zeits.*, **9**, 807, 1908, and **12**, 439, 1911; *Phil. Mag.*, **18**, 180, 1909, and **19**, 476, 1910. *See also* Richmond, *Phil. Mag.*, **18**, 771, 1909.

[3] Marsh and Nottage, *Phys. Soc. Proc.*, **23**, 264, 1911; Robinson, *Phys. Soc. Proc.*, **25**, 256, 1913; Barton and Kilby, *Phil. Mag.*, **24**, 728, 1912.

[4] *Journ. Opt. Soc. Amer.*, **14**, 431, 1927.

[5] *Ann. d. Physik*, **153**, 102, 1874, and **157**, 42, 1876.

[6] *Phil. Mag.*, **5**, 580, 1928 and **7**, 525, 1929; *see also* Cook, *Phys. Rev.*, **36**, 1,098, 1930 and **37**, 1,189, 1931; Pringle, *Phil. Mag.*, **10**, 139, 1930; Hutchinson and Morgan, *Phys. Rev.*, **37**, 1,155, 1931.

[7] *Roy. Soc. Proc.*, **134**, 445, 1931; *Phil. Trans.*, **230**, 413, 1932.

[8] *Phil. Trans.*, **175**, 1, 1883. *See also* Schlichting, *Phys. Zeits.*, **33**, 329, 1932.

in Fig. 83*b* (bottom left) running along the wall from loop to node, in to centre, and back along the axis of the tube. Andrade has published some very beautiful photographs both of these circulations and of the antinodal discs. The upper drawing (*a*) shows two separate half circulations traced from such photographs, the clarity of which is in the main due to freedom from harmonics in the oscillator which drove the tube and careful water-jacketing of the tube to minimize convection currents. The circulations were best set in evidence by tobacco smoke. If an obstacle such as a ball bearing was placed on the axis of the tube, secondary oscillations were set up in its vicinity like large vortices (Fig. 83*c*, bottom right).

Robinson and Stephens [1] have discovered an interesting variant on

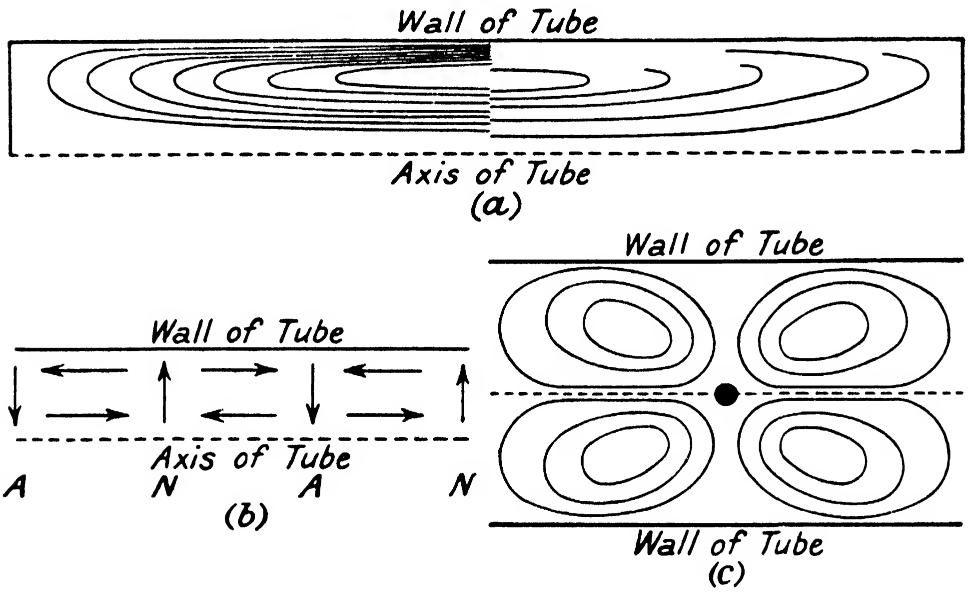


FIG. 83.—Circulations in Kundt's Tube (*Andrade*).

the dust tube. In place of particles, they introduce a series of soap films, formed across the tube at intervals. Those near an antinode burst, while the remainder settle in the nodes when the source is set in vibration.

C. ABSOLUTE PRESSURE MEASUREMENTS

Three attempts have been made to produce absolute instruments. Raps [2] utilized the optical interference method previously described (see p. 183). One of the interfering beams of light passed through the atmosphere in the neighbourhood of an intense sound source, the other passed through a closed vessel. Measurable oscillations of the bands were obtained, but he does not state what

[1] *Phil. Mag.*, **17**, 27, 1934. [2] *Ann. d. Physik*, **50**, 193, 1891.

precautions were taken to ensure that the sound did not pass through the walls of the control vessel.

Altberg[1] measured the pressure due to the sound. Dvorak[2] found that when sound waves in air impinged upon a solid wall they exerted a steady pressure upon it. Such a pressure is common to all forms of radiation, and is to be distinguished from the oscillatory change of pressure observed at a node of stationary waves. Lebedew[3] has extended his researches in light-pressure to this discovery of Altberg, and has verified the law which ascribes this pressure to the sound energy arriving at the wall per second, i.e., to $\frac{I}{c}$, or $\frac{2I}{c}$ when the waves reflected from the wall are also counted, I being the energy per second per unit area, and c the velocity of sound. On account of this radiation pressure a resonator experiences a repulsive force away from a tuning fork or source of sound placed at its mouth, owing to the resultant pressure which acts at the rear wall of the resonator. But to return to Altberg's application of this principle to sound intensity measurements. A hole is bored in the wall on which the sound impinges, and is nearly closed by a loose plug which is suspended from one arm of a very delicate torsion balance, serving to measure the force on the plug in the presence of the source of sound. The constants of the torsion fibre having been determined, it is possible to measure the force on the plug by the deflection of the torsion balance, as exhibited by a spot of light (Fig. 84). To get a readable deflection intense sources of sound were necessary. Altberg used the longitudinal vibrations of a glass rod excited by mechanical rubbing; the sound was unendurable unless the ears were stopped. The average pressure on the plug was estimated to be 0·24 dynes per sq. cm., whence $I = 4{,}100$ ergs. per sec. per sq. cm. This is the energy that crosses unit area per second and is known as the strength of the sound (per unit area). For simple harmonic waves the kinetic energy in unit volume at any instant $= \frac{1}{2}\rho\left[\frac{d\xi}{dt}\right]^2$, $\frac{d\xi}{dt}$ being the velocity in the vibration. Integrating over the complete

[1] *Ann. d. Physik*, **11**, 405, 1903. *See also* Waetzmann, *Phys. Zeits.*, **21**, 449, 1920; Stefanini, *N. Cimento*, **7**, 1, 1930.

[2] *Akad. Wiss. Wien. Ber.*, **72**, 213, 1875, and **84**, 710, 1882.

[3] *Ann. d. Physik*, **59**, 116, 1896, and **62**, 158, 1897. *See also* Meyer, *Ann. d. Physik*, **71**, 567, 1923; Hippe, *ibid.*, **2**, 161, 1927; Thomas, *ibid.*, **83**, 255, 1927; Waetzmann and Schuster, *ibid.*, **1**, 556, 1929.

period, we find the energy in unit volume given by $\frac{1}{2}\rho(2\pi na)^2$. Hence the energy crossing unit area per second is :—

$$I = \tfrac{1}{2}\rho(2\pi na)^2 c = \tfrac{1}{2}\rho\left[\frac{d\xi}{dt}\right]^2_{max} c \quad . \quad . \quad . \quad . \quad (71)$$

so that the pressure measurement gives a value of a if n be known.

Barus[1] employs a resonator with capillary neck, which he calls a pin-hole resonator, whose response he measures on this principle. A tube leads from the body of the resonator to a mercury surface. This surface is depressed a minute distance when the resonator sounds, requiring optical interference methods for the determination of the change of level of the surface due to the pressure on it.

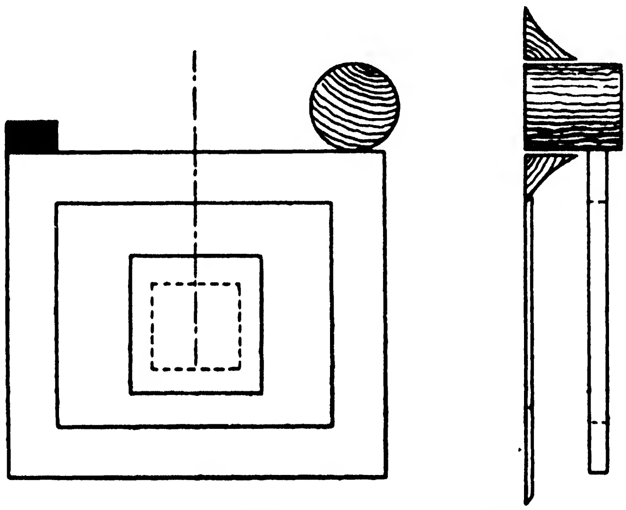


FIG. 84.—Pressure of Sound (*Altberg*).

Raps' measurement depends on an oscillating indication, Altberg's on a steady one. Each has its advantages and disadvantages. A steady change in the conditions, e.g., a change of density will merely shift the zero of Raps' manometer, whereas a corresponding increase of pressure on Altberg's plug, as, for example, from a draught, will upset the readings. On the other hand, the latter indicator gives the total sound energy due to the source, independent of the wave-form, whereas the interpretation of the movements of the interference bands is difficult when they are not simple harmonic.

The Null Method of Absolute Measurement. Resonant action of the diaphragm detracts from the reliability of measure-

[1] *Science*, **53**, 489, 1921 ; *Nat. Acad. Sci. Proc.*, **7**, 207, 1921, and **8**, 66 and 163, 1922, and **12**, 137 and 223, 1926. *See also* Carrière, *J. de Physique*, **4**, 413, 1923 ; Barus, *Nat. Acad. Sci. Proc.*, **13**, 52 and 579, 1927.

ments of sound intensity made by an instrument involving such a system. In order to avoid this trouble Gerlach[1] has adopted the ingenious idea of compensating the forces on the diaphragm due to the impinging sound waves, by measurable electrodynamic forces, so that the diaphragm remains at rest under the joint action of the opposing mechanical and electrical forces. The value of the latter forces, in mechanical units, gives the value of the fluid forces acting on the diaphragm, and hence the intensity of the impinging sound. Since the membrane is prevented from vibrating, it makes no difference whether the frequency is equal to, or far removed from, the natural frequency of the system, indeed, the inventor claims greater reliability for the instrument at resonance, since any slight out-of-balance between the opposing forces will be rapidly magnified by resonance into a large vibration, making the instrument very sensitive to this frequency. The principle on which the compensation is accomplished is to pass an alternating current of the same frequency across the diaphragm which is placed in a magnetic field. Both the frequency and the phase of the current must be adjusted until complete compensation is reached, and no movement of the diaphragm can be detected. This adjustment is made on the alternator which gives the current, and a listening tube behind the diaphragm leading to the ears in stethoscopic fashion serves to test the condition of compensation by the absence of sound from the membrane; the original sound waves must of course be prevented from reaching the hearing system. At present the apparatus is confined to S.H. sound waves and S.H. alternating currents.

The diaphragm consists of a thin rectangular band of aluminium held by two clamps at its ends, serving as terminals for the A.C. A field across and in the plane of the diaphragm is produced by a permanent magnet. The sound waves are allowed to affect only the central portion of the band where the magnetic field is uniform, the rest being kept fixed by rubber clamps. The force per unit area of the exposed part of the diaphragm, which balances that due to the sound, is $\frac{Bi}{b}$ (in c.g.s. units) where B is the induction in the aluminium due to the magnet, i the instantaneous current strength and b the width of the conducting band, assuming that

[1] *Wiss. Veröfft. d. Siemens-Konzern*, **3**, 139, 1923; *see also* Smith, *Phys. Soc. Proc.*, **41**, 496, 1929; Grützmacher, *Zeits. f. tech. Phys.*, **10**, 572, 1929; Hartmann, *ibid.*, **10**, 553, 1929.

the lines of magnetic force are perpendicular to the current lines. As long as the strength of the magnet is maintained, B and b are constants for the instrument which can be determined once for all; then the "root mean square" value of the compensating A.C. as given by an ammeter determines the mean pressure per unit area on the membrane, according to the above formula, which therefore gives the mean pressure due to the sound waves. The apparatus is used by the Siemens firm for testing the acoustical output of telephone transmitters and loud-speakers; but unfortunately no figures are yet available to demonstrate the sensitivity or facility of working of this instrument, so that the method must be regarded as in the tentative stage.

In the simple theory of the Rayleigh and Altberg discs (cf. pp. 218 and 223), two by no means negligible factors were ignored, viz., (1) the diffraction of the radiation by a small disc and its effect on the sound field, (2) the fact that the disc tends to swing with the field. The necessary corrections have been determined theoretically by King,[1] while Wood[2] has independently examined both theoretically and by experiment the magnitude of the second factor. The formula for the moment on the disc (69) must be multiplied by the inertia factor $(1-\beta^2)$ where β is the ratio of the velocity amplitude taken up by the disc to that of the air ($\bar{V}$) which excites it into vibration. Thus the relative velocity of fluid and disc is $\bar{V}(1-\beta)$, and this term replaces $\bar{V}$ in (69). The term $\frac{4}{3}\rho r^3$ is the equivalent fluid load on a thin disc set at 45° to the stream. If M is the mass of the disc, m that of the fluid displaced by the disc (in the Archimedean sense), the ratio of velocities

$$\beta = \frac{m + \frac{4}{3}\rho r^3}{M + \frac{4}{3}\rho r^3}.$$

In the experiments, Wood used a family of discs of the same radius but different mass in water at a point in the sound field of unvarying intensity, and verified the applicability of the correction term. The possibility of flexural vibrations is also considered, but it is shown that a disc 2 cm. in diameter and 2 mm. thick will be unaffected by resonance in the audible region.

[1] *Roy. Soc. Proc.*, **153**, 1, and 17, 1935.

[2] *Phys. Soc. Proc.*, **47**, 149, and 779, 1935. *See also* King, *Roy. Soc. Proc.*, **147**, 212, 1934, and **153**, 1, 1935; Lehmann, *Zeits. tech. Phys.*, **18**, 309, 1937; Merrington and Oatley, *Roy. Soc. Proc.*, **171**, 505, 1939.

CHAPTER TEN

ACOUSTIC IMPEDANCE

Acoustic Impedance. When a steady force is applied to a body of gas in a conduit, tending to move it along the conduit, there exists a definite relation between the applied pressure and the current produced, called the resistance; this resistance is due to viscosity. When the applied pressure is alternating, viscosity is not the only force opposing the movement of the gas; there is also a factor dependent on the frequency. The ratio between the applied pressure and the velocity produced by it, in the more general case, is known as the acoustic impedance, by analogy with the electrical impedance, which is the applied alternating electromotive force divided by the current produced by it. Taking the ideal case in which the displacement ξ of the gas is uniform over any one section (area A) and viscous resistance may be neglected, if the applied pressure is $P \sin \omega t$, the velocity produced $\frac{d\xi}{dt}$ can be written in the form $\frac{1}{A} \cdot \frac{dX}{dt}$, where X is of the dimensions L^3, and is a species of volume displacement. Then actually the impedance (Z) is defined by the relation

$$\frac{dX}{dt} = \frac{P \sin \omega t}{Z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

$\left[\text{the corresponding electrical equation is } \frac{dQ}{dt}(= I) = \frac{E \sin \omega t}{Z}\right].$

Suppose we have a cylindrical volume of gas, area of cross-section A, mass m, elasticity k, free from viscous forces, and subjected to an applied alternating force $F \sin \omega t$. The equation of motion will be

$$m\frac{d^2\xi}{dt^2} + k\xi = F \sin \omega t \quad \text{(cf. 20)}$$

Further let the instantaneous displacement be uniform over any one section of the gas, though varying from section to section,

so that we can put $\xi A = X$ and $F = PA$, where P is the maximum value of the applied *pressure*. Then :—

$$\frac{m}{A^2}\frac{d^2X}{dt^2} + \frac{k}{A^2}X = P \sin \omega t.$$

Case 1. Let $k = 0$ (restoring force negligible in comparison with inertia),

$$\frac{m}{A^2}\frac{d^2X}{dt^2} = P \sin \omega t$$

Integrate and :—

$$\frac{m}{A^2}\frac{dX}{dt} = -\frac{P}{\omega}\cos \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

Comparing with (72), we find $Z = \frac{\omega m}{A^2}$. $\frac{m}{A^2}$ is called the inertance (L) of the body of gas.

Case 2. Let $m = 0$ (mass negligible compared with elastic force),

$$\frac{k}{A^2}.X = P \sin \omega t.$$

Differentiate and :—

$$\frac{k}{A^2}\frac{dX}{dt} = \omega P \cos \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

Therefore $Z = \frac{k}{\omega A^2} = \frac{1}{\omega C}$, where C is called the capacitance.

Case 3. Both m and k finite. The solution is :—

$$\frac{dX}{dt} = \frac{P \cos \omega t}{\frac{k}{\omega A^2} - \frac{\omega m}{A^2}} = \frac{P \cos \omega t}{\frac{1}{\omega C} - \omega L} \quad \text{(cf. 25)}$$

$Z = \frac{1}{\omega C} - \omega L$ precisely as in the electrical case, save that the electrical analogue of L is called the inductance. The minus sign indicates that the effects of inertance and capacitance oppose each other. This expression will be a maximum when $\frac{1}{\omega C} - \omega L$ is a minimum, i.e., when $\omega^2 = \frac{1}{LC}$. This is the value of ω, which will excite the system to resonance; in fact the natural frequency of the system is given by :—

$$n = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

The reader who is familiar with alternating current theory will have recognized by this time that we have been developing equations similar to those used in that branch of electrical theory. Equation (75) with L and C given their electrical significance, gives the natural frequency of electrical circuits which we have often employed in connection with the maintenance of vibrations.

Helmholtz Resonator as Inertance and Capacitance in Series.

Mass of air in neck $= \rho l A$.

Hence $L = \frac{m}{A^2} = \frac{\rho l}{A}$, where l is to include the end-correction.

Capacitance of air in the cylindrical body is given by $\frac{1}{C} = \frac{k}{A_1^2}$ where k is the restoring force for unit volume-displacement, therefore $k = A_1 \delta p$. Also, the change in volume, produced by such displacement $\delta v = A_1 \times 1$.

Therefore
$$\frac{1}{C} = \frac{k}{A_1^2} = \frac{\delta p}{A_1} = \frac{\delta p}{\delta v} = \frac{\gamma p}{v} = \frac{c^2 \rho}{v}$$

And the natural frequency
$$n = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi}\sqrt{\frac{A}{\rho l}\cdot\frac{c^2\rho}{v}} = \frac{c}{2\pi}\sqrt{\frac{A}{lv}}$$

which is the same as (68).

It was pointed out in the first proof (p. 214), that this theory assumes all the inertance to be concentrated in the neck, and the capacitance in the body of the vessel. Helmholtz single and double resonators consist therefore, theoretically at least, of inertances and capacitances in series.[1]

A nondescript body of air such as that in a tube possesses in general both capacity and mass. Such a body must be regarded as an inertance and a capacitance in parallel. The reciprocal of the impedance due to such is given by the sum of the reciprocals of its constituents, or

$$\frac{1}{Z} = C\omega - \frac{1}{L\omega} = \frac{LC\omega^2 - 1}{L\omega}.$$

Conduits with Branches. We will now, following Stewart, consider a number of equal impedances, (Z_1) in a conduit, separated by branches containing other equal impedances, (Z_2) (Fig. 85). The tubes must be so short that no appreciable phase differences

[1] *See* Hahnemann and Hecht, *Phys. Zeits.*, 22, 353, 1921.

exist, at least over any one section. It will be shown that any simple harmonic vibration impressed on one end of the network will fail to "get through" unless the ratio of the impedance values for this particular frequency, Z_1/Z_2, lies between 0 and -4.

Let the value of the applied alternating pressure be $P_A \sin \omega t$ at A, $P_B \sin \omega t$ at B. Further let $\dot{X}$ be the velocity or rate of change of volume-displacement produced by the applied pressure, having amplitude $\dot{X}_{j-1}$ over EA, $\dot{X}_j$ over AB and $\dot{X}_{j+1}$ over BF. Then since the algebraic sum of the currents meeting at

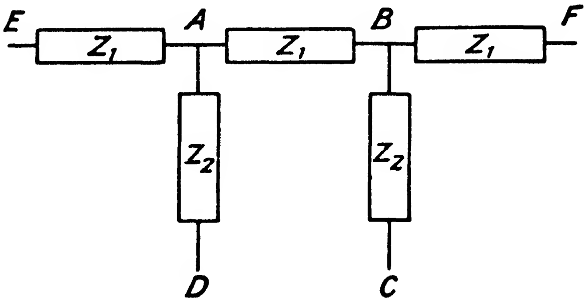


FIG. 85.—Conduit with Branches.

any junction must be zero (cf. "Kirchhoff's law" in electricity), the amplitude over AD must be $\dot{X}_{j-1} - \dot{X}_j$ and over BC, $\dot{X}_j - \dot{X}_{j+1}$ (C and D are terminals of constant pressure).

Then the alternating pressure-difference over :—

$$AB = (P_A - P_B) \sin \omega t = Z_1\dot{X}_j \text{ by (72)},$$
$$AD = P_A \sin \omega t = Z_2(\dot{X}_{j-1} - \dot{X}_j) \text{ (by 72)},$$
$$BC = P_B \sin \omega t = Z_2(\dot{X}_j - \dot{X}_{j+1}) \text{ by (72)}.$$

Therefore $Z_1\dot{X}_j = Z_2(\dot{X}_{j-1} - \dot{X}_j) - Z_2(\dot{X}_j - \dot{X}_{j+1})$

or
$$\frac{\dot{X}_{j+1}}{\dot{X}_j} + \frac{\dot{X}_{j-1}}{\dot{X}_j} = \left(\frac{Z_1}{Z_2} + 2\right) \quad . \quad . \quad . \quad . \quad . \quad (76)$$

Owing to the impedance, there will be a fall in velocity amplitude down the conduit, and considerations of the symmetry of the circuit point to there being the same relative loss over each section EA, AB, etc., so that we may write :—

$$\frac{\dot{X}_j}{\dot{X}_{j-1}} = \frac{\dot{X}_{j+1}}{\dot{X}_j} = \quad . \quad . \quad . \quad = e^{\alpha}, \text{ (say)}$$

where α represents the rate of decay of flow amplitude with distance down the main conduit. Substituting in (76), we find :—

$$e^{\alpha} + e^{-\alpha} = \frac{Z_1}{Z_2} + 2$$

or
$$\cosh \alpha = 1 + \frac{1}{2}\frac{Z_1}{Z_2}.$$

When α works out as an imaginary quantity, this means that there is no attenuation of amplitude. This is so when cosh α lies between $+1$ and -1, or :—

$$0 > \frac{Z_1}{Z_2} > -4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

In words, if ω is such that the impedance values satisfy this condition, a tone of this frequency will traverse the conduit with undiminished amplitude (friction disregarded); otherwise, if the conduit is long enough, its amplitude will be reduced to zero in transit.

Acoustic Filters. The effective impedance being a function of the inertance, capacitance and *frequency*, the statement expressed symbolically in (77) means that if a conglomeration of tones are led into such a conduit, all those having frequencies which do not satisfy this condition will be rapidly attenuated, and only those covering a certain range and satisfying (77) will get through. The system therefore acts as a sound filter. Analogous electric circuits have been employed in telephony for the past two decades, but Stewart[1] was the first to envisage the possibility of, and to construct, acoustic filters. Three types are possible:

1. High-pass filters.

Z_1 consists of capacitance, $\dfrac{1}{\omega C_1}$, only.

Z_2 consists of inertance, ωL_2, only.

If $\dfrac{Z_1}{Z_2} = 0$, $\omega = \infty$; if $\dfrac{Z_1}{Z_2} = -4$, $\omega = \dfrac{1}{2\sqrt{L_2 C_1}}$, so that the filter passes all frequencies between $\dfrac{1}{4\pi}\sqrt{\dfrac{1}{L_2 C_1}}$, and infinity. In practice this consists of a wide tube with holes, which may have short necks (L) surrounding them.

[1] *Phys. Rev.*, **20**, 528, 1922, and **22**, 502, 1923, and **23**, 520, 1924, and **25**, 90, 1925, and **26**, 688, 1925, and **27**, 487 and 494, 1926, and **28**, 1,088, 1926. *See also* Peacock, *Phys. Rev.*, **23**, 525, 1924; Mason, *Bell Syst. Tech. J.*, **6**, 258, 1927; **9**, 332, 1930; *Phys. Rev.*, **31**, 283, 1928; Stewart, *ibid.*, **31**, 696, 1928; Stewart and Sharp, *Opt. Soc. Amer. J.*, **19**, 17, 1929; Lindsay, *Phys. Rev.*, **34**, 652 and 808, 1929; Lindsay, White, etc., *Acoust. Soc. J.*, **4**, 155, 1932; **5**, 196 and 212, 1934; Schuster, *Ann. d. Physik*, **4**, 513, 1930; Levy, *Phys. Zeits.*, **31**, 358, 1930; Waetzmann and Noether, *Ann. d. Physik*, **13**, 212, 1932; West, *Phys. Soc. Proc.*, **46**, 186, 1934.

2. Low-pass filters.

Z_1 an inertance, and Z_2 a capacitance merely.

If $\frac{Z_1}{Z_2} = 0$, $\omega = 0$; if $\frac{Z_1}{Z_2} = -4$, $\omega = \frac{2}{\sqrt{L_1C_2}}$, so that the filter passes all frequencies between 0 and $\frac{1}{\pi}\sqrt{\frac{1}{L_1C_2}}$.

For this filter, a tube having branch cavities of considerable size (C) set close together, will serve.

3. Medium-pass or band filters.

These like the high-pass filters have capacitance C_1 in the main, but both capacitance C_2 and inertance L_2 in the branch lines. Their limits are :—

$$\omega = \frac{1}{\sqrt{(C_2 + 4C_1)L_2}} \text{ and } \omega = \frac{1}{\sqrt{L_2C_2}}.$$

The main tube of such a filter has side holes with necks, but the

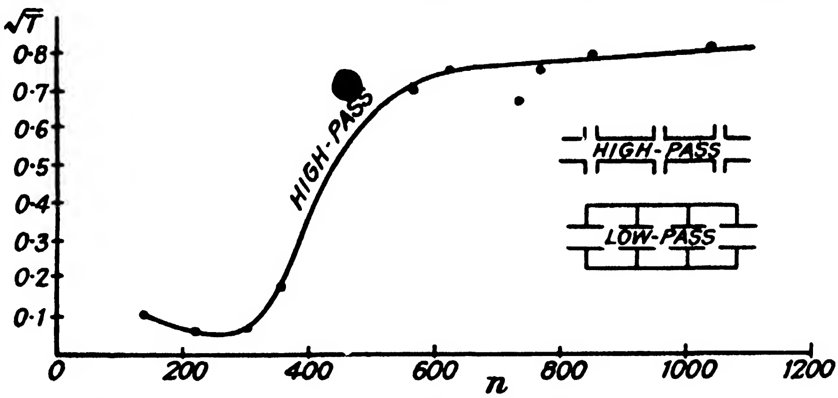


FIG. 86.—Energy transmitted by High-Pass Filter.

necks are broken at the centre, and communicate with closed vessels which surround them.

Besides Stewart, Canac[1] has been successful in constructing such filters. A test of the filtration possibilities of a high-pass filter is shown in Fig. 86 in which the intensity of sound T from a telephone transmitter, excited at various frequencies, after passing through the filter is shown. Sections of the filters are shown as insets to Fig. 86.

These filters should be very useful in sound experiments for purifying a note, and could also be employed commercially. For instance, a low-pass filter in the "tone-arm" of a gramophone should be useful in filtering out the "scratch" of the needle.

[1] *J. de Physique*, 7, 161, 1926.

Schuster and Kipnis [1] have also examined the exhaust silencers of motor vehicles from the aspect of filters. The ordinary baffle silencer consisting of a long tube divided off by baffles pierced with coaxal holes is essentially a low-pass filter as Davis [2] points out. As the most considerable engine noises are coperiodic with the revolutions, or a small multiple thereof, it is not very efficacious.

Both Hurst [3] and Erwin Meyer [4] have independently treated the problem of transmission through a manifold partition, and each has brought out the filter characteristics of this type of sound insulator. Hurst develops from first principles the theory of transmission through a series of plane infinite rigid partitions with constant separation, and shows that as the frequency of the oncoming sound is raised the system exhibits a series of alternate attenuation and transmission bands, the high-pitch terminals of the latter having heads like band spectra corresponding to the condition $l = j\lambda/2$, where l is the spacing and j an integer. The latter author is more concerned with the frequency limits and the actual value of the attenuation which such a filter can be expected to exhibit in the lower part of the gamut, i.e., when the spacing l is small compared to the wave-length, the obverse condition to that postulated by the Canadian scientist. With this proviso, the system will form a high-pass filter of limiting pulsatance $\omega = 2c\,.\,(lm/\rho)^{\frac{1}{2}}$, m being the mass per unit area of the partition and ρ the density of the air. Experiments with 5, 10, 15 element model systems made of cellophane or cardboard confirmed this view, at least in its general features, small discrepancies in the expected attenuation being ascribed to resonant drumming of the air pockets. This could, of course, be reduced by straw or similar packing.

A number of mechanical models have been built to demonstrate the properties of acoustic (and electric) filters. Most of these employ rods for the main line loaded by collars to act as branches. Lindsay and his collaborators [5] have examined the theory of a number of these. One constructed by West [6] is useful for demonstration purposes. It consists of a series of equal masses m suspended on springs of restoring force k per unit displacement, distance a apart. The line consists of an elastic cord under ten-

[1] *Ann. d. Physik*, **14**, 123, 1932. [2] *Phil. Mag.*, **16**, 787, 1933.
[3] *Canad. J. of Research*, **12**, 398, 1935.
[4] *Elekt. Nach. Tech.*, **12**, 393, 1935.
[5] *Acoust. Soc. J.*, **4**, 155, 1932; **5**, 196, 1934; **8**, 42, 1936; **10**, 41, 1938.
[6] *Phys. Soc. Proc.*, **46**, 186, 1934.

sion F and threaded through holes in the masses. The input is furnished by an electric motor coupled to one end of the cord by a rocker arm which contributes a transverse simple harmonic motion to the system, which then acts as a band-pass filter whose limits are (*a*) the period of a single mass on its spring, and (*b*) the period which has a half wave-length equal to the distance between consecutive masses. The one limit corresponds then to resonance in the branch, the other to resonance in the line element of an acoustic (or electric) filter. Between these limits the length of the wave transmitted progressively changes, the relation between the frequency and wave-length being

$$n = (2\pi)^{-1} . \left\{(4\mathrm{F}/ma) \sin^2 \pi a/\lambda . + k^2/m\right\}^{\frac{1}{2}},$$

which checks well with experiment. Considerable dispersion is shown by the filter, the wave velocity varying from 265 to 1,080 cm./sec. within the range of the experiments.

Impedance of Pipes. Up to the present we have considered the motion in the system to be wholly in phase. If we have to deal with conduits or sections of filters of length comparable to the wave-length of the transmitted sound we must, of course, use the wave equations. The electrical analogue will now be the cable, without attenuation, if we neglect viscous resistance.[1] It will also be more convenient in dealing with pipes to write the simple harmonic variation with time in the exponential form, viz., $e^{i\omega t}$ instead of $\sin \omega t$ or $\cos \omega t$. The particle velocity at any section along the pipe at a distance x from the end then takes the form

$$\dot{\xi} = \left(A \sin \frac{\omega}{c}x + B \cos \frac{\omega}{c}x\right)e^{i\omega t}$$

Whence $$\xi = (i\omega)^{-1}\left(A \sin \frac{\omega}{c}x + \mathrm{B} \cos\frac{\omega}{c}x\right)e^{i\omega t} \quad \text{(cf. 53)}$$

But $$-\frac{\partial(\delta p)}{\partial x} = \gamma p\frac{\partial^2 \xi}{\partial x^2} \quad \text{(cf. p. 3)}$$

[1] *See* Fleming, *Electric Wave Teleph. and Telegraphy*, p. 295, 1919; Mallett, *Phys. Soc. Proc.*, **39**, 251, 1927; Kennelly and Siskind, *Amer. Phil. Soc. Proc.*, **66**, 89, 1927; Waetzmann and Schuster, *Ann. d. Physik*, **84**, 507, 1927; **3**, 314, 1929.

$$\text{Whence } \delta p = -\rho c^2 \frac{\partial \xi}{\partial x} = i\rho c\left(\mathrm{A} \cos \frac{\omega}{c}x - \mathrm{B} \sin \frac{\omega}{c}x\right)e^{i\omega t}$$

$$Z_l = \frac{\delta p_l}{\dot{X}_l} = \frac{i\rho c}{S}\left[\frac{A \cos \frac{\omega}{c}l - B \sin \frac{\omega}{c}l}{A \sin \frac{\omega}{c}l + B \cos \frac{\omega}{c}l}\right]$$

But when $x = 0$; $Z_0 = \frac{i\rho c}{S}\left[\frac{A}{B}\right]$

Therefore
$$Z_l = \frac{i\rho c}{S}\left[\frac{Z_0 \cos \frac{\omega}{c}l - \frac{i\rho c}{S} \sin \frac{\omega}{c}l}{Z_0 \sin \frac{\omega}{c}l + \frac{i\rho c}{S} \cos \frac{\omega}{c}l}\right] \quad . \quad . \quad (78)$$

This gives the impedance "looking in" at the end $x = l$ in terms of that at $x = 0$.

If the pipe is stopped at $x = l$, $Z_l = \infty$ and then

$$Z_0 = -\frac{i\rho c}{S} \cot \frac{\omega}{c}l \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

If, on the other hand, it is completely open at $x = 0$, as well as at $x = l$,

$$Z_0 = +\frac{i\rho c}{S} \tan \frac{\omega}{c}l \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

These expressions equated to zero give the resonant frequencies corresponding to those on p. 162, obtained by the "classical" means. For the case of a partially stopped pipe we may substitute the impedance of an orifice for Z_0 in (78), i.e., $Z_0 = i\rho\omega/\kappa$ where κ is the conductivity (p. 215).

Alternatively, we consider the partially stopped pipe in resonance as being the sum of two impedances, pipe + orifice in series, and find the resonant frequency by equating the sum to zero. Thus for a pipe partially stopped at one end ($x = 0$) and completely stopped at the other ($x = l$)

$$Z_l = \frac{i\rho\omega}{\kappa} - \frac{i\rho c}{S} \cot \frac{\omega}{c}l.$$

or, putting $k = \omega/c$, resonance will occur when

$$\tan kl = \kappa/kS$$

This formula can be solved graphically by plotting $\tan kl$ and κ/kS against $1/kl$ and finding where the curves intersect.[1] Note that when κ is big this formula reduces to $\cot kl = 0$ for the stopped pipe (cf. 79), while when it is small enough for $\tan kl$ to be put equal to kl, it reduces to the Helmholtz resonator formula (68), i.e., $k^2 = \kappa/lS = \kappa/v$.

Fig. 87 shows the theoretical values (dotted line) compared with some actual values obtained by tuning resonant lengths of a pipe

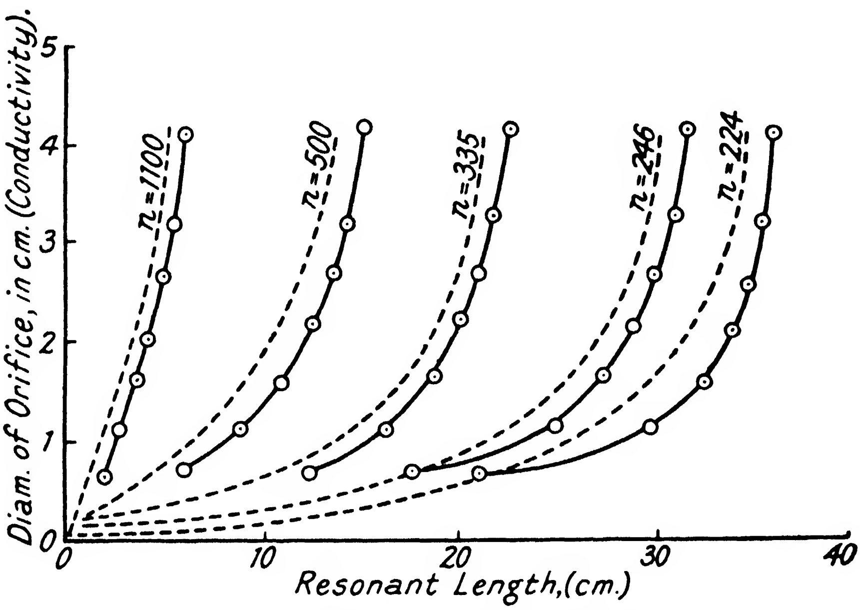


FIG. 87.—Resonance of Partially Stopped Pipe.

to a tuning fork while the upper end of the pipe was covered with a metal plate having holes of various sizes drilled in it. In this case κ = the diameter of the orifice.

Pipes with Side Holes. A rather similar case in which the resonant frequencies can be calculated by putting the net impedance equal to zero is that of a pipe with a hole in its side. Thus for a pipe with a hole distant L from one end and l from the other, the impedance can be considered as made up of an orifice (impedance Z_0) added to two pipes of lengths L and l (Z_L and Z_l) *in parallel*. So that the resonant frequency is given by

$$Z_0 + \frac{1}{1/Z_L + 1/Z_l} = 0$$

or

$$\frac{1}{Z_0} = -\frac{1}{Z_L} - \frac{1}{Z}.$$

[1] Richardson, *Phys. Soc. Proc.*, **40**, 206, 1928; *see also* Warren, *ibid.*, p. 296; Hanson, *ibid.*, **42**, 43, 19C0.

If the complete pipe is open at each end

$$-\frac{\kappa}{i\rho\omega}=\frac{S}{i\rho c}\left[\cot\frac{\omega}{c}L+\cot\frac{\omega}{c}l\right],\ \text{from (80)}$$

or

$$\frac{\kappa}{kS}=-\cot kL-\cot kl \quad . \quad . \quad . \quad . \quad . \quad (81)$$

This corresponds to the flute with one side hole open; the resonant frequency is not the same as that of a pipe terminating at the first open hole, as an approximate theory would indicate. For a stopped pipe with one side hole open the corresponding formula is

$$\frac{\kappa}{kS}=\tan kL-\cot kl \quad . \quad . \quad . \quad . \quad . \quad (82)$$

This corresponds to the clarionet with one side hole open. The case of the oboe (closed cone) can be dealt with by the same formula (81) as the flute. The formulæ (81) and (82) give good agreement with measurements on actual wind instruments.[1]

If the instrument has a number of side holes open, as is often the case in playing wood-wind instruments, the equations become a rather awkward series of continued fractions but, as the amplitude in the orifices lower down the tube is small, the contribution of those beyond the first two or three open ones may be neglected. The system will in fact tend to act as a filter when a large number of side holes are opened.

Compound Pipes. Cases in which two pipes or a pipe and resonator are joined in *series* or *parallel* have been treated theoretically by Irons. Several of these cases are important in practice as they are used as organ stops, e.g., the Boys' resonator which consists of a Helmholtz resonator on the end of a pipe—the resonator may alternatively be let into the side of the pipe—and the *flûte-à-cheminée*, in which a narrow pipe is a continuation or re-entrant upon a wider one. In the latter case, the resonant frequencies are given by a formula originally due to Aldis,[2] which Bate[3] has verified experimentally, and which we may write as the sum of two open pipe impedances in series, i.e.,

$$i\rho c\left[\frac{1}{\pi R^2}\tan kL+\frac{1}{\pi r^2}\tan kl\right]=0$$

L being the length and R the radius of the wide pipe, l and r those of the narrow pipe.

Taber Jones[4] has treated the theory of the Haskell organ pipe, an open pipe into which a shorter closed pipe is inserted (Fig. 88) excited in the usual edge-tone method of flue pipes. In terms of the symbols

[1] Richardson, *Acoustics of Orchestral Inst.*, *App.*; *see also* Irons, *Phil. Mag.*, **10**, 945, 1930, and **11**, 535, 1931.

[2] *Nature*, **114**, 309, 1924.

[3] *Phil. Mag.*, **16**, 562, 1933.

[4] *Acoust. Soc. J.*, **8**, 196, 1937.

shown on the figure, he proves that the wave-number k of the natural frequency is given by

$$q_1 \cot ka + q_2 \cot kb = (q_1 + q_2) \tan kl \quad . \quad . \quad . \quad . \quad (4)$$

The system of acoustic impedances is that of a stopped pipe with two open pipes in series branched upon the open end, and serves to produce bass notes in a comparatively restricted space, more so, in fact, than

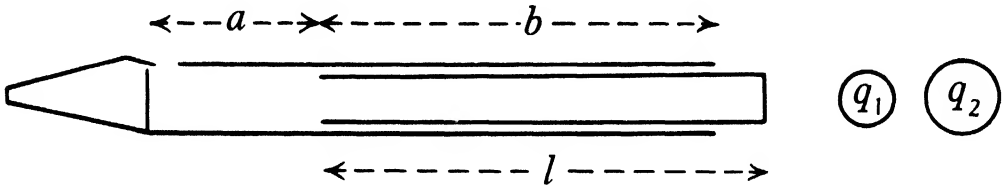


FIG. 88.—Haskell Organ Pipe.

if a single stopped pipe were used. To make this formula agree with the experimentally observed frequencies, it is of course necessary to make certain assumptions with regard to end corrections.

A rather more complicated case is that of a wide pipe with a constriction somewhere inside it. This we may regard as two pipes connected in series by an orifice. Irons[1] has treated this case both theoretically and experimentally, using the method of Kundt's dust figures to determine wave-lengths. The tube being virtually separated into two stopped pipes of length L and l by the constriction, the resonant frequencies will be given by

$$\frac{kS}{\kappa} - \cot kL - \cot kl = 0$$

Moreover the formulæ for compound pipes can be readily extended to meet cases where the two lengths consist of media of different properties by inserting, e.g., $\rho' c'$ in place of ρc for the " characteristic impedance " (cf. p. 239) in the second medium, a case worked out on classical lines by Lees.[2] Further, if ρ, ρ' and c are known, the determination of the natural frequency of such a compound system will enable c', the velocity of sound in the second material to be measured. In this way it is possible to measure the velocity of sound in soft and brittle solids, by making a compound rod with a length of steel or ash joined to the specimen end to end, a method first used by Stefan.[3]

Characteristic Impedance. Consider the case of plane waves incident normally on a boundary between two media. Let the equation of the incident wave, moving from left to right, be

$$\xi_1 = f(ct - x)\,; \quad \dot{\xi}_1 = cf'(ct - x)$$

[1] *Phil. Mag.*, 7, 523 and 873, 1929.

[2] *Phys. Soc. Proc.*, 41, 204, 1929 ; *see also* Irons, *J. Sci. Inst.*, 7, 323, 1930.

[3] *Akad. Wien. Ber.*, 57, 697, 1868.

Then $$s_1 = -\frac{d\xi_1}{dx} = f'(ct - x)$$

so that $\dot{\xi}_1 = cs_1$, where s_1 is the condensation. For the reflected wave,
$$\xi_2 = f(ct + x)$$

and $$\dot{\xi}_2 = -cs_2$$

while for the transmitted wave, $\xi' = f(c't - x)$
$$\dot{\xi}' = c's'$$

Since the net particle velocity normal to the boundary must be the same on each side,
$$\dot{\xi}_1 + \dot{\xi}_2 = \dot{\xi}'$$
or
$$cs_1 - cs_2 = c's'.$$

Since the normal pressure at the boundary must be the same in each medium
$$es_1 + es_2 = e's'.$$

(Note that the elasticity = applied pressure/resulting condensation.) Eliminating s' between these two equations, we obtain

$$\frac{s_2}{s_1} = \frac{e'c - ec'}{e'c + ec'} = \frac{\rho'c' - \rho c}{\rho'c' + \rho c} \quad . \quad . \quad . \quad . \quad (83)$$

since elasticity = (velocity)^2 $\times$ density.

This ratio expresses the relative pressure amplitudes in the reflected and incident sound waves. There will in fact be no reflection if $\rho'c' = \rho c$. This product of the density and velocity of sound is a characteristic of the medium and the type of wave (plane), and we have already met it in connection with the pressure exerted on a solid boundary by the sound (p. 224) where it has been called the energy density or intensity crossing unit area per second. In connection with sound transmission it is more often known as the "characteristic impedance of the medium." Considering a plane wave crossing unit area of the medium

$$z = \frac{\delta p}{\dot{X}} = \frac{es}{cs} = \rho c$$

This impedance, being real and independent of frequency, is a pure resistance, but this is not true of spherical waves, to which we must turn our attention for a while.

In the development of our fundamental wave equation, we

introduce a quantity ϕ whose derivative with respect to any direction is to represent the particle velocity in that direction, e.g., in the present case, $\frac{\partial \phi}{\partial r}$ represents the radial velocity. It can then be shown that $\dot{\phi} = -c^2 s$.[1] For a spherical wave we write, satisfying (62),

$$rs = e^{ik(ct-r)}, \text{ where as before } k = \omega/c$$

$$\phi = -c^2 \int s dt = -\frac{c}{ikr}.(rs)$$

$$\frac{\partial \phi}{\partial r} = \frac{c}{ikr}\left(\frac{1}{r} + ik\right).(rs)$$

Also (cf. p. 163) $\qquad \delta p = \rho c^2 s = \frac{\rho c^2}{r}(rs).$

Dividing these last expressions, we obtain the characteristic impedance to spherical radiation, viz.,

$$z = \rho c\left(\frac{ik}{1/r + ik}\right) = \rho c\left(\frac{1}{1 - i/kr}\right) \quad . \quad . \quad . \quad (84)$$

or in a form which allows one to separate the real and imaginary parts

$$z = \rho c\left(\frac{ikr}{1 + ikr}\right) = \rho c\left(\frac{k^2 r^2 + ikr}{1 + k^2 r^2}\right) \quad . \quad . \quad (85)$$

The first term in the numerator is a resistance, the second a reactance. If r is very large, (84) reduces to ρc as for plane waves. If, on the other hand, we are considering a place at a small distance (compared with the wave-length) from the source, so that kr is small, (85) shows that the impedance is approximately

$$\rho c k^2 r^2 = 4\pi^2 n^2 \rho r^2 / c$$

which is sometimes termed the "radiation resistance."

In giving the impedance theory of pipes we should properly have included this radiation impedance as a consequence of the change over from plane to spherical waves at the mouth. Its influence on the natural frequencies of the system is small however, and the effect is bound up with the end correction of the tube. A more important aspect of this matter appears in the calculation of the ratio of the transmitted to the reflected energy. Not only will the considerations leading to equation (83) apply to a material change in the medium, but in addition whenever

[1] Cf. Lamb, p. 205.

there is a sudden change in the *total* acoustic impedance there will be reflection. Such change may occur at a constriction in virtue of a change in S, or at the end of a pipe in virtue of the change from plane to spherical waves. Energy will be radiated most efficiently, i.e., there will be least reflection, if the impedances of the tube and of the open air at the end are matched; this cannot be done with a cylindrical pipe.

The Horn as Radiator. Consider a tube with cross-section increasing along its length having a diaphragm vibrating in piston fashion at its narrow end, and terminating at the other in a wide flare. The piston will work most efficiently into the horn if the acoustic impedance of the air at the throat is matched to its mechanical impedance. This can usually be done by constructing an air cavity in the throat, one side of which is closed by the piston, while the other leads through a narrow aperture into the body of the horn. The inertance and capacitance of the throat can be adjusted by calculation (cf. the Helmholtz resonator) to equal the mass and stiffness respectively of the diaphragm. Incidentally, the throat acts as an acoustic transformer, the product of pressure and velocity amplitudes on either side of the constriction being approximately the same. The waves then pass along the widening tube and are let into the atmosphere from an opening whose diameter is large. They therefore start out from this place as spherical waves of considerable radius and, provided the wave-length is not too large, the acoustic impedance is nearly the same as for plane waves. Radiation then takes place efficiently, there is little reflection back into the horn except at low frequencies, and moreover at wave-lengths small compared with the aperture there will be a directive effect. In spite of the tendency of the waves to spread, interference will ensure that most of the energy is sent out as a beam parallel to the axis of the horn, as in the corresponding optical case.

Looked at from another point of view, for spherical waves of small radius and therefore diverging from a narrow aperture, the particle velocity and pressure are nearly 90° out of phase, and therefore the impedance of the spherical waves is nearly all reactance—a function of the frequency—whereas that of the plane waves inside the tube is a pure resistance independent of frequency. Even if their absolute values are equal they are not thereby matched. As the frequency is raised and the aperture increased in diameter, these two factors come more nearly into

phase, until the impedance of the medium outside is a pure resistance like that inside.

A cylindrical or *slightly* conical pipe is essentially then a resonator. Its function is to have marked natural frequencies in virtue of the large amount of reflection at the open end. Orchestral wind instruments are of this type; as radiators they are inefficient except at high frequencies, in spite of the flare at the end of most of them, which is too "sudden" to be of use acoustically. At the other extreme, the loud-speaker horn is designed with a gradual flare to be an efficient radiator without markedly detached resonances.

Horns of types other than the cone were first studied theoretically by Webster,[1] and latterly from both practical and theoretical aspects by the staff of the Bell Telephone Laboratories. It is found that the exponential horn, one whose section at a distance x from the throat of area S_0 is given by $S = S_0e^{\alpha x}$, is the best, in that the impedance at the open end becomes a pure resistance at much lower frequencies than that of the conical horn —$S = S_0(1 + \alpha x)$—of the same initial and final cross-sections. It therefore radiates low frequency sounds more efficiently.

The Annular Effect. It has been assumed up to the present that the motion across the section of a pipe or orifice is uniform. It is obvious that in the immediate neighbourhood of the solid confines of a conduit there will be a drop in the displacement amplitude to zero, but before considering this effect of viscous resistance, there is another effect, more important in practice, which invalidates this assumption. It was discovered by the author in experiments with a hot wire traversed across an orifice or conduit in which aerial oscillations were taking place, that the velocity amplitude was much *greater* in the outer annuli just before the wall was reached than at the centre of the tube. There is in fact a peak of alternating velocity whose magnitude (relative to the central velocity) increases with frequency, but whose distance from the wall decreases as this factor goes up. This circumstance, which gives the distribution of velocity across the tube or orifice quite a different trend to that of one-way air flow, immediately calls to mind the skin effect in electrical technology, shown by high-frequency alternating currents flowing in a magnetic conductor. Fig. 89 shows some typical results. This "annular effect" can be explained by considering the equations of

[1] *Nat. Acad. Sci. Proc.*, 5, 275, 1919.

motion of the air in the tube, and is due to a combination of inertia and viscosity. It appears from the theory that the distance (δ) of the peaks from the walls of the tube is given by the formula $\delta\sqrt{n} =$ a constant.[1]

This law, that δ is proportional to $n^{-\frac{1}{2}}$ is borne out in practice; the position of the peaks is shown by the dotted line on Fig. 89. The existence of this annular effect has been confirmed by Carrière[2] who noted the distances traversed by smoke particles executing

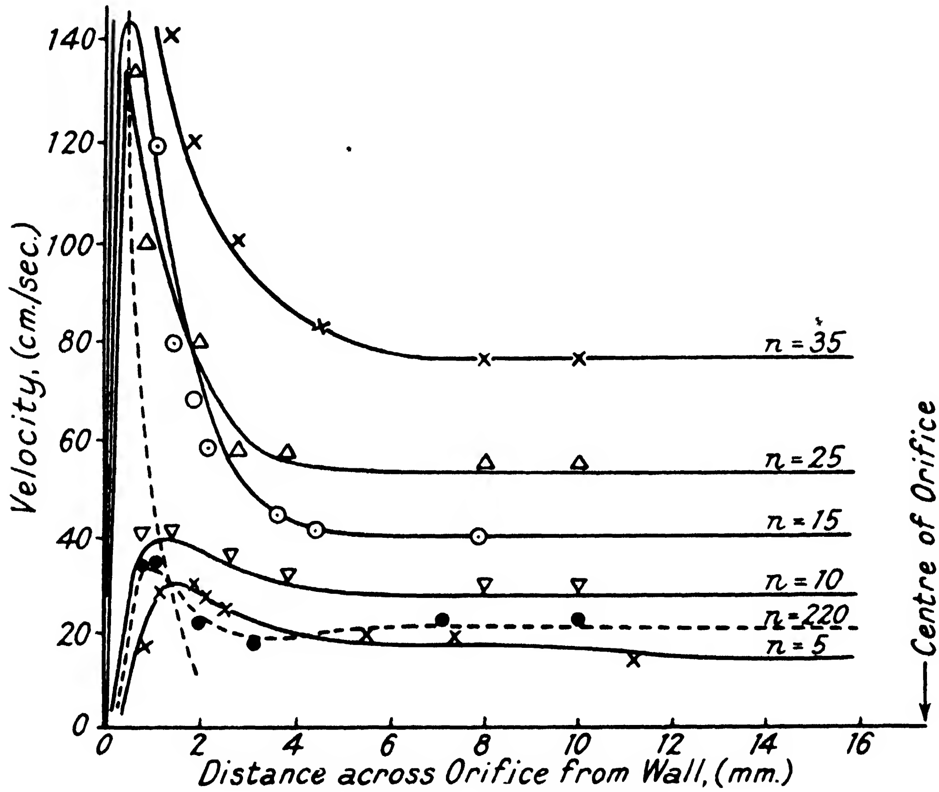


FIG. 89.—The Annular Effect.

S.H.M. in a pipe. The amplitude was a maximum at points a short distance from the walls of the pipe.

Imperfect Reflectors. If, in equation (78) for the pipe, we set Z_0 equal to the impedance of an imperfect reflector, and can determine the impedance Z_l looking in at the other end, we can, with the aid of equation (83), determine the reflection coefficient of the specimen, for the first equation with Z_l and ω known will give us the ratio of ρc to Z_0, i.e., the ratio of the characteristic

[1] Richardson, *Phys. Soc. Proc.*, **40**, 206, 1928; Richardson and Tyler, *ibid.*, **42**, 1, 1929; *Phys. Zeits.*, **32**, 509, 1931; Sexl, *Zeits. f. Phys.*, **61**, 349, 1930.

[2] *J. de Physique*, **9**, 187, 1928.

impedances of the wide pipe and the pervious material respectively, while this ratio, if substituted in the second equation, will give us s_2/s_1, the amplitude reflection coefficient of the medium. This method was first suggested and used by Wente and Bedell[1] to measure the absorption coefficient of a specimen to sound. It has also been applied by Penman and Richardson[2] to test the velocity of sound in narrow tubes, the specimen in this case being a bundle of glass tubes of diameter 0·2 mm. forming the imperfect stop at one end of a pipe $1\frac{1}{2}$ in. wide, driven at the other end by a telephone, maintained with constant frequency.

Keibs[3] has a simple method of measuring the impedance, and hence the absorption coefficient, of a reflector at the end of a tube of length l with attenuation constant α and propagation constant β, provided βl is not less than 3 cot $(\alpha + i\beta)l$. He measures the pressure at the sending end when the other is closed successively by a hard stop and by the unknown impedance z. The ratio of these two pressures

$$\frac{p_1}{p_2} = \frac{z}{z + \cot(\alpha + i\beta)l} = \frac{z}{z+1}, \text{ approximately.}$$

He uses this simple formula to measure the impedance of the ear drum placed at the end of a short tube, a microphone at the other end measuring pressures, while just behind it is a thermophone wire as source.

Practical Measurement of Impedance.

1. *Direct Method* (*Richardson*[4]). The velocity distribution across the orifice is measured by the hot wire method, and the pressure gradient through it by means of the manometric capsule described on p. 182. Thus, in the case of a sounding Helmholtz resonator, if this membrane is inserted into the back of the reservoir and the velocity is integrated across the mouth by means of the hot wire, the ratio of the two gives the apparent impedance.

2. *Acoustic Method* (*Stewart*[5]). Two telephone diaphragms are placed at one end of each of two conduits, and supplied with alternating currents of the same frequency. At their other ends are attachments leading to the ears as in the stethoscope. One of these is fixed in position, but the distance of the other attachment from the source can be varied by telescoping two parts of the tube over one another. The positions are adjusted until no sound is heard in the stethoscope; this means that the vibrations

[1] *Bell. Syst. tech. J.*, **7**, 1, 1928. [2] *Acoust. Soc. J.*, **4**, 322, 1933.
[3] *Ann. d. Physik*, **26**, 585, 1936.
[4] *Phys. Soc. Proc.*, **40**, 206, 1928. [5] *Phys. Rev.*, **28**, 1038, 1926.

are arriving at the ear out of phase. The system whose impedance is to be measured is now inserted as a branch on one conduit. Both the currents in the telephone and the position of the sliding attachment have to be changed in order to restore the silence condition. From the first the change of pressure amplitude, and from the second the change of phase due to the insertion of the branch can be calculated. This method, though less direct, therefore gives both magnitude and phase factor of the impedance, whereas the first gives the absolute value only.

3. *Pressure Method (Schuster*[1] *and Robinson*[2]*)*. This method involves the equating of the pressure at some point in an impedance which is supposed known, or taken as standard—e.g., a wide pipe—with the pressure on one side of the impedance to be measured, both this and the standard being driven at the other end

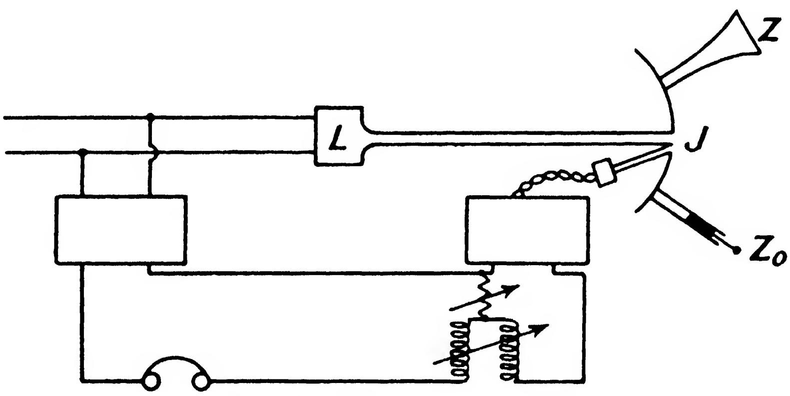


FIG. 90.—Impedance Meter (*Flanders*).

by the same telephone diaphragm. In Robinson's apparatus the position of the point of attachment of the stethoscope in the wide pipe can be varied by a sliding piece, or the length of the wide pipe itself can be varied until equality of pressure is attained. The apparatus is then like an acoustic Wheatstone Bridge in that a variation in one arm is made until the stethoscope which takes the place of the galvanometer indicates equality of pressure across the junctions. Robinson has measured the impedance of orifices and constrictions in pipes by this method (cf. p. 247).

4. *Electrical Method (Flanders*[3]*)*. This is an impedance meter in which the known impedance Z_0 is a tube one-eighth of a wavelength long, which is joined, in turn with the unknown impedance, to a tube of fixed length, the R.M.S. pressure at the junc-

[1] *Phys. Zeits.*, **35**, 408, 1934. [2] *Phys. Soc. Proc.*, **46**, 772, 1934.
[3] *Bell Syst. tech. J.*, **11**, 402, 1932.

tion being read on a condenser microphone. The apparatus (Fig. 90) has the advantage that the ultimate readings are made on a potentiometer. Sound from the loud-speaker L passes along the fixed tube to the junction J, where a side branch leads to the microphone. The electric amplitude of transmitter and receiver after amplification are compared on the potentiometer, consisting of a variable resistance and a variable mutual inductance in series. The voltage across the output amplifier is balanced by a null method against that across the resistance and secondary of the inductance. By analogy, the corresponding electric case comprises an electromotive force E in series with an impedance T up to the junction (more precisely, "the impedance at the junction looking towards the same when on open circuit," the latter *proviso* implying a rigid stop at J in the acoustic case) and an attached impedance beyond J. The pressure e at J is, like the "potential difference between the terminals" at J and the other end of the cell, equal to $\frac{E}{T+Z}.Z$.

We first attach the "known" impedance Z_0, next a rigid stop, and finally the "unknown" impedance Z and have :—

$$Z = Z_0, \; e_1 = \frac{EZ_0}{T + Z_0}$$

$$Z = \infty, \; e_2 = E$$

$$Z = Z, \;\; e_3 = \frac{EZ}{T + Z}$$

(The method will recall to the mind of the reader the well-known experiment of the elementary physics course : "to find the internal resistance of a cell.")

Eliminating E and T, we have :—

$$Z/Z_0 = (e_2/e_1 - 1)/(e_2/e_3 - 1)$$

The pressures e are proportional to the respective currents through the resistance and primary. The drop in voltage across the resistance and secondary is equal and opposite in phase to E, when no current passes through the headphones. If then z is the electrical impedance value of the resistance and inductance (which can be measured by the usual electrical methods), ez is constant, and the equation becomes :—

$$\frac{Z}{Z_0} = \frac{z_1/z_2 - 1}{z_3/z_2 - 1} = \frac{z_1 - z_2}{z_3 - z_2}$$

5. *Bridge Methods* (*Robinson*[1]).—Robinson constructed an acoustic impedance bridge, by analogy with the apparatus familiar in A.C. electrical work. To take the place of the ratio arms of a Wheatstone Bridge, reactances were introduced consisting of wide tubes of length l and area S, for which the impedance was taken to be $-\frac{i\rho c}{S} \cdot \cot 2\pi c/\lambda$, ($\rho$ being density of air, c velocity, and λ wave-length of sound used). The source of sound was a loud-speaker (outside the room) connected to one corner of the diamond network, the opposite corner being a dead-end or earthed into the atmosphere.

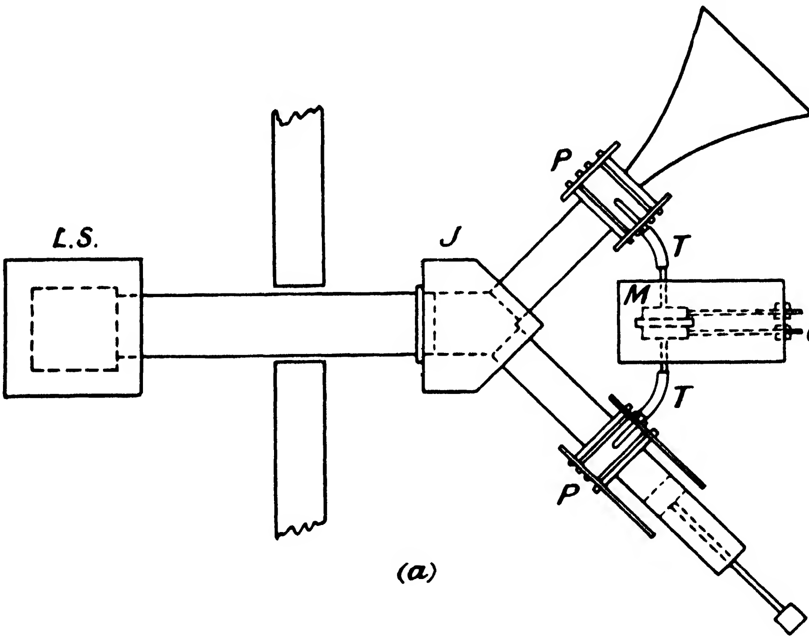


FIG 91.—Acoustic Bridge (Robinson).

The arrangement is shown in Fig. 91. The loud-speaker feeds to a junction box J, then follow the ratio arms, and beyond the connecting pieces P, P, the unknown impedance (shown as a horn) and the known impedance (a tube of adjustable length). Rubber tubes T, T lead to the galvanometer, a microphone M, with compensating tubes and pistons C, C permitting an adjustment of the acoustic impedance on either side of the diaphragm. The apparatus was first used to measure standard forms of impedances such as the Helmholtz resonator. Difficulty in deciding what end correction to ascribe to an orifice which does not debouch into the open air, but into a co-axial pipe of different section, led eventually to the use of a second method to determine such corrections. Even

[1] *Phys. Soc. Proc.*, **46**, 772, 1934; *Phil. Mag.*, **23**, 665, 1937.

when the orifices were used alone, without the reservoir, higher values for the conductivity were obtained than the theoretical value, which for a circular hole in a thin plate is equal to the diameter.

The second apparatus is shown in Fig. 92. It is designed for the measurement of the " conductivities " of orifices. The loud-speaker source is at one end of a tube, divided by the listening-tube at C into two parts, l_1 to the left, l_2 to the right. The terminating impedance on the right (Z_2) is either the orifice or a rigid stop. With the stop *in situ* so that $l_2 = m\lambda/4$, where m is an odd integer, the impedance to the right is zero, and the sound in the microphone a minimum. This sound may be further reduced to zero by the adjustment of the impedance to the left, which is effected by altering the position of the piston in the side tube B. The unknown impedance is then introduced in place of A and l_2 adjusted to restore silence. The effective impedance to the right is again zero, and the value of the unknown impedance may be calculated by equating

$$Z_1 = \frac{i \cos kl_2 . Z_2 - \frac{\rho c}{S} \sin kl_2}{\frac{S}{\rho c} \sin kl_2 . Z_2 - i \cos kl_2}$$

to zero, from which $Z_2 = \frac{i\rho c}{S} \tan kl_2$. For the unknown impedance Robinson substituted circular orifices in metal plates, either forming constrictions in a stopped tube or debouching directly on the atmosphere.

It was found that the conductivity of a hole of diameter d in a tube of diameter D could be represented by

$$C = \frac{0{\cdot}787d}{(1 - d/D)^{1{\cdot}895}}$$

if it formed a constriction in a tube and $C = d(1 + d/D)^{1{\cdot}19}$ if it were a terminal. A formula of the latter type was first suggested by Bate [1] on the basis of experiments on the natural frequencies of pipes terminated by circular orifices of various diameters.

The conductivity at the junction of two tubes of different diameter d and D could be represented by approximately twice

[1] *Phys. Soc. Proc.*, 48, 100, 1936.

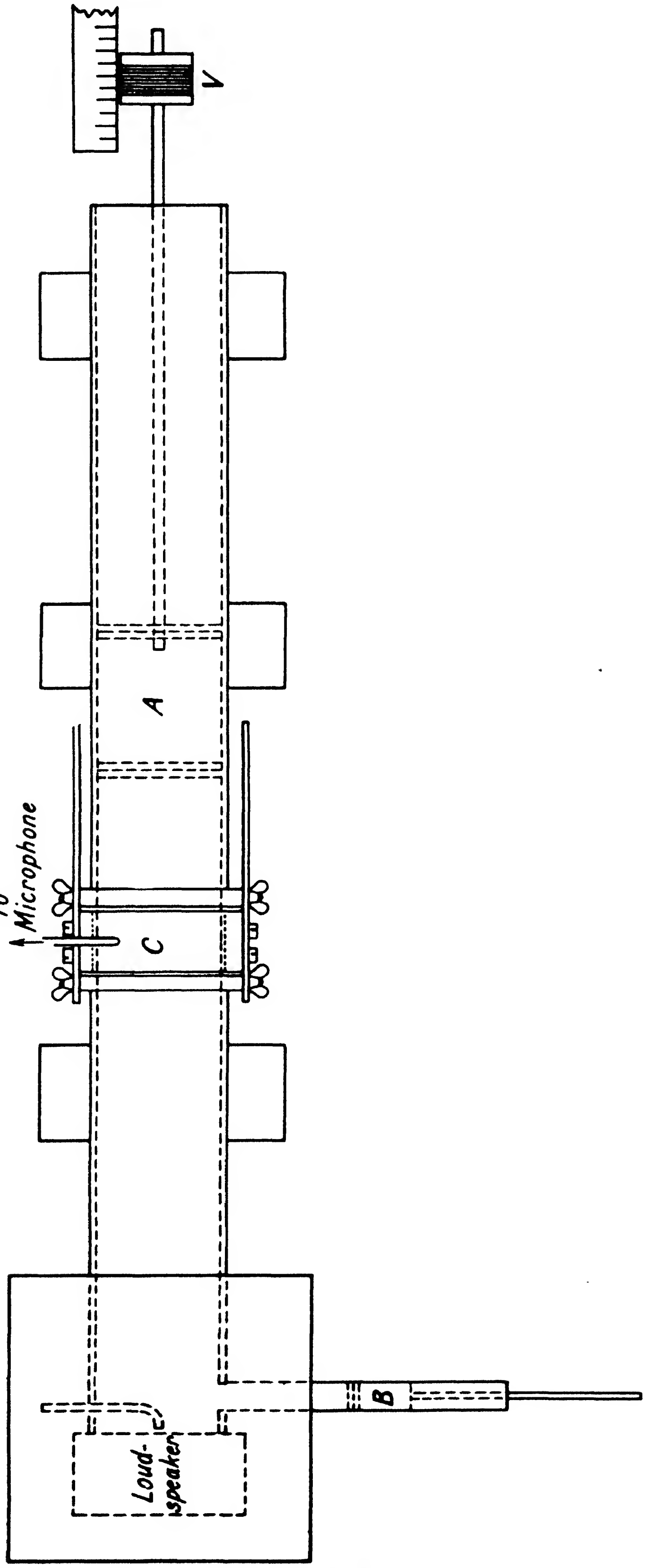


FIG. 92.—Acoustic Bridge (Robinson).

the conductivity of a thin orifice of the same diameter as the smaller tube, or more accurately by $C = \frac{2 \cdot 0d}{(1 - d/D)^{1 \cdot 45}}$. These relations were verified with the first impedance bridge.

Finally, a simplified form of the bridge was built, consisting of a single tube 10 in. long, with a single telephone source on a branch at the centre. Two pressure tubes were brought to the axis of the main tube, one either side of the T junction, 1 in. from the ends of the main tube. The differential microphone was then attached to the two pressure tubes and unknown and known impedances joined to the ends of the main tube. The bridge has proved to be convenient and accurate for determining acoustical reactances, and work on resistances of the slot type is now in progress.

CHAPTER ELEVEN

SUPERSONICS

Piezo-electric Source of High Frequency. It was discovered by Haüy that a mechanical pressure exerted on certain crystals could give rise to a difference of electric potential in a perpendicular direction capable of giving rise to a current in a circuit connected to the faces of the crystal. A tension on the crystal reverses the direction of the current. The converse of this "piezo-electric effect" has been demonstrated, in particular if an alternating current be applied, rapid alternations of compression and distension occur in the two perpendicular directions. These forced vibrations will usually be insignificant unless their frequency coincides with one or other of the natural frequencies for longitudinal waves in the crystal, applying the word "longitudinal" to compressional waves in either of the two directions in the crystal, which are at right angles to the current.

The fundamental frequency for longitudinal waves of such a vibration is given by formula (36). When the crystal is of quartz, the velocity of such waves is found to be $5{\cdot}5 \times 10^5$ cm./sec., so that (36) becomes

$$n = \frac{1}{2l} \times 5{\cdot}5 \times 10^5,$$

that is, when each face is free to vibrate, and the centre of the crystal is a node. Thus, by cutting from a quartz crystal a section parallel to the optic axis, about 3 cm. deep, and mounting it between metal contacts or electrodes so that the faces are free to move, Cady [1] constructed a source of sound whose fundamental frequency was of the order of 100,000, having, of course, a series of overtones of even higher frequencies.

In use, this source is placed in the tuning circuit of a triode

[1] *Opt. Soc. Journ.*, **10**, 475, 1925. *See also* Glebe and Scheibe, *Zeits. f. Phys.*, **33**, 335, 1925; v. Laue, *Zeits. f. Phys.*, **34**, 347, 1925; Dye, *Phys. Soc. Proc.*, **38**, 399, 1926; Meissner, *Zeits. tech. Phys.*, **7**, 585, 1926.

valve, in fact, it takes the place of the inductive circuit of Fig. 49. Pierce himself used the circuit shown in Fig. 94, in which the quartz is self-excited and placed directly in the anode circuit of the valve. To get more power in the vibrations of the quartz, the resonant circuit shown in Fig. 93 may be used, employing two valves in "push-pull." The A.C. mains supply the two filaments through a transformer, the quartz is connected across the two grids, and the resonant circuit consists of the inductance L and the variable condenser C which lies across the anodes. To the mid-point of L is connected the high-tension battery through the milliammeters A_1 and A_2. It is convenient for the methods of measurement in fluids, described in the next section, to have the normal reading of A_2 reduced to zero so that only changes in the

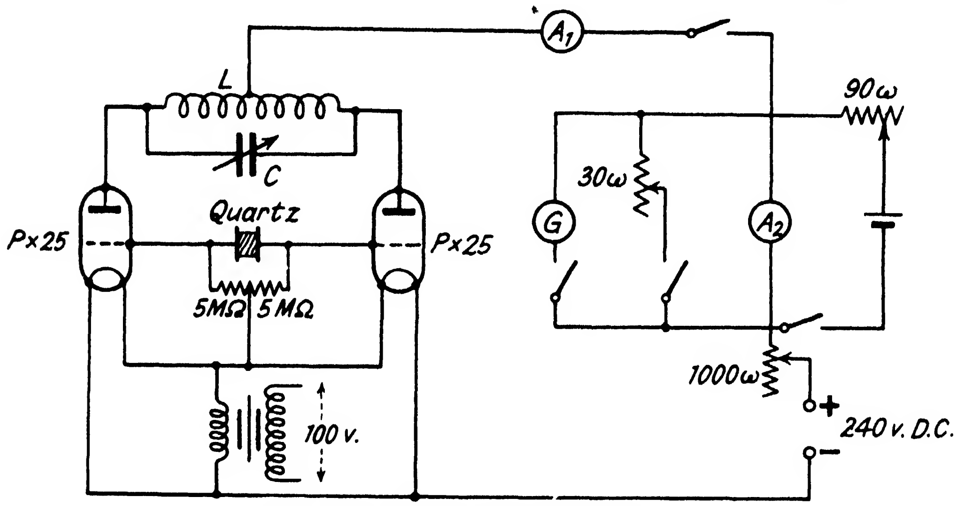


FIG. 93.—Supersonic Oscillation.

amplitude of the oscillator are recorded. This is secured by a "backing battery" working through the 90 ohms resistance. By operating the switches the galvanometer G and its shunt may take over the duty of A_2, and give greater sensitivity to the measurements.

For laboratory or lecture demonstration purposes a slab of 100,000 cycles per second or a disc of 200,000 cycles per second is recommended. These can be obtained, in England, from the Quartz Crystal Co., New Malden. Osram PX 25 valves will be found effective, or American valves of similar characteristics. A coil of 200 turns wound on a former of 6 in. diameter and a variable condenser (up to 0·01 microfarad) will cover the range of frequency required. To exhibit the effects described in the final paragraph of this chapter, 250-watt valves for which the usual mains supply must be transformed up to 2,000 or 4,000 volts, and an oil-filled condenser will be a necessity.[1]

[1] *See also* Bosch and Allée, *Amer. Physics Teacher*, 6, 272, 1938.

Since piezo-electric oscillators are used as standards of high frequency, it is important to be able to locate their natural frequencies. If an acoustical method is used, the upper electrode is made as small as possible and central, the plate being sprinkled with lycopodium powder and the circuit varied until resonance and the consequent Chladni figures appear. Sometimes luminous appearances may be seen along the nodal lines. The frequency of the mode is then calculated from the known elasticity and density of quartz at the temperature in question. It is preferable, however, to find the natural frequency of the circuit in which the crystal is oscillating. For this purpose, a circuit containing a known inductance L and a variable condenser is brought near the maintaining circuit and the capacity C of the latter varied until the two circuits are in tune. This is indicated by an alternating current galvanometer in the subsidiary circuit; then $n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ (cf. p. 228).

The quartz resonator under proper temperature control at once became a reliable standard of frequency and was adopted as such in stabilizing broadcasting frequencies and chronometers, competing in these spheres with the valve-maintained tuning fork. In stabilizing frequencies of audible pitch it is usual to exercise the control by synchronizing the supersonic oscillator with one of the high harmonics of the low-frequency vibrator. Dye [1] has given a very thorough examination, both theoretical and experimental, of the quartz resonator and its circuit, and has examined the modes of vibration of the crystal itself in a Michelson (light) interferometer.

Propagation of Supersonics in Gases. If a reflector (R, Fig. 94) e.g., a board, is placed at a distance opposite one face of the crystal, stationary waves will be set up in the space between if this is a multiple of half the wave-length. This condition will in fact be indicated by the response of the milliammeter (A) in the circuit, whose function is to indicate that portion of the valve current which is *direct*, for it is not able to follow the high-frequency oscillations. At positions such that the waves return in phase with those being sent out, the oscillations of the crystal will be encouraged and a minimum reading of A will be shown; if the contrary is the case, A will indicate a maximum

[1] *Roy. Soc. Proc.*, **138**, 1, 1932; *see also* Osterberg, *Phys. Rev.*, **43**, 819, 1933; *Rev. Sci. Inst.*, **5**, 183, 1934; Straubel, *Phys. Zeits.*, **35**, 179, 1934.

since the alternating current is least. This apparatus, designed by Pierce,[1] accordingly serves as an interferometer for high-frequency waves, a series of maxima and minima being shown on A as the board is moved out, occurring at distances $\lambda/4$ apart. The radiation may be detected by its pressure on the reflector, particularly if the latter consists of a light disc [2] at one end of a suspended lever (cf. p. 224). Under certain conditions, the amplitude of the peaks may be used to measure the absorption in the medium, but a better way to do this is to follow Pielemeier [3] and use a quartz of identical frequency to the sender in place of the

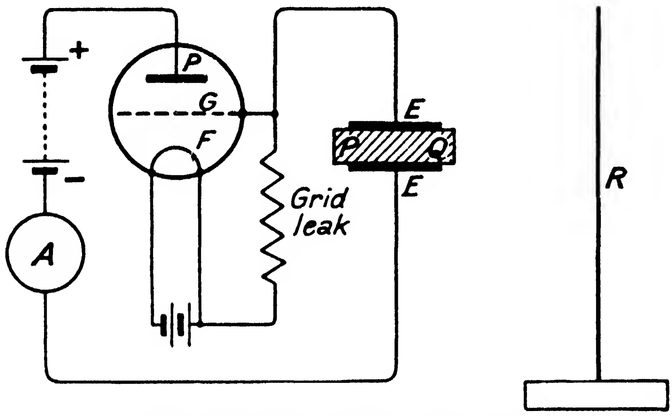


FIG. 94.—Piezo-Electric Quartz Resonator (*Pierce*).

reflector and measure the reaction upon an oscillating circuit connected to the receiving quartz, as the separation between the two is varied.

Most of the detectors described in Chapters VII and X may be used with equal facility in the supersonic region provided their bulk is small; torsion discs,[2] resistance thermometers [4] and hot

[1] *Amer. Acad. Sci. Proc.*, **60**, 271, 1925; *see also* Griffiths, *Phys. Soc. Proc.*, **39**, 300, 1927; Vautier, *Comptes Rendus*, **189**, 1253, 1929; Reid, *Phys. Rev.*, **35**, 814, 1930; Thompson, *ibid.*, **36**, 77, 1930; Klein and Hershberger, *ibid.*, **37**, 760, 1931; Hubbard, *ibid.*, **38**, 1,011, 1931, and **41**, 523, 1932; Hershberger, *Acoust. Soc. J.*, **3**, 263, 1931, and **4**, 273, 1933; Kao, *Comptes Rendus*, **193**, 21, 1931; Grabau, *Acoust. Soc. J.*, **5**, 1, 1933.

[2] Abello, *Nat. Acad. Sci. Proc.*, **13**, 699, 1927; *Phys. Rev.*, **31**, 1083, 1928; Bourgin, *ibid.*, **34**, 521, 1929; **42**, 720, 1932, and *Phil. Mag.*, **7**, 821, 1929; Richards, *Nat. Acad. Sci. Proc.*, **15**, 310, 1929, and **17**, 611, 1931; Paolini, *Alta. Freq.*, **1**, 857, 1932.

[3] *Phys. Rev.*, **34**, 1184, 1929; **36**, 1,005, 1930; **38**, 1,236, 1931; **41**, 833, 1932; *see also* Groszmann, *Ann. d. Physik*, **13**, 681, 1932; *Phys. Zeits.*, **35**, 83, 1934; Eucken and Becker, *Zeits. f. phys. Chem.*, **20**, 467, 1933; Barnes, *Acoust. Soc. J.*, **3**, 579, 1932. [4] Malov, *Hochfreq.*, **42**, 115, 1933.

wires,[1] are particularly suitable. Brandt and Freund[2] showed that dust figures could be set up in the stationary wave-system between quartz and reflector, exhibiting the Kundt's tube phenomena on a small scale. In view of the small wave-length of the radiation relative to the aperture (formed by the electrodes) through which the energy is usually radiated, the confining tube of the low-frequency experiments is superfluous as the waves travel ahead in a narrow beam, as in the analogous system of a beam of light shining through a hole. In fact, many of the features of light propagation, such as diffraction, interference, passage through gratings, etc., may be illustrated by the behaviour of radiation from a quartz oscillator of sufficiently high frequency.

To obtain simultaneous values of both the absorption coefficient and velocity in a fluid medium it is best to keep the source and reflector in fixed relative positions, in order not to disturb the oscillations of the former, and to traverse a hot-wire detector through the intervening space. In this way the stationary wave-system will be traced out.

The absorption is measured in terms of an absorption coefficent a defined by $I=I_0e^{-ax}$, where I_0 is the initial amplitude (at the quartz face) and I that remaining after a distance x has been traversed. This will affect the stationary waves, and the equations will need modification for this decay of amplitude. Let us suppose the progressive and retrogressive waves of a dispersive gas are given by the expression

$$y = Be^{-\alpha x}e^{i(\omega t - \beta x)} + Ce^{\alpha x}e^{i(\omega t + \beta x)}$$

Since $y = O$ at $x = l$, viz., at the reflector,

$$O = Be^{-(\alpha + i\beta)l} + Ce^{(\alpha + i\beta)l}$$

Therefore $y = Ce^{(\alpha + i\beta)l}\left[e^{(\alpha + i\beta)(x - l)} - e^{-(\alpha + i\beta)(x - l)}\right]e^{i\omega t}$

$$= 2Ce^{(\alpha + i\beta)l} \sinh \{(\alpha + i\beta)(x - l)\} e^{i\omega t}.$$

Whence (putting $A = Ce^{\alpha l}$)

$$y^2 = 2A^2[\cosh \{2\alpha(x - l)\} - \cos \{2\beta(x - l)\}].$$

As α is usually small compared to β, the maximum and minimum values of y as x varies are given by $2A \cosh \{\alpha(x - l)\}$ and $2A \sinh \{\alpha(x - l)\}$, so that by tracing out the peaks and troughs in the pseudo-stationary waves, both α and β can be determined.

[1] Bücks and Müller, *Zeits. f. Phys.*, **84**, 75, 1933; Richardson, *Roy. Soc. Proc.*, **146**, 56, 1934.

[2] *Zeits. f. Phys.*, **92**, 385, 1934; **94**, 348, 1934. *See also* Pearson, *Phys. Soc. Proc.*, **47**, 136, 1935; Parker, *ibid.*, **49**, 95, 1937.

Measurements in dry air have shown but slight differences between the velocity of propagation at supersonic frequencies and that normal to the audible range, except close to the source, when the enhanced velocity is apparently due to an amplitude effect. Hitchcock[1] found that he could vary the velocity in this region by altering the manner of excitation of the crystal. At very high frequencies of the order of 200,000 cycles/sec., anomalies are found in certain gases of which carbon dioxide is the most noteworthy. Pierce himself had noticed that his interferometer

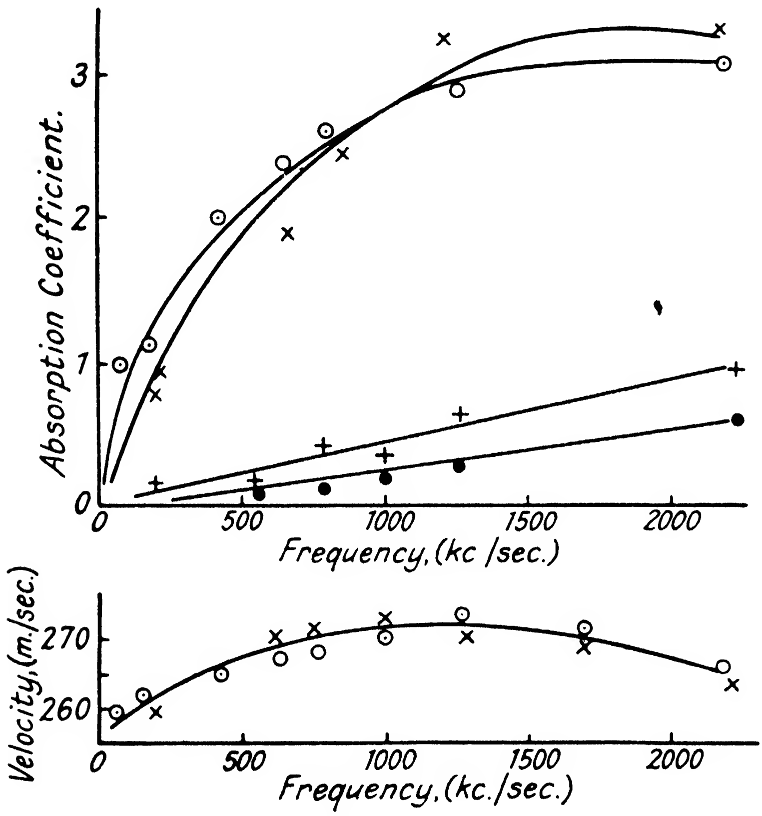


FIG. 95.—Supersonic Dispersion in Gases (● O_2, + SO_2, × N_2O, ⊙ CO_2).

gave diminishing response as the reflector was moved back in this gas. The experiments have been repeated by a number of investigators. Change of velocity with frequency, i.e., dispersion, is shown by these results (see Fig. 95) as well as absorption.

The principal gases in which this excessive absoprtion is observed are carbon dioxide—at 10^5 cycles/sec. it amounts to 40 times that which the classical theory, based on dissipation due to viscosity and heat conduction, would indicate—nitrous oxide and sulphur dioxide; but dispersion of the velocity is also ascribed to carbon

[1] *Inst. Rad. Eng. Proc.*, 15, 906, 1927.

monoxide, on which very careful measurements at two frequencies have been made by Sherratt and Griffiths[1] and to ammonia by Steil.[2]

The dispersion curve—variation of velocity with frequency—shows often a slight dip below the normal value followed by a more considerable rise and, occasionally, another slight dip as the frequency goes to still higher values. It does not, however, return to normal, at least, within the range of frequency attainable. It

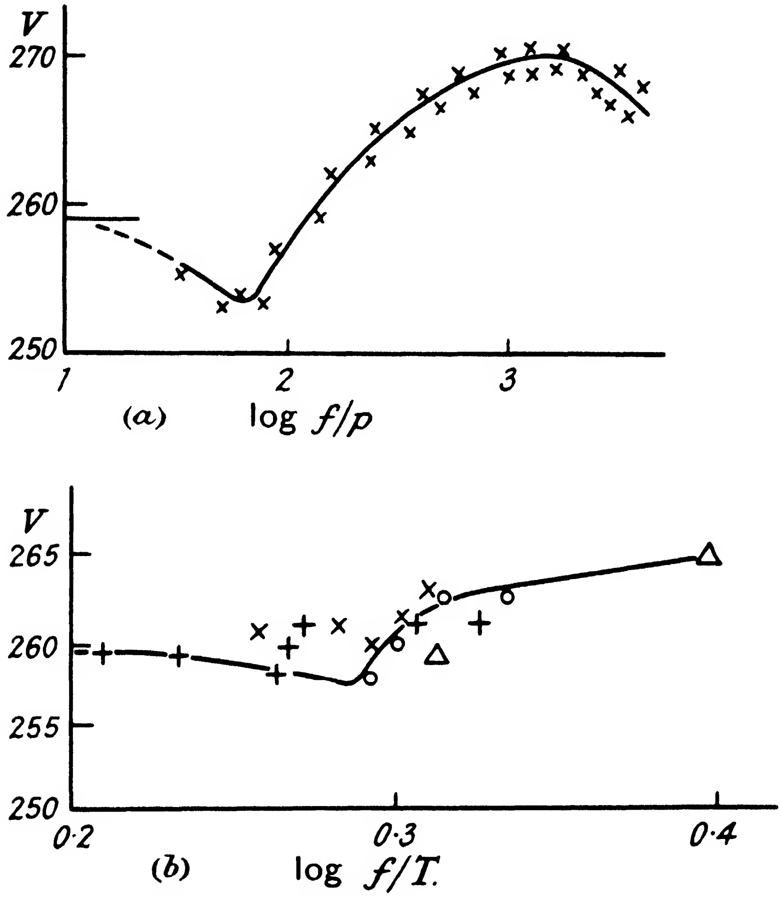


FIG. 96.—Effect of (*a*) Pressure and (*b*) Temperature on Supersonic Dispersion.

must be borne in mind that the accuracy with which the wavelengths can be measured decreases as they get smaller. It is important to know how pressure and temperature affect the dispersion curve. Richards and Reid[3] first obtained some data for the three first-mentioned gases, and more comprehensive data, enabling the complete curves to be plotted, were obtained by

[1] *Roy. Soc. Proc.*, **147**, 292, 1934.
[2] *Zeits. phys. Chem.*, **31**, 343, 1936.
[3] *J. chem. Phys.*, **2**, 193, 1934.

Railston and Richardson [1] for the pressure effect and by Penman [2] and by Warner [3] for the temperature effect. If the velocity (reduced to 0° C.) is plotted against frequency divided by pressure (Fig. 96 *a*) the results for carbon dioxide fall nicely on a single curve. Plotting the other results against the quotient of frequency and temperature is not so successful (Fig. 96 *b*), though the general trend of the data can be outlined by a single curve; moreover, the crux of the velocity rise occurs at a value which increases in linear fashion. (It should be reiterated that this change of velocity has nothing to do with the change of density of the gas, which has been allowed for by the reduction of the values to 0° C. in accordance with the usual formula (6)).

Other instances of excessive absorption occur when a gas, which in the pure state behaves normally, has a small admixture of another gas or vapour. The most notable case is that of air containing water vapour. Knudsen [4] noticed that, even at audible frequencies, moist air absorbed sound to a much greater extent than dry air. It has since been observed in the supersonic region, while Pielemeier [5] and Mokhtar [6] have found dispersion of the velocity in moist air. There is usually for each gas or mixture of gases one value of the humidity for which the radiation suffers its maximum of absorption and of velocity.

Theories of Absorption and Dispersion. This phenomenon has been explained by Herzfeld and Rice [7] in terms of a delay in the change of translational energy into vibrational energy of the molecules as the wave is propagated. When the time period of vibration becomes comparable with this "relaxation time" or "mean life of a sound quantum" the molecules become stiffer to the vibration (to use the expressive phrase of Hubbard) with a consequent absorption of energy and rise in γ, the ratio of specific heats. Probably the simplest mathematical treatment is that of Henry [8] as follows :—

[1] *Proc. Phys. Soc.*, **47**, 533, 1935. [2] *Phys. Soc. Proc.*, **47**, 543, 1935.

[3] *Acoust. Soc. J.*, **9**, 30, 1937.

[4] *Acoust. Soc. J.*, **3**, 126, 1931, and **5**, 112, 1933; Kneser, *ibid.*, p. 122; Rogers, *Phys. Rev.*, **45**, 208, 1934.

[5] *Acoust. Soc. J.*, **10**, 87, and 313, 1939 [6] *Thesis, Durham*, 1939.

[7] *Phys. Rev.*, **31**, 691, 1928; *see also* Kneser, *Ann. d. Physik*, **11**, 761 and 777, 1931; **16**, 337, 1933; and Zuhlke, *Zeits. f. Phys.*, **77**, 649, 1932; Heil, *ibid.*, **74**, 31, 1932; Rutgers, *Ann. d. Physik*, **16**, 350, 1933; Bourgin, *Acoust. Soc. J.*, **4**, 108, 1932; **5**, 57, 1933; Luck, *Phys. Rev.*, **40**, 440, 1932; Richards and Reid, *several papers in J. Chem. Phys.*, 1933–4; Teeter, *ibid.*, **1**, 251, 1933; Rose, *ibid.*, **2**, 260, 1934.

[8] *Camb. Phil. Soc. Proc.*, **28**, 249, 1932.

Let E_x be the actual energy and E_T the energy which the molecules would have if at equilibrium at temp. T, and suppose that

$$\frac{dE_x}{dt} = \frac{1}{\beta}(E_T - E_x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

where β is the period of relaxation of the vibrational energy. If the gas is subject to adiabatic variations of temperature of frequency $\omega/2\pi$

$$T = T_0 + T_1 e^{i\omega t}$$
$$E_T = E_0 + C_\omega T_1 e^{i\omega t}$$

C_ω being the specific heat of vibrations. Substituting in (86) and solving, we find :—

$$E_x = E_0 + C_\omega T_1 (1 + i\omega\beta)^{-1} e^{i\omega t}$$

If the total specific heat be regarded as made up of two parts, C_ω due to vibration and C_1 due to translation (+ rotation)

$$C' = C_1 + C_\omega (1 + i\omega\beta)^{-1}$$

where C' is complex ; also, as usual, $\gamma' = 1 + R/C'$, (R = gas constant), and the velocity of plane waves V is $\sqrt{\gamma' p/\rho}$.

Hence $\qquad \gamma' = 1 + R[C_1 + C_\omega/(1 + i\omega\beta)]^{-1}$

and is also complex.

To find the real part we write :—

$$\frac{1}{C_1 + C_\omega/(1 + i\omega\beta)} = \frac{1 + i\omega\beta}{C_\omega + C_1 + i\omega\beta C_1}$$

of which the real part is

$$\frac{C_0 + \omega^2\beta^2 C_1}{C_0^2 + \omega^2\beta^2 C_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

where $C_0 = C_\omega + C_1$, i.e., the total specific heat when $\omega \longrightarrow 0$. Thus the usual specific heat at constant volume in the expression for the velocity of sound is replaced by the reciprocal of (87). When ω approaches the value $C_0/(C_1\beta)$ a rise in velocity occurs from the normal ; $V_0^2 = (1 + R/C_0).p/\rho$ to an ultimate value $V_\infty^2 = (1 + R/C_1)p/\rho$.

There is also absorption in the critical region due to a change of relative phase between pressure and condensation in the wave. From observations such as those of Fig. 91 it is then possible to predict the relaxation time β. Thus for carbon dioxide at N.T.P. $\beta = 10^{-5}$ sec., for nitrous oxide $\beta = 10^{-6}$ sec.

It will be noted that the theory as set out demands that the velocity should remain at its low-frequency level until the critical frequency is approached, when it should rise fairly steeply to its ultimate level and stay there.

Railston[1] has tested organic vapours. The work is difficult, for condensation tends to take place on the quartz and impede its oscillations. Though a number show abnormal absorption, dispersion of

[1] *Acoust. Soc. J.*, **11**, 107, 1939.

velocity has only been confirmed in carbon disulphide and benzene vapours.

Anomalous Dispersion.—Another type of dispersion is found when any type of radiation encounters a system which contains a set of resonators tuned to a common frequency, or—what amounts to nearly the same thing in acoustics—a set of obstacles spaced at wave-length distances. Such a phenomenon is accompanied by scattering of the radiation, even when the obstacles are much smaller than the wave-length of the radiation as when supersonics pass through a cloud of dust particles, and this scattering of the radiation causes a diminution of the intensity which may be likened to absorption. When, in addition, the size of the obstacles approaches the wave-length of the radiation, there is a characteristic see-saw in the velocity which is more

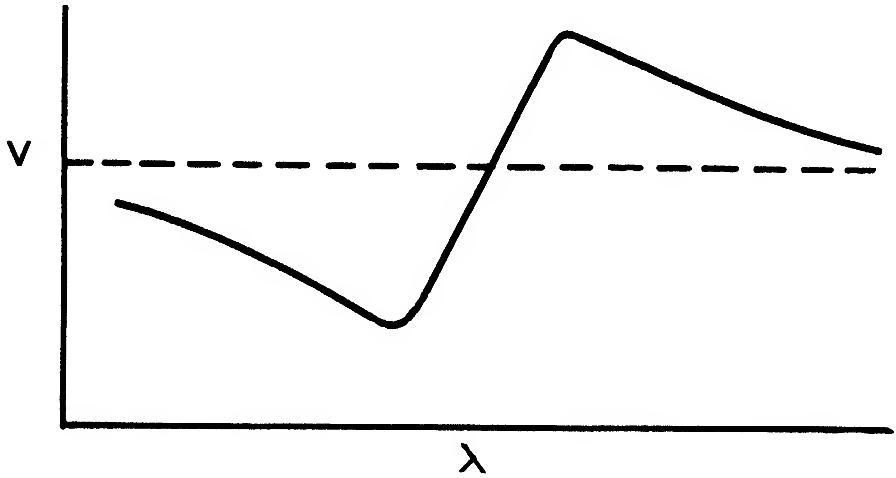


FIG. 97.—Anomalous Distension of Sound.

familiar in light than in sound. This anomalous dispersion was first shown for sound by Belikov.[1] With supersonics it can be demonstrated by passing a beam of the radiation from sources of various but near-by frequencies through a lattice made up of equidistant and parallel wire nails. The effect on the velocity as the wave-length passes through a value equal to the spacing of the lattice is shown in Fig. 97. It will be noticed that it bears a marked resemblance to that of Fig. 96 for carbon dioxide. It is also a matter of observation that carbon dioxide scatters the radiation from a quartz oscillator to either side of the direct path when the frequency and pressure are such as to correspond to a point in the dispersion region. This leads one to suspect that a similar process, i.e., selective absorption, is taking place when a gas shows supersonic dispersion, but the further development of this speculation awaits more data both on the theoretical and experimental side.

Viscous Absorption at High Frequencies. It is by no means certain that the classical theory of viscous and thermal absorption

[1] *Zeits. f. Phys.*, **39**, 933, 1926.

of sound, as outlined on p. 168, is applicable at high frequencies, though experiment suggests that it is no further wide of the mark than at lower frequencies. The experiments in question were performed by Norton [1] and by May,[2] employing nickel needles in magneto-strictive oscillation inside narrow tubes as sources for the determination of the velocity and absorption therein. The latter, in particular, used tubes down to 0·6 mm. in diameter. The absorption coefficient was about as much (up to 3 times) greater than the theoretical value as at low frequencies. The velocity diminishes slowly with rise of frequency (cf. eqn. 63*a*, p. 168). If, then, the S-shaped curve of relaxation (*vide supra*) were overlaid by a steady fall due to viscosity, with rising frequency, the net effect might well resemble the observed ones of Figs. 96 and 97. This is put forward as an alternative to the selective resonance theory of the last section.

Propagation of Supersonics in Liquids. These studies were initiated by Boyle [3] and his collaborators in Canada, using the Pierce method and torsion vanes as well as dust figures. Hopwood [4] has exhibited a number of reflection and diffraction effects on a small scale by this method. Naturally, considerably more energy has to be put into the source than in the corresponding case of a gas, and considerable dissipation of energy as heat may occur, but no dispersion such as that described in the last section has yet (1939) been attested. In using the Pierce method for liquids the physical properties of the reflector have to be seriously considered. It was hoped by Boyle that it would be possible to detect an iceberg in sea water by the reflection of supersonic energy sent out beneath the sea from a ship. The reason why this method must fail is an interesting result of the theory we have given on p. 239. Sea water and ice have, in fact, nearly the same characteristic impedance although water and steel are sufficiently different in properties to make it possible to detect the steel hull of another ship, in time of fog, by the use of a submarine supersonic beam. In the table the approximate velocities of sound, densities, characteristic impedances and in the final column the reflection coefficients: water-ice, water-steel, are set out.

[1] *Acoust. Soc. J.*, 7, 16, 1935.

[2] *Phys. Soc. Proc.*, 50, 563, 1938.

[3] *Several papers in Roy. Soc. Can. Trans.*, 1925–32, cf. p. 303; *see also* Hubbard and Loomis, *Phil. Mag.*, 5, 1,177, 1928; Biquard, *Comptes Rendus*, 188, 1,230, 1929; 193, 236, 1931; Swanson, *Rev. Sci. Inst.*, 4, 603, 1933.

[4] *Sci. Inst. J.*, 6, 34, 1929.

	c	ρ	ρc	r
Water	$1{\cdot}4 \times 10^5$	1	$1{\cdot}4 \times 10^5$	—
Ice	1 ,,	0·9	0·9 ,,	0·03
Steel	5 ,,	8	40 ,,	0·93

Another factor which has a bearing on the reflection is the thickness of the reflector. Boyle and Froman [1] have found that the ratio of incident to reflected energy is least when the thickness equals an even multiple, and is greatest when it is an odd

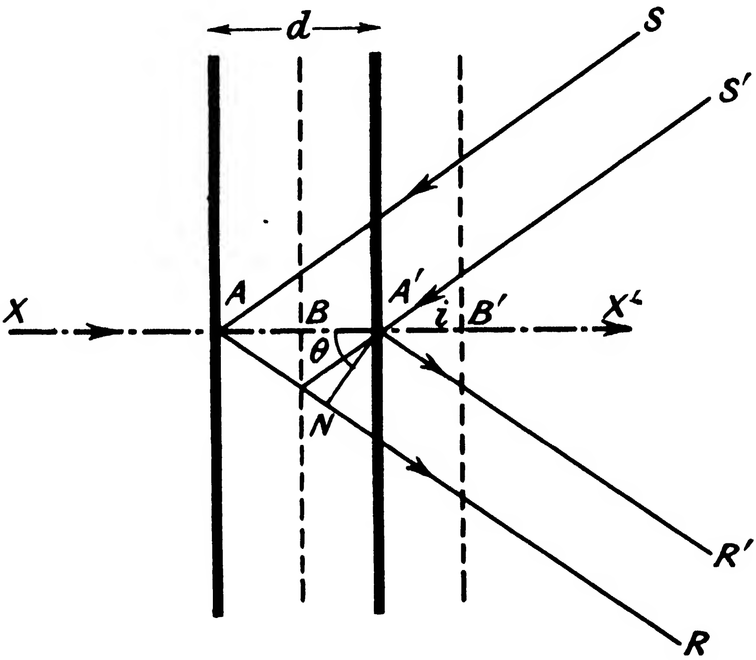


FIG. 98.—Diffraction of Light by Supersonic Waves.

multiple of a quarter wave-length for the supersonic radiation in the material, an experimental fact which is readily confirmed by theory.

About a decade ago Brillouin predicted that if a liquid were penetrated by *progressive* waves of compression of short wave-length at the same time as it was irradiated by light, there would be diffraction of the light by the regular pattern of density variations in the liquid in analogous fashion to the diffraction of X-rays by a crystal in the familiar Bragg experiment. Thus in Fig. 98 suppose that a sinusoidal supersonic wave is moving from left to right in the direction XX' giving at the instant pictured

[1] *Canad. J. Res.*, 1, 405, 1929, and 2, 3, 1930.

maxima of compression distant d apart at AA', with rarefactions at BB', and let light of wave-length λ be incident at an angle i to the direction of propagation and be reflected along paths such as SAR, and let $\theta = \pi/2 - i$ be the angle between the perpendicular $A'N$ from A' on to AR and the direction of propagation. Then the path difference $= 2AN = 2d \sin \theta$ and the expression for equality of phase in the light reflected along AR, $A'R'$ is, as in the Bragg experiment, $j\lambda = 2d \sin \theta$ with the restriction that for a sinusoidal wave $j = \pm 1$, instead of any integral value. The condition for a maximum in the diffracted light is therefore $\sin \theta = \pm \lambda/2l$, l being the wave-length of the sound. If then a beam of light cross the path of the supersonic beam, we should see on the far side the central image, with two diffraction maxima, one on each side. Notice that l must be small to produce a reasonable separation, θ. If $\lambda = 0{\cdot}6\mu$ and $l = 10\mu$, $\theta = 2°$. It should also be noted that the theory is independent of the motion of the layers A, A', as long as they conserve their separation. We are not here dealing with stationary waves. The necessary experiments have recently been undertaken by Debye and Sears [1] on the one hand and Lucas and Biquard [2] on the other, using light passing from a linear slit, through a trough of liquid and out to a photographic plate. The simplest pattern observed is made up of a central undeviated image of the slit from which the light comes and a diffracted line on either side; but others comprise five or more images. In fact, the above theory is too simple to explain all the complexities of the phenomenon, though it suffices for the fundamentals. Raman and Nath [3] have given a more precise theory, but this is of interest to the student of optics and does not concern the applications of the discovery with which we are now dealing.

When stationary waves are formed between a quartz oscillator and a reflector in a liquid medium the compressions which act as diffracting centres to the light are, of course, immobile, and an even simpler apparatus can be set up to show their locations. A parallel beam of light of aperture sufficient to comprise a number of supersonic half-wave-lengths is cast athwart the trough con-

[1] *Nat. Acad. Sci. Proc.*, **18**, 409, 1932; *Comptes Rendus*, **198**, 237, 1934.
[2] *Comptes Rendus*, **194**, 2,132, 1932; **195**, 121, 1932; **196**, 257, 1933; **197**, 309, 1933; *J. de Physique*, **3**, 464, 1932.
[3] *Ind. Assoc. Sci. Proc.*, **2**, 406, 1935; **3**, 75, and 119, 1936.

taining the liquid and made to fall on a viewing screen on the other side, whereon the nodal planes appear as shadows whose separation can be read off with a travelling microscope. The experiment is essentially of the same nature as the Schlieren method of Toepler (p. 17), and has been greatly developed by Hiedemann[1] and his collaborators, one of whom, Seifen,[2] has recently brought the technique to a high state of precision for supersonic velocity measurements. They have also used it on progressive waves, interrupting the (polarised) light from the lamp at the same frequency as the supersonic source by controlling a Kerr cell from some of the electric potential developed in the driving circuit. The Kerr cell interrupts the light at the same frequency, thus acting as an optical stroboscope.

In place of a slit, Bär and Meyer[3] use an opaque screen studded with holes giving a large number of light sources. Their plates have somewhat the appearance of Laue X-ray photographs, the dots being drawn out where the light has passed through the irradiated liquid. Since the intensity of the diffracted light is proportional to the intensity of the supersonics at any point it is possible to estimate the absorption suffered by the latter in the liquid. Bär and Meyer have also verified the wave-length of the supersonics by letting them fall on a wire grating, set in a plane parallel to the wave front. This causes diffraction of the supersonics in the liquid, changing their direction through the angle ϕ. There remains, of course, still the direct radiation. The light from the section of the spotted source which passes through the diffracted beam *alone* then will have each spot drawn out in the same direction, viz., at an angle ϕ to the original beam. Measuring this angle and knowing the spacing of the wire grating they were able to calculate the supersonic wave-length, independently of a knowledge of the light wave-length.

All liquids show a greater absorption than that which theory would indicate (effects of viscosity and heat conduction alone), but some show an enormous absorption (e.g., benzene and chloroform), which recalls the behaviour of carbon dioxide and nitrous oxide gases. It is indeed striking that those substances which behave abnormally to supersonic radiation are just those which

[1] *Zeits. f. Phys.*, **87**, 442, 1934 *et passim*, 1935-8.

[2] *Zeits. f. Phys.*, **108**, 681, 1938.

[3] *Phys. Zeits.*, **34**, 393, 1933; Wyss, *Helv. Phys. Acta*, **7**, 406, 1934; Bergmann, *Phys. Zeits.*, **34**, 76, 1933.

scatter light most readily, which leads one to speculate that there must be some connection between the mechanisms concerned. Possibly the common factor is the tendency to aggregation. Kneser[1] has shown that the attenuation is much greater than possible relaxations could explain.

On the basis of the absorption measurements of Biquard[2]—who, incidentally, has shown that the radiation is "scattered" to a large extent—Lucas[3] envisages a liquid as being a medium always in the throes of casual fluctuations in density, caused by a series of thermal waves which ricochet from one boundary to another. The interactions of the supersonics with such a "speckled" medium—to use a homely phrase—cause a diversion from the straight path and diminution of amplitude at increasing distances from the source. The mechanism propounded is similar to that by which light is scattered from a fog or colloidal suspension.

The question of whether dispersion of the velocity takes place in liquids is still *sub judice.* Certainly, nothing like the rise of velocity which occurs in carbon dioxide has been observed in any liquid. Parthasarathy[4] has examined a large number of organic liquids. Bär[5] has pushed the frequency up to the enormous value of 8×10^7 cycles/sec., but cannot detect dispersion in water, within the limits of experimental error, which mount up at such short wave-lengths (about 0·02 mm.). A change of velocity of the order of 1 per cent. is suspected by other workers in acetic acid, acetone, benzene and toluene.[6] Biquard,[7] too, has shown that the velocity rises in linear fashion as the pressure on the liquid is increased. We must await refinements in the technique of measurement of supersonic wave-lengths before these and similar results can be confirmed.

Miscellaneous Effects of Supersonic Waves. Besides purely acoustical measurements with supersonic waves, other experiments have been performed of a rather spectacular nature. Wood and

[1] *Ann. d. Physik*, **32**, 277, 1938.

[2] *Ann. de. Physique*, **6**, 195, 1936.

[3] *J. de Physique*, **8**, 41, 1937.

[4] *Proc. Ind. Acad. Sci.*, **2**, 497, 1935; **3**, 285, 482, and 519, 1936.

[5] *Helv. Phys. Acta*, **10**, 332, 1937; **11**, 472, 1938.

[6] Spakovsky, *Acad. Sci. U.S.S.R.*, **18**, 169, 1938; Dutta, *Phys. Zeits.*, **39**, 186, 1938; Rao, *Proc. Ind. Acad. Sci.*, **7**, 163, 1938.

[7] *Comptes Rendus*, **206**, 897, 1938.

Loomis [1] fed 2 kilowatts of electric power to an oscillator under oil, which allows of much greater power being employed without breakdown of the insulation. The pressure at the surface of the oil was so great as to cause the oil to rise in water-spout form, emulsifying as it did so. The hydrostatic pressure is sufficient to drive all air out of the solution in the form of bubbles rising from nodal planes. Even if there is no dissolved air, cavitation [2] in the form of bubbles may occur in which the liquid is apparently evaporated by the intensity of the rapidly alternating forces. A catalytic effect on chemical reactions has been noted, reactions taking place at lower temperatures, when irradiated by intense supersonics, than under ordinary conditions; and Szalay [3] has been able to break down certain polymeric organic molecules into less complex ones by irradiation of aqueous solutions by supersonics.

The waves also possess marked biological action in which the immediate cause seems to be the intense energy dissipation in the track of the radiation. Their disruptive actions on a number of living specimens, plants, vaccines, suspensions of bacteria and of blood corpuscles have been examined by Wood and Loomis,[1] Hopwood [4] and others, and in every case their action is to destroy living matter. An industrial application of this process is the sterilization of milk by supersonic energy.

[1] *Phil. Mag.*, **4**, 417, 1927; Richards, *Amer. Chem. Soc. J.*, **49**, 3,086, 1927; **51**, 1,724, 1929; Blancani and Dognon, *Comptes Rendus*, **197**, 1,070, 1933.

[2] Boyle, Taylor, etc., *Roy. Soc. Can. Trans.*, **23**, 91 and 187, 1929.

[3] *Zeits. f. phys. Chem.*, **164**, 234, 1933; *Phys. Zeits.*, **35**, 293, 1934.

[4] *Sci. Inst. J.*, **6**, 34, 1929.

CHAPTER TWELVE

SUBJECTIVE SOUND

The Voice. The human organs engaged in the production of sound consist of (1) the lungs which by their expansion and contraction provide a blast of air, delivered through the miscalled (2) vocal cords in the larynx at which the vibration originates, (3) the cavities formed by the larynx itself, mouth, nose, and sinuses behind the forehead, which function as resonators to the sound produced in the larynx, and of which two at any rate, the mouth and larynx, are capable of adjustment or tuning at will. This system has often been compared to a reed instrument, in which the lungs form the bellows, the vocal cords the reed in its tube, the head cavities make up the resonant column of air; but the analogy fails in likening the vocal cords to a reed stretched across the larynx.

This organ consists of two flat membranous bands stretched across the larynx, spanned by two muscles which are able to close the slit which forms the air passage, and at the same time to put tension on the bands. The idea that these bands vibrate transversely in the fashion of the cords of a stringed instrument to produce the voice, is as old as Galen, but the range of tension and thickness actually available, even allowing for possible partial vibrations, seems too small to form, in accordance with the expression for the natural tones of strings, proper vibrations covering the two octaves over which the average voice ranges. It seems preferable to regard the sound as being engendered by a sort of jet tone (p. 156), in which however the membranous sides of the slit themselves take part, like the lips of a player on a brass instrument. By means of the laryngoscope invented by the famous singer Manuel Garcia [1]—a small mirror by which the cords can be viewed in the glottis while the patient is singing —it can be demonstrated that not only is the tension varied,

[1] *Phil. Mag.*, **10**, 218, 1855.

but that the width of the slit between them is altered as the frequency of the note or the velocity of expulsion of the breath is changed, agreeing with our formula (55), p. 150. Records of the vibrating cords by stroboscopic methods,[1] or by the painful process (for the singer) of a recording instrument in the glottis,[2] show that these membranes do indeed execute a S.H.M., while the note emitted from the mouth is complex.

The reed-pipe analogy led scientists to ascribe the production of sound, or of the pitch characterizing a sung vowel, to the vocal cords, while to the neighbouring air-cavities, mouth, nose, throat, was ascribed the function of modifying the quality of the note to form the different vowel sounds. Apart from these modifications, there are two varieties of larynx tone production. In the lower part of a singer's range of pitch the "chest voice" is employed. The laryngoscope shows that the cords form a long fine slit by their juxtaposition in this range, while the walls of the upper part of the chest seem to be set in forced vibration—hence the name. The upper notes are produced by the "head voice," in which a part of the slit is completely closed by the small tensioning bones, which strain the vocal cords to a greater extent than in the chest voice. Between these two "registers" there is a break in the voice, in disguising which the art of the virtuoso has to be exercised. A rapid alternation between the registers on a note in their limited overlapping region, results in that peculiar sound known as "yodelling." In whispering, the vocal cords do not take part in the vibration—anyone may test this, by holding his "Adam's apple"—the mouth resonator is feebly excited merely by blowing breath into it through the narrow orifice of the throat.

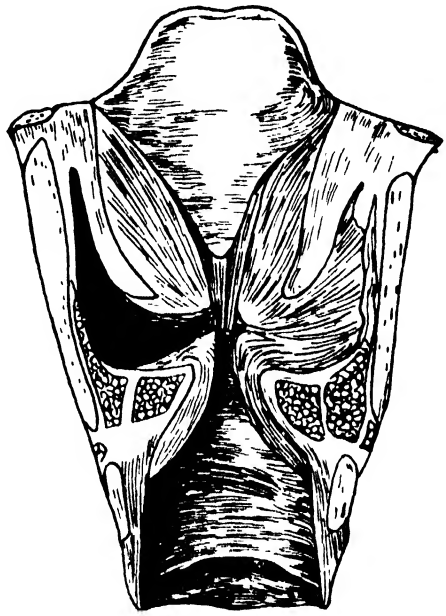


FIG. 99.—The Larynx.

Vowels. Granted that vowels are characterized by modification of the size and shape of the mouth and throat, we have yet to

[1] Musehold, *Arch. f. Laryng.*, 7, 1898.

[2] Katzenstein, *Beit. z. Anat. d. Ohres*, etc., 3, 291, 1910.

consider to what extent the vibration of the air in these cavities is coupled to the larynx-notes which we have just been discussing. The theory, the oldest and best borne out by experiment, ascribes certain fixed simple tones, characteristic of each vowel, to the complex note produced by singing the vowel to any pitch, and goes back to Willis,[1] 1832. This is what one would expect if the tonal modifications have their origin in the mouth which is set to a more or less definite position for each vowel. The alternative idea, that the quality of vowels depends on the *relative* number and intensity of the overtones to each fundamental larynx-note has fallen into disrepute. Helmholtz [2] envisaged the action of the mouth as a resonator amplifying certain partials in a *complex* larynx note, and so characterizing each vowel. This is again a modified fixed-region theory, but invites objections on two counts; firstly it may be asked what happens when the natural frequency is near but not equal to one of the partials in the larynx note, and secondly, no account is taken of the reaction of the resonator on the larynx note. For these reasons Hermann [3] prefers to regard the vocal apparatus as a coupled system, somewhat like the wood-wind instruments, but differing in this respect that not only the length of the vibrating column of air, but also its relative shape can be altered. Into all the arguments for and against these differing views we cannot enter here. Suffice it to say, that scientific opinion supports the view of the vocal cords and the adjacent air cavities as mutually reacting systems, in which however the larynx note is primarily responsible for the notion of the pitch of a sung vowel, the air cavities, on the other hand, for the vowel type which reaches the ears of a listener, a type dependent on partial tones covering a certain fixed region.

[1] *Ann. d. Physik*, **24**, 397, 1832.

[2] *Sensations of Tone*, p. 105.

[3] *Pfluger's Archiv.*, 1897 to 1902, *passim. See also* Waetzmann, *Phys. Zeits.*, **10**, 313, 1909; Fiorentino, *N. Cimento*, **5**, 61, 1913, and **7**, 186, 1914; Paul Sabine, *Phys. Rev.*, **22**, 303, 1923; Crandall and Sacia, *Bell. Tech. Journ.*, **3**, 232, 1924; Lullies, *Pfluger's Archiv.*, **211**, 373, 1926; Trendelenburg, *Wiss. Ver. Siemens-Konzern*, **4**, 1, 1925; Crandall, *Bell Syst. Tech. Journ.*, **4**, 586, 1925; Riegger and Trendelenburg, *Zeit. tech. Phys.*, **7**, 187, 1926; Jung, *Phys. Zeits.*, **27**, 716, 1926; Crandall, *Bell Syst. Tech. J.*, **6**, 100, 1927; Waetzmann, *Naturwiss.*, **15**, 97, 1927; Stefanini, *N. Cimento*, **8**, 213, 1931; Backhaus, *Zeits. f. tech. Phys.*, **13**, 31, 1932; Obata and Tesima, *Jap. J. Phys.*, **8**, 132, and **9**, 510, 1933; Lenk, *Akad. Wien. Ber.*, **142**, 313, 1933; Gemelli and Pastori, *Rev. d'Acoust.*, **2**, 169, 1933; Cotton. *Acoust. Soc. J.*, **5**, 208, 1934; Steinberg, *ibid.*, **6**, 16, 1934.

Scripture[1] has emphasized that a Fourier analysis does not help much to characterize a vowel sound. The fact is, there are so many high overtones present that such an analysis even if it could be expeditiously carried out would not enable one to "see the wood for the trees." It is nevertheless possible from a mere inspection of the trace to recognize certain vowel types. The vowel *ah*, for instance, appears as a reiterated series of rapidly diminishing peaks of the relaxation type (p. 57) in which the vibrations of the air in the mouth cavities are started by the vocal cords, then relax in a series of rapid and damped oscillations, until caught up again by the succeeding impulse from the vocal cords.

Consonants. These may be divided into classes. Some are characterized by a complete temporary stoppage of sound (*B*, hard *C*, *D*). Others involve an incomplete cessation, the remaining sound anticipating the following vowel by a weak note of corresponding quality (*F*, *V*, *L*, *M*, *N*). *R* and *S* show a very small drop in intensity with anticipation of the following vowel (Gianfranceschi).[2]

Analysis of Speech. Phonetics. Almost all the analysers described in the last chapter have been used, in experiments too numerous for detailed mention, for tracing the sounds of speech and song in the neighbouring air. Miller,[3] who has done much work recently on the subject, finds his results to be in agreement with the fixed-region vowel theory, and also finds that the mouth, as a resonator, exhibits multiple resonance. Paget[4] has constructed models of the vocal system by ingenious combinations of resonators, formed of plasticine with an artificial larynx formed by an orifice over which a rubber band is stretched edgewise. These, if blown, produce good imitations of vowels. The tongue divides the mouth into two cavities, the larger of which has reso-

[1] *Phys. Zeits.*, **29**, 911, 1928; *Acoust. Soc. J.*, **5**, 148, 1933; *see also* Curry, *Thesis, Durham*, 1934.

[2] *N. Cimento*, **16**, 161, 1918. *See also Acad. Lincei. Atti.*, **22**, 216, 1913; Waetzmann, *Phys. Zeits.*, **10**, 503, 1909; Paget, *Roy. Soc. Proc.*, **114**, 98, 1927.

[3] *Science of Musical Sounds*, 215, 1922. *See also* Devaux-Charbonnel, *Comptes Rendus*, **149**, 630, 1909; Laudet, *Comptes Rendus*, **144**, 1,258, 1907, and **147**, 1,311, 1908; Trendelenburg, *Zeits. tech. Phys.*, **5**, 236, 1924.

[4] *Roy. Soc. Proc.*, **102**, 752, 1923, and **106**, 150, 1924, and **119**, 157, 1928; *see also* Panconcelli-Calzia, *Ann. d. Physik*, **85**, 483, 1928; Wegel, *Bell Syst. tech. J.*, **9**, 207, 1930; Riesz, *Acoust. Soc. J.*, **1**, 273, 1930.

nances between 300 and 850, and the smaller between 600 and 2,500, depending on the position of the tongue. The resonance pitches of this double resonator lie farthest apart for *I* and nearest for *A* (as in calm). Most vowels involve a throat resonance, in addition.

One respect in which the human vocal cavities differ from those found in a laboratory is that the softer walls induce much greater rates of damping. Cotton[1] has done some experiments on this aspect of vocal acoustics. The scheme involved the obtaining of resonance curves for cylindrical vessels lined with various soft substances and actuated by a telephone oscillator of constant output, adjusted to give the same ground-level for all frequencies, as measured by a condenser microphone. He obtains the resonance curves of the resonator when lined with (*a*) beef steak, (*b*) cotton waste, (*c*) plasticine. The latter approaches the theoretical resonant frequency for a cylinder with rigid walls most closely. The resonances of the soft-walled cavities are perceptibly higher in pitch and have low, broad crests.

The pitch and intensity at which a vowel is uttered naturally affects the wave-form of the resulting sound. This makes it more difficult to recognise a vowel from its wave-form, though with many vowels the change is not great at moderate ranges of pitch. Actually, it is not possible to utter certain vowels at extremes of pitch; singers have to do the best they can by changing the vowel to some closely related one when such is the case. The effect of pitch on wave-form has been recently examined by Riddell[2] and Stout.[3]

The human being finds the correct position of the mouth for different sounds by instinct and imitation. It is the business of the science of phonetics to aid him in this process, especially when he is mastering a foreign language with foreign sounds, by discovering for him the positions and movements of the mouth in speech. These positions are found by probing instruments, when the system is set to produce the given sounds, and records or models are made. The pupil then endeavours to mould his mouth into the same shape.

Marage[4] has made important contributions to the scientific

[1] *Acoust. Soc. J.*, **5**, 208, 1934. [2] *Thesis, Durham*, 1938.

[3] *Acoust. Soc. J.*, **10**, 137, 1938.

[4] *Comptes Rendus*, **144**, 1,175, 1907, and **147**, 921, 1908, and **148**, 110 and 1,118, 1909, and **149**, 936, 1909, and **152**, 1,265, 1911. *See also* Daniel Jones, *Nature*, **99**, 285, 1917, and **100**, 96, 1917.

study of the voice. He observes that the mouth is not kept fixed by a normal person when the same vowel is sung to a number of notes, as the shape of the larynx is altered with pitch, and corresponding adjustments of the mouth have to be made to preserve the same vowel sound. Deaf-mutes—using the term in a relative sense—generally move the mouth only. To this fact Marage ascribes the poor quality of such voices.

Hearing. Ohm's Law. Before introducing the mechanism of the ear and possible explanations of the action of hearing, we shall describe the salient observations, physical and psychological, of the functioning of the human auditory system, considering the ears merely as two receivers of sound located on either side of the head, connected with the brain by a nervous system.

It is a matter of common observation that the ear is able to perform, qualitatively at least, the Fourier analysis of a complex note, meaning that we are able, within limits depending on our natural ability and training, to say what partial constituents are present in the note. Assistance in this direction may be obtained by sounding the expected partial alone, before listening to the note of which it forms a constituent, or by amplifying the partial by a suitable resonator. Ohm [1] enunciated this principle in 1843 in a statement generally known as Ohm's law of acoustics, to distinguish it from his more famous electrical law: "Every simple harmonic motion of the air is perceived by the ear as a simple tone; all others are resolved by the ear into a series of simple tones of different periods." That vague concept which we term the quality of the note, depends then on the number and relative magnitude of the partial simple tones which the ear can find in it. When these are inharmonic and scattered indiscriminately through the audible pitch range, we describe the impression as a noise. As far as weak sounds are concerned, physically and physiologically this theory has stood the test of time fairly well; the instances where it seems to be untrue can be explained in the main as aural illusions, that is to say, that their cause is psychological. Early in this book it was shown that the response of an asymmetric recorder like the ear-drum to intense sounds is non-linear, and may therefore not be a faithful copy of the air vibrations. This is not a refutation of Ohm's law, at least in principle, but may simply imply the intrusion of other simple

[1] *Ann. d. Physik*, 59, 497, 1843.

tones not in the external sound, into the quality of the note as perceived by the ear. An outstanding example of such intrusions are the subjective combination tones.

Phase and Quality. If Ohm's law be true, the question is naturally asked, what influence have the respective phases of the components of the note on the impression of its quality? Helmholtz[1] and also König[2] attacked this problem experimentally. Helmholtz employed a double siren by which he could produce two tones of different pitch to form a complex note of two components. The phase relation between them depended of course on the relative time of opening of the holes in the two discs of the siren; provided these rotate at the same rate the phase difference remains constant. But by a handle one disc could be made to advance slowly upon the other causing a progressive change of relative phase. No difference of quality could be detected. If phase has no influence on quality, synthetic wave forms, made up of the same components but differently spaced as to phase, will produce the same effect on the ear, though the resultant complex waves are of different shape. König constructed a siren having templates cut to such shapes and passing over the holes, and reached the same conclusion. Finally with the development of polyphase alternating currents, Lloyd and Agnew[3] were able to reverse the phase of one component of a vibration imposed on a diaphragm transmitter; again no change in the quality heard.

It is curious that, in listening to an orchestra, no difficulty is experienced in assigning a particular partial tone in the mass of tone colour to the instrument which is evolving it. Localization plays perhaps a part in this, and the difference in duration of different notes helps, but the ear seems to have an uncanny facility in this respect. Possibly because we are used to hearing the solo part at the top of the pitch scale, the ear tends to pick out the topmost part in the musical piece, and characterize this as the "tune" of the piece. On the other hand, the pitch of the note of a single instrument or voice is characterized by that of the fundamental, or lowest tone in the note, this being generally of much greater intensity than the upper partials. Fletcher[4] believes that he has shown by recent experiments that

[1] *Sensations of Tone*, 103, etc.

[2] *Ann. d. Physik*, **12**, 344, 1881, and **57**, 339, 1896.

[3] *Bureau of Standards Bull.*, **6**, 255, 1909; *see also* Beasley, *Acoust. Soc. J.*, **1**, 385, 1930; Chapin and Firestone, *ibid.*, **5**, 173, 1934.

[4] *Phys. Rev.*, **23**, 427, 1924.

the fundamental does not determine what pitch we mentally assign to a note. A diaphragm being excited by an alternating current of complex type, certain partials were removed by suddenly switching-in suitable filters to the electric circuit. The corresponding diaphragm and air tones being presumably removed by this action, a musician who was listening was asked to say whether he thought the "pitch of the note" had altered. Whenever one of the *lower* constituents had been removed the listener made answer that not the pitch, but only the quality of the note had been altered. If this experiment bears a physical or physiological explanation, it is of far-reaching consequence for our ideas of the function of the ear, but it is important to emphasize the control conditions, and to inquire to what extent the effect is psychological. The procedure was to remove the lower tones from the note and ask what *change* in pitch resulted; not to produce an isolated note with the fundamental missing and ask what pitch the note had.

Physiological Intensity. Our instruments which measure intensity of sound, actually measure the energy which falls upon them. The energy of a vibrating particle is proportional to the square of the velocity and therefore, in S.H.M., to n^2a^2 (cf. 10); or, if the frequency of the tone is fixed, to the square of the amplitude. When however we consider the human instrument, the case is altered, for our sensation of the loudness of a tone is not a linear function of the intensity as received by the ear-drum. Experiments to determine on what power or function of the amplitude the sensation of loudness or physiological intensity depends are fraught with difficulty. Early attempts were based on a gratuitous assumption of the inverse square law. For example, a tuning fork would be maintained at a certain amplitude a_1; then removed to a distance, and the original loudness at the stationary observer's ear restored by increasing the amplitude to a_2. Then if the inverse square law holds, and loudness varies as the x'th power of the amplitude: $\dfrac{a_1^x}{r_1^2} = \dfrac{a_2^x}{r_2^2}$ to determine x. The problem has been much discussed and explored by Stefanini,[1] who holds the view that the ear measures the momentum and

[1] *N. Cimento,* **19**, 5, 1920, and **26**, 137, 1923. *See also* Ercolini, *N. Cimento,* **16**, 45, 1908, and **17**, 265, 1909; Michotte, *Arch. Néerl. de Physiol.,* **7**, 579, 1922.

not the energy falling upon it; so that physiological intensity is to be proportional to the amplitude itself.

The "Weber-Fechner law" relating to the stimulation of the senses may be stated, as follows: The increase of stimulus necessary to produce a just perceptible increase of the sensation δE bears a constant ratio to the total stimulus Σ, or :—

$$\delta E = k\frac{\delta \Sigma}{\Sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

or, in the integrated form: $E = k \log_e \Sigma$, the sensation is proportional to the logarithm of the stimulus. This law applies to all our senses. Steinberg [1] has found that k is not truly constant at all frequencies, as it is a function of the minimum stimulus required to produce any sensation of sound at all in the ear, and this varies with pitch, as we shall find shortly. This introduces a complication into the problem as we have not only to determine how the stimulus depends on the amplitude, but also the sensation is not itself a linear function of the stimulus. It is difficult to see how these two factors are to be separated.

Pitch Limits. It is a matter of common observation that when a body is vibrating sufficiently slowly the alternate condensations and rarefactions impressed by it on the air and received by the ear are perceived by it as distinct pulses, but that when these pressure changes take place sufficiently rapidly, the sense of their isolation is lost and they blend into a musical tone. This point represents the lower limit to the ear's power of analysis. This lower pitch limit depends on the individual ear and may be detected by a siren rotated at slow and then faster speeds. It is important to remember that a pulse in the mathematical sense of a compression followed by a single rarefaction is very rare in nature, and even the "pulse" produced by the air when a single hole of the siren is uncovered may involve tones continuing for a short while. In such an experiment these tones of higher pitch must be mentally excluded from perception. The lower pitch limit is about 16 vibrations per second.

Slow vibrations involving considerable movements of the air remain unperceived as tones if their rate of pulsation falls below this limit, though their existence can be demonstrated by manometric flames, etc. Such "infrasonic" waves have been exten-

[1] *Phys. Rev.*, **26**, 507, 1925. *See also* Fletcher and Steinberg, *Phys. Rev.*, **24**, 306, 1924; Fletcher, *Acoust. Soc. J.*, **6**, 59, 1934.

sively studied by Esclangon,[1] for this type of vibration is propagated from the muzzles of guns at the instant of firing. Though no sound in the strict sense of the word is heard, yet there is a sensation of detonation, when the initial compression made by such a wave is sufficiently precipitate. Such a sensation is merely one of pressure on the ear-drum, and its intensity is a function of the abruptness of the discontinuity in the air; the ear is in fact acting as a manometer. Thus large explosions may produce a small sensation if their rate of development is slow. On the other hand, proximity to the bursting of a shell may involve temporary loss of hearing, of memory, and more complicated nervous disorders due to the violent shock of pressure to the auditory system.

There is an upper frequency limit to the ear's powers of tone perception. When the frequency of vibration exceeds this limit, no sound is heard at all; it is as though the sounding body were absent. This limit lies round 20,000 vibrations per second, so that the audible pitch range covers 10 or 11 octaves. The upper limit falls with increasing age. Fundamental tones of this high order of pitch can of course be produced only by very small vibrating bodies; the noise made by the wings of a grasshopper is a well-known example. For laboratory work on this question, a number of sound sources have been employed.

(1) The longitudinal tones of short glass rods, estimated by Kundt's tube (cf. p. 27).

(2) The overtones of small and thin glass plates, clamped at their edges, determined by sprinkled sand.[2]

(3) The Galton[3] whistle.

The last instrument is most favoured by psychologists. It consists essentially of a very short cylindrical pipe (Fig. 100), blown from an annular nozzle (cf. p. 173) of which the "height of the mouth," f, can be varied by turning the lower micrometer screw; so that as nf is constant and n is always one of the partial tones of the little pipe, a series of high-pitched notes is obtainable. The length of the pipe can also be varied in Edelmann's form of the instrument,[4] by twisting the upper screw.

Another instrument of the experimental psychologist, though

[1] *Comptes Rendus*, **168**, 699, 1919.

[2] Schulze, *Ann. d. Physik*, **68**, 99 and 869, 1899, and **24**, 785, 1907.

[3] *Inquiries into Human Faculty*, p. 375, 1883.

[4] *Ann. d. Physik*, **2**, 469, 1900. *See also* Ercolini, *N. Cimento*, **19**, 44, 1910.

not used for such high frequencies, may conveniently be described now. This is the "ton-variator" designed by Stern.[1] It consists of an adjustable resonator excited by blowing across the mouth (Fig. 101). A piston is raised or lowered in the resonator by means of a graduated cam, of such a shape that the angle of rotation is proportional to the change in frequency produced. The instrument must be used at constant blowing pressure.

Minimum and Maximum Audibility. Beside the frequency limits there are intensity limits to the sounds which the ear can perceive. The minimum or threshold audibility of the normal ear has been measured most carefully by Fletcher.[2] A common

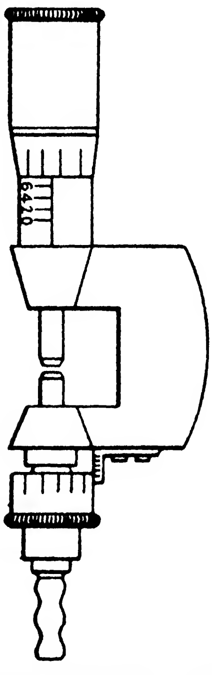


Fig. 100.—Galton Whistle (*Edelmann*).

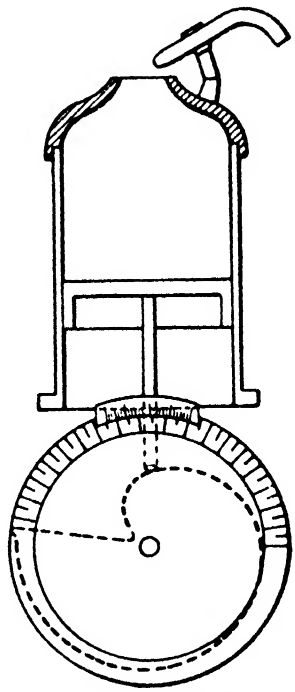


Fig. 101.—Ton-Variator (*Stern*).

method for making the test on an individual ear is to compare the time during which a damped tuning fork is heard by the ear with that of a normal ear. Less intensity for a sound to be audible is required in the middle of the pitch range; more is required at each end.

When a sound stimulus is very intense, the sensation becomes

[1] *Deut. phys. Ges. Verh.*, p. 302, 1904. *See also* Beryl Love and Margaret Dawson, *Phys. Rev.*, **14**, 49, 1919.

[2] *Frank. Inst. Journ.*, **193**, 720, 1922, and **196**, 289, 1923. *See also* Lane, *Phys. Rev.*, **19**, 492, 1922; Swan, *Am. Acad. Proc.*, **58**, 425, 1923; Kranz, *Phys. Rev.*, **21**, 573, 1923, and **22**, 66, 1923; Huizing, *Thesis, Gröningen*, 1932.

painful, and above a certain limit cannot be perceived as sound; there is merely an unpleasant feeling of pressure on the ear-drum. Exposure to such intense sounds causes temporary tinnitus, or "ringing in the head," which in some persons, due to a lesion of the auditory nerves, amounts to a permanent or recurring affection of the auditory system. The two lines forming the upper and lower limits of intensity for tone perception are shown on an audition diagram (Fig. 102, after Wegel [1]), plotted against the frequency. The region enclosed between the two lines represents the auditory region of the ear, both as regards intensity and pitch, and the shaded area represents the region commonly employed in speech.

Andrade and Parker [2] have devised a pipe maintained by a loud-

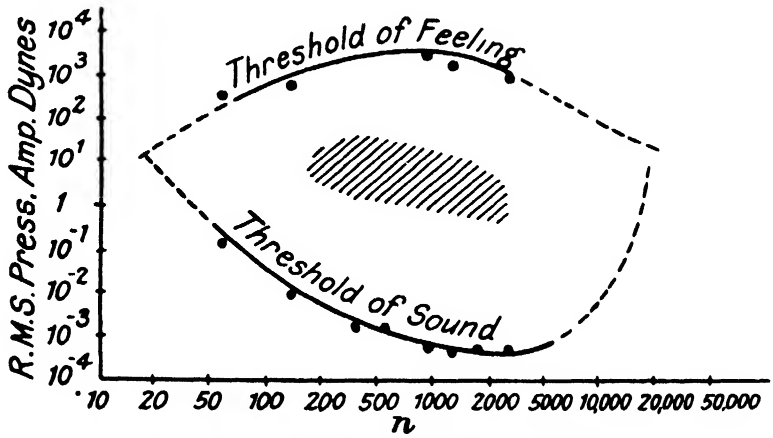


FIG. 102.—Audition Diagram (*Wegel*).

speaker unit at one end, while the other end is open to the atmosphere, as a standard source suitable for making measurements of audibility. The intensity of the source is derived from observations of the amplitude of smoke particles held in suspension in the glass tube forming the pipe (cf. p. 186). The source was mounted on the top of a building facing another across a court, the floor of which was padded to avoid reflections. A listener stationed himself on the roof of the building opposite with his head in a specified position, and his threshold of audibility was determined in terms of the energy which would be radiated to the distant station from a piston source having an amplitude equal to that at the open end of the pipe. Since draughts would have prevented an actual

[1] *Deut. phys. Ges. Verh.*, p. 302, 1904. *See also* Beryl Love and Margaret Dawson, *Phys. Rev.*, 14, 49, 1919.

[2] *Roy. Soc. Proc.*, 159, 507, 1937.

measurement of the amplitude taken up by particles in the mouth, the pipe was run at its second harmonic and the measurement made at the internal antinode, one-third of the way along from the diaphragm. The averages of the minima for five auditors at two frequencies (410 and 646 cycles/sec.) agree with the American measurements (Fig. 102).

Under abnormal conditions, the threshold intensity may be higher than the average shown in Fig. 102.[1] This occurs especially when the tone is masked by the simultaneous loud sounding of another, or of an incoherent noise. Wegel and Lane[2] have conducted experiments in which one pure tone was masked by another pure tone, defining the masking as the logarithm of the ratio of the threshold intensity of the masked tone to that of the same tone unmasked. The effects are complicated by the additional masking introduced by the combination tones formed between the two tones. The question is of importance in connection with the intelligibility of speech in the presence of noise, as in listening to a telephone in a noisy office or works.

The Bel : Measurement of Loudness. Since the loudness of a noise is measured nowadays in terms of the masking which it can produce upon a note of variable intensity, it is important to be able to measure such an intensity in physiological units. A scale of loudness should be logarithmic, if the Weber-Fechner law (88) be true, and must bear a relation to the minimum audible loudness at the same pitch. The standard generally accepted is that of a tone whose objective intensity is ten times that of the just audible sound of the same pitch. This is called the *bel* after the telephone pioneer. Alternatively if I_1 and I_2 are two intensities the difference in sensation level in bels is given by : $\delta E = \log_{10}[I_1/I_2]$. More often a unit one tenth of the bel called the *decibel* (db.) is used.[3] If reference is always made to the threshold of audibility at 1,000 cycles/sec. instead of to the threshold *at the same pitch*, the units are called *phons*.

The simple method of Davis[4] for measuring the sensation level

[1] *See* Minton, *Phys. Rev.*, **19**, 80, 1922, and **221**, 506, 1923; Campbell, *Phil. Mag.*, **19**, 152, 1910; Knudsen, *Phys. Rev.*, **26**, 133, 1925; Marage, *Comptes Rendus*, **158**, 438, 1914, and **162**, 175, 1916.

[2] *Phys. Rev.*, **23**, 266, 1924. *See also* v. Wesendonck, *Phys. Zeits.*, **10**, 506, 1909, and **12**, 231, 1911; Barkhausen and Tichner, *Zeits. tech. Phys.*, **8**, 215, 1927; Guttman and Harr, *Acoust. Soc. J.*, **2**, 83, 1930.

[3] *See* Fletcher and Munson, *Acoust. Soc. J.*, **5**, 82, 1933.

[4] *Nature*, **125**, 48, 1930; *see* Baron, *Rev. d'Acoust.*, **2**, 116 and 189, 1933.

of a noise consists in striking a tuning fork at a determined amplitude in the vicinity and observing the time which elapses before its decaying amplitude is masked by the noise in question. There are noise meters now available in which the intensity of a "warble tone," viz., one whose frequency wanders over a certain range at about four periods per sec., can be varied until it is just masked, the masking level being read off in decibels on the meter. Noises vary from about 10 db. for a whisper to 100 db. for a steam siren.

The method of measuring loudness employed at the Bell Telephone Laboratories is of considerable interest.[1] The experiments in essence involve a subjective judgment of "twice as loud," although it is not certain that this ratio possesses the same significance as the octave does in pitch perception. The first experi-

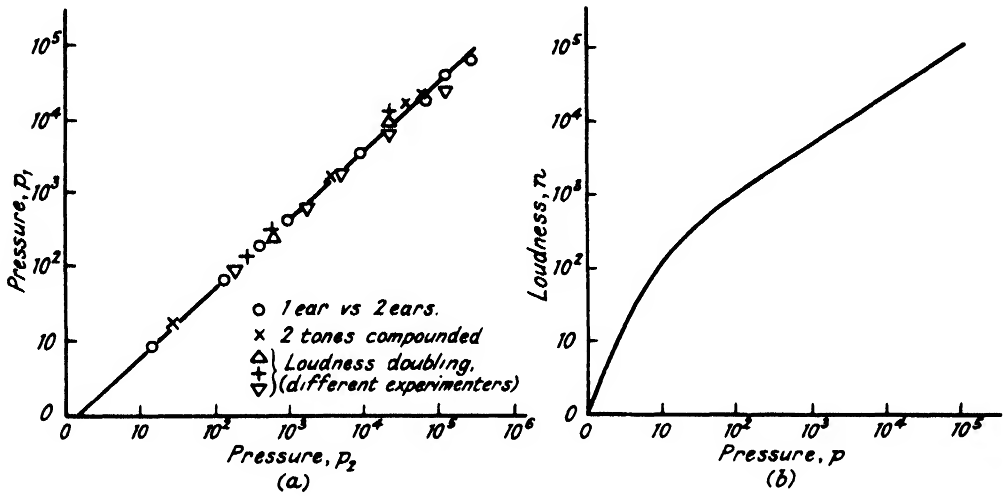


FIG. 103.—Experimental Basis of Loudness Scale (*after Fletcher*).

ment deals with monaural versus binaural hearing. If the observer hears first with both ears and then with one only, it is assumed that the loudness has decreased to one-half. The r.m.s. sound pressure at the single open ear being p_1, it is necessary to raise it to p_2 before the loudness seems equal to what it was with both ears open. With a number of listeners, corresponding pairs of values of p_1 and p_2 were measured, and are shown as o on Fig. 103 (*a*). In the second experiment a 1,000-cycle reference tone is balanced at r.m.s. pressure p_1 against a pure tone of remote frequency until they sound equally loud. They are then sounded together, and again it is assumed that the loudness is doubled. The 1,000 cycle-tone sounding alone is now raised to pressure p_2

[1] Fletcher, *J. Franklin Inst.*, **220**, 405, 1935; *Acoust. Soc. J.*, **9**, 274, 1938.

until it sounds equally loud to the combination. Such points are marked × on the figure. In the third experiment, which has been used by four different sets of observers, although only three are shown in the graph, the reference tone alone is listened to at pressure p_1, and then raised to p_2, such that the loudness appears doubled. The consistency in the results obtained by these different methods is considered sufficient to warrant a loudness scale, based on successive doubling. Thus we can calculate from this figure the relative pressure increases corresponding to increases of loudness $= 2^n$. This has been done, taking a pressure ratio of 100 to correspond arbitrarily to an increase of loudness of 1,000 times ($n = 3$), and the result shown by the curve on Fig. 103 (*b*). It will be noted that in accordance with the well-known relation between stimulus and sensation, the scales have been plotted logarithmically, and the straight lines obtained—at least over the range which matters in telephony, broadcasting, etc.—justify the choice of a logarithmic scale of loudness. It is important, however, as A. H. Davis has emphasized, to *specify the threshold* in every case, if we are to understand each other's scales.

A discussion at one of the Physical Society's meetings during 1935, initiated by a paper by Campbell and Marris,[1] which the authors tried to keep within the province of physics, contrarily enough proved to be a field day for psychologists. The sponsors of the paper were not concerned with what scale worked best in practice, but questioned whether any scale at all could be built up on the facts as we now know them. In marshalling the facts of audition—although a number of these were discredited by various speakers—and examining their relation to the mental estimate of loudness, they have done considerable service in clearing the air of many false premises which mar the efforts of a number of workers in this field. The impression gained, however, was that they endeavoured to establish loudness-measurement on the same basis as a purely physical measurement, and failed in that " they seemed to regard the sense organs as physical instruments in which, when you apply a certain stimulus at one end, you will always get the same response at the other "—to use the words of one speaker—that is, they failed to make due allowance for the influence of earlier stimuli on the response of the ear to the latest one. Another point in the discussion which deserves emphasis is the difference between a sensation and the stimulus which produces it. While it

[1] *Phys. Soc. Proc.*, **47**, 153, 1935.

appears to be correct that sensations are proportional to the logarithms of the stimuli which produce them, the decibel scale must be recognized as a scale of stimulus-energy and not one of sensation.

Stevens[1] has shown that with a change of intensity, the mind associates a change of pitch. In the middle of the gamut, where the ear is most sensitive, no apparent change of pitch is produced by making a tone louder, but down at 150 cycles/sec. a rise of intensity of 50 db. involves a subjective *fall* of as much as 12 per cent., whereas at 12,000 c./sec. the pitch appears to rise by an equal amount. This subjective change of pitch is then paralleled by the decrease in sensitivity of the ear. The author suggests that excessive intensity acting at the ends of the basilar membrane changes the spatial stimulus pattern on it. Montgomery[2] stresses the effect of the method used on the results in such sensitivity measurements; particularly has the type of judgment demanded of the patient an important effect on the results. When the step in intensity or pitch is reduced until the subject reports "no change," the criterion is most difficult to determine. A large number of observations on different people is required to get a good statistical average ear.

Sensitivity of Ear. Psychologists measure sensitivity by the ratio $\frac{\delta\Sigma}{\Sigma}$ in the Weber-Fechner law (88). Both intensity sensitivity and pitch sensitivity have been measured. These have been defined as $\frac{\delta I}{I}$ and $\frac{\delta n}{n}$ respectively, where δI and δn are the least perceptible changes in I or n. From this form of definition arises the paradox that the smaller the sensitivity, the greater the numerical value of the quantity which defines it. For determining $\frac{\delta n}{n}$ the original method was to move a weight on a tuning fork until the observer indicated that the pitch had been changed thereby—he was not asked to determine whether it had been raised or lowered; a more difficult test. Max Wien,[3] who determined the relation between the amplitude of a telephone diaphragm in terms of the current through the exciting circuit (cf. p. 209), introduced the general method for $\frac{\delta I}{I}$, i.e., a sudden small change

[1] *Acoust. Soc. J.*, **6**, 150, 1935. [2] *Acoust. Soc. J.*, **7**, 39, 1935.
[3] *Ann. d. Physik*, **36**, 834, 1888.

in intensity produced by change of current, to be judged by the observer. Knudsen[1] has recently made careful measurements of both sensitivities. The average results are shown in Fig. 104. The changes δI and δn were made by switching over the exciting circuits of valve-maintained telephone transmitters to alternate circuits having slightly different resistance (whereby I was altered), or having different inductance end capacity (causing an alteration of n in the circuit).

Minimum Number of Vibrations for Perception. It is easy to form an idea of the pitch of a sound, if it lasts several seconds; would the perception of pitch be possible if the sound lasted only long enough for the ear to receive a few vibrations? This question was first put by Savart, who satisfied himself that two vibrations only were enough to give an idea of the pitch, but more

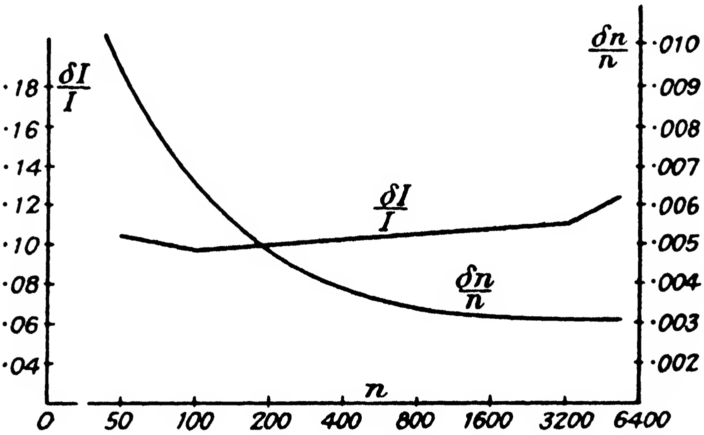


FIG. 104.—Sensitivity of Ear to Intensity and Pitch (*Knudsen*).

were necessary to perceive the quality of the note. More recent experimenters have used a telephone membrane excited for a very short period, i.e., by momentarily "making" the current circuit, or a siren in which all but a few consecutive holes were stopped. Stefanini[2] verifies Savart's statement by several methods,

[1] *Phys. Rev.*, **21**, 84, 1923. *See also* Birnbaum, *Ann. d. Physik*, **49**, 201, 1916; Fletcher and Wegel, *Phys. Rev.*, **19**, 553, 1922; McKenzie, *Phys. Rev.*, **20**, 331, 1922; Stefanini, *N. Cimento*, **19**, 5, 1920; Abraham, *Comptes Rendus*, **144**, 1099, 1907; Stücker, *Akad. Wiss. Wien. Ber.*, **116**, 367, 1907; Barkhausen and Lewicki, *Phys. Zeits.*, **25**, 537, 1924; Waetzmann, *Phys. Zeits.*, **27**, 455, 1926; Allen and Macdonald, *Phil. Mag.*, **9**, 827, 1930; Riesz, *Phys. Rev.*, **31**, 867, 1928; *Acoust. Soc. J.*, **4**, 211, 1933; Shower and Biddulph, *ibid.*, **3**, 275, 1931; Lifschitz, *ibid.*, **5**, 31, 1933; Geiger and Firestone, *ibid.*, p. 25.

[2] *N. Cimento*, **13**, 65, 1917.

at least up to a limiting frequency. Leimbach [1] puts this limit at about 1,000 vibrations per second, so that the shortest time for pitch perception is about a thousandth of a second. On the other hand Gianfranceschi [2] concludes that the criterion for pitch perception is the absolute time during which the ear hears the tone, so that it depends on the absolute pitch; he sets this time at 0·025 second down to 0·01 second for the most sensitive part of the audible region, corresponding closely to the figures obtained for the duration of the after-sound.

Interrupted Sounds. If a pure tone were interrupted for definite intervals and at definite instants, say a tone of frequency n interrupted m times per second, according to König a tone of frequency m would be perceived by the ear. This was thought to be an infringement of Ohm's law, but doubt has been cast on the observation by later investigations. König [3] employed a siren with, for example, 2 holes of every 5 in the ring plugged, so that m equalled $\frac{n}{5}$. This interruption-tone can be picked out by a resonator, so that it appears to be the siren which is producing it. Indeed Schaefer and Abraham [4] failed to detect it with a less dubitable apparatus, and remarked only a change in the quality when the sound was interrupted, not a new tone.

As the number of interruptions is increased, the intermittence ceases to be apparent; the sound seems to continue without break. This critical pulsation has been determined for different frequencies by Mollie Weinberg and Allen [5] by rotating a stroboscope disc in front of a hole in a box, from which a pure tone from a Stern " ton-variator " was issuing. At a critical speed of rotation of the disc the interruptions ceased to be audible. The incidence of this critical pulsation is dependent on the rate of damping of the aural mechanism, and the time during which the tone is interrupted at the critical pulsation represents the time during which the ear continues to respond after the tone itself has ceased. It is analogous to the " after image " effect on the eye, and, as with

[1] *Ann. d. Physik,* 39, 251, 1912. *See also* Abraham, *Ann. d. Physik,* 60, 55, 1919.

[2] *Accad. Lincei. Atti.,* 23, 704, 1914. [3] *Ann. d. Physik,* 157, 228, 1876.

[4] *Archiv. f. ges Physiol.,* 183, 207 and 85, 536. *See also* Schulze, 26, 217, 1908, and 88, 475, 1901; Budde, *Phys. Zeits.,* 16, 62, 1917; Tricca, *N. Cimento,* 16, 55, 1918; Valle, *Accad. Linc. Atti.,* 29, 268, 1920.

[5] *Phil. Mag.,* 47, 50, and 126 and 141 and 941, 1924.

the after image, the time increases when the ear has been fatigued by the steady drone of the same or a nearby tone. The results show that for a tone of average intensity, this after-response falls from 0·02 second at a frequency of 140 to 0·015 second at 280.

Binaural Location. The problems discussed above concern the ear as an individual receiver. There are others which involve the relation between the two ears. The most important of these is that faculty by which we determine the direction of a source of sound. Often we are aided in this by the senses of sight and touch, but in the absence of these we can form a good estimate of the direction of a sound. This has been explained in two ways:

(1) That there is a difference of *intensity* at the two ears, the sound being judged on the side of that ear which receives the greater intensity.

(2) That there is a difference of *phase* at the two ears, the sound being deemed on that side where the phase is in advance.

Since in practice there is both phase and intensity difference at the ears, except when the source is directly in front or behind, the relative weight of the two factors has been a prolific source of investigation. Those of Stewart[1] demand detailed mention for his painstaking. Sounds from a tuning fork were conducted to the two ears severally by two rubber tubes whose openings lay at equal distances from the fork. The observer then judged the sound to lie in the median plane through the head. The intensity of one component was changed without altering the phase difference, by pushing the mouth of one tube further from the fork, and the observer pointed out on a large protractor encircling his head, the angle ϕ through which he judged the sound to have turned. The relation :—

$$\phi = \mathrm{K} \log \frac{I_R}{I_L} \qquad \qquad (89)$$

[1] *Phys. Rev.*, **15**, 425, and 432, 1920. *See also* Rayleigh, *Phil. Mag.*, **13**, 214, 1907; More and Fry, *Phil. Mag.*, **13**, 452, 1907; Boulker, *Phil. Mag.*, **15**, 318, 1908; Mallock, *Roy. Soc. Proc.*, **80**, 110, 1908; Rayleigh, *Roy. Soc. Proc.*, **83**, 61, 1909; More, *Phil. Mag.*, **18**, 308, 1909; Timpe, *Phys. Zeits.*, **20**, 396, 1919; v. Hornbostel, Wertheim and Künze, *Phys. Zeits.*, **21**, 437, 1920; Künze, *Zeits. f. tech. Physik*, **3**, 49, 1922; Marjorie Simpson, *Phys. Rev.*, **15**, 421, 1920; Hartley and Fry, *Phys. Rev.*, **18**, 431, 1921; Pérot, *J. de Physique*, **2**, 97, 1921; Stefanini, *N. Cimento*, **23**, 5, 1922; Bekesy, *Phys. Zeits.*, **31**, 824 and 857, 1930; Kurchara, *Acad. Tokyo Proc.*, **6**, 310, 1930; Stewart, *Acoust. Soc. J.*, **1**, 344, 1930; King and Laird, *ibid.*, **2**, 99, 1930; Firestone and Wightman, *ibid.*, **2**, 260 and 271, 1930; Reich and Behrens, *Zeits. f. tech. Phys.*, **14**, 1, 1933.

was found, I_R and I_L being the intensities received at the right and left ear respectively, and K a constant for the individual. To produce phase-differences alone Stewart used an instrument which he named a "phaser." A toothed iron wheel revolved in front of two independent electro-magnets, somewhat as in the phonic motor (p. 116). As the teeth passed they induced alternating currents in the coils of the electro-magnets, and excited telephones held to each ear. By altering the relative positions of the electro-magnets, any desired phase-difference δ between the tones heard could be produced. He found :—

$$\phi = K'\delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

The two effects, intensity and phase, were discrete, for they did not interfere with each other's effects when both were employed together. But it was concluded—and the results of other workers generally support the conclusion—that up to a frequency between 1,000 and 1,500 vibrations per second, direction is judged almost entirely by phase-difference. At higher frequencies the phase effect fails and the intensity difference must be called upon to explain binaural location.

An illuminating experiment was performed by Lo Surdo.[1] He led the sound from a tuning fork of frequency below 600 to both ears by tubes, and then lengthened one tube so much that the phase at the left ear was in advance of that on the right. In spite of the diminished intensity at the left ear due to the longer tube which this component had to traverse, the sound was judged to come from the left.

If two notes of slightly different pitch are led to the two ears severally, "binaural beats" [2] are heard, involving rapid changes of the relative phase at the two ears. This gives an impression of a sound hovering or revolving round the head, unless the beats become too rapid.

The idea that the brain itself can appreciate a difference of phase between the sensations brought by the nervous mechanism of two independent organs has been a stumbling-block to physiologists. Some, e.g., Myers and Wilson,[3] have therefore supposed that the phase difference is apprehended as a difference of intensity at the two ears due to conduction across the head of the sound from the

[1] *Accad. Lincei. Atti.*, **30**, 125, 1921.

[2] Stewart, *Phys. Rev.*, **9**, 502 and 514, 1917. *See also* Lane, *Phys. Rev.*, **26**, 401, 1925.

[3] *Roy. Soc. Proc.*, **80**, 261, 1908.

side on which the source lies. Banister,[1] who has thoroughly examined the pros and cons of this question, considers that the existence of such "bone conduction" from side to side through the head of aerial vibrations impinging on one side remains to be demonstrated.

Anatomy of the Ear. Externally the ear presents a trumpet-shaped channel to the sound. This narrows down to a nearly cylindrical tube at the end of which is found the membrane known as the ear-drum, which receives the aerial vibrations. The ear is shown in section in Fig. 105. To the back of the membrane M and at its centre is attached one end of the stapes, S, a couple of levers which transmit the vibrations of M to the "oval window" O, which communicates with the inner ear. The inner ear, or labyrinth, is filled with a liquid, the endolymph, having physical

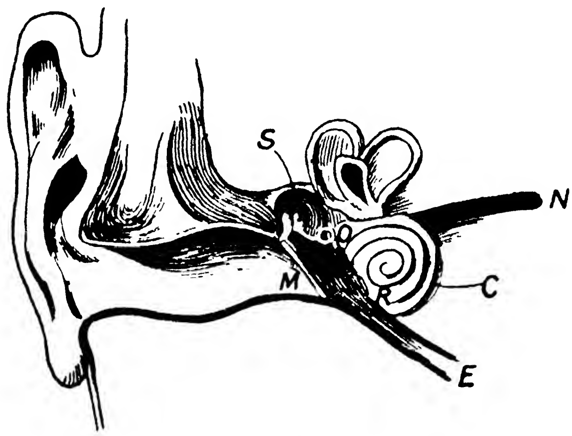


FIG. 105.—The Ear.

properties similar to water. The principal part of this inner organ is the cochlea C, which consists of two galleries, one over the other, divided by the basilar membrane, and communicating with each other at the far end, where there is a break in the basilar membrane. At the nearer ends of the galleries are the oval window O, in the upper gallery, and the "round window" R in the lower, which debouches on the upper end of the Eustachian tube E. Instead of stretching straight out from the two windows the whole of the cochlea is coiled round on itself like the shell of a snail, hence the name. A system of nerves, N, connects the cochlea with the brain. The pressure on both sides should be normally

[1] *Phil. Mag.*, **2**, 144 and 402, 1926; *see also* Bekesy, *Ann. d. Physik*, **13**, 111, 1932, and **14**, 51, 1932; Dean, *Acoust. Soc. J.*, **2**, 281, 1930; Watson, *ibid.*, **9**, 294, 1938.

atmospheric. To secure this the lower end of the Eustachian tube can be occasionally opened to the atmosphere, e.g., in the act of swallowing.

The inside of the cochlea must be considered in detail. Fig. 106 shows a section across one of the folds, showing the two galleries with the narrow basilar membrane *B* lying between. The size of the section decreases three-fold from the near to the far end of the galleries, with corresponding diminution in the width of the basilar membrane. This membrane has embedded in it a number of tendons or " strings " which therefore decrease in length from

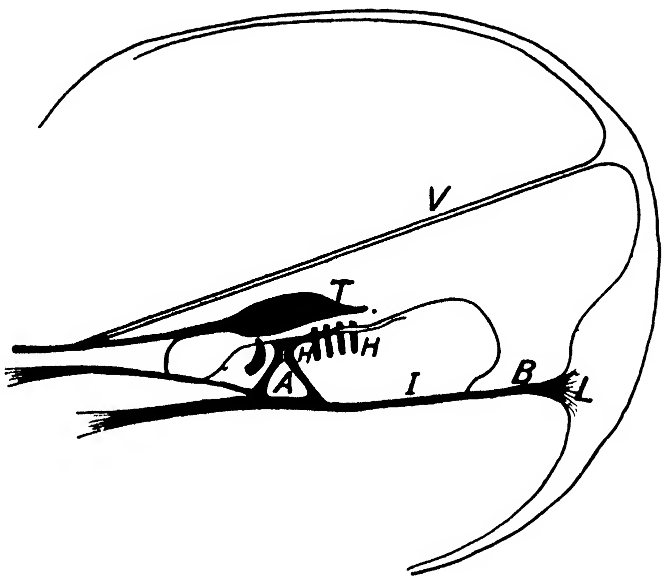


FIG. 106.—Section of the Cochlea.

one end of the membrane to the other ; their direction lying in the plane of the figure. This membrane is attached to the wall of the cochlea by a series of fine fibres, known as the spiral ligament *L*. These serve to put tension on the basilar membrane, and on the fibres embedded in it. Gray [1] has shown that this ligament decreases in stoutness from the lower to the upper end of the cochlea, leading him to suggest that the fibres in the membrane are graduated both for length and tension from end to end of the cochlea. Overshadowing the membrane lie " Corti's arches " *A*, rods set at an angle to the membrane and which support a structure containing minute hair cells *H*, over which the " tectorial membrane " *T* is stretched. Higher up still and distinct from both of these lies the " vestibular membrane " *V*. All these structures progressively

[1] *Journ. Anat. and Phys.*, **34**, 324, 1900.

diminish in size from the base to the apex of the cochlea. Nerve bundles lead away from the neighbourhood of the roots of the hair cells.

The Resonance Theory of Hearing. This theory is generally associated with the name of Helmholtz, but the germ of it is to be found in Cotugno, *De acquaeductibus aurae humanae internae* of 1760, though a hint had been given by Du Verney 80 years earlier. Two sentences translated from Cotugno will serve to show his idea: "It appears that the sensation of hearing consists in the vibration of acoustic tendons (*nervi*) exactly corresponding to the vibrations of the sounding body. . . . The acoustic tendons are like cords oscillating with the vibrations of the sounding body, so that the impressions which the brain receives are as many as the vibrations of the sonorous body."

Helmholtz[1] elaborated this idea into what is known as "the resonance theory of hearing," likening the cochlea to an instrument containing a series of resonators, which could be excited by vibrations of appropriate frequency communicated to the cochlea via the stapes, the corresponding nerves then transmitting the sensation to the brain. At first he postulated Corti's arches as the resonators, but on finding that these were absent in certain creatures, he adopted a suggestion of Hensen that the fibres of the basilar membrane were the resonators. Objections to this theory were not wanting. The tenuity of the "strings," the strong damping due to the fact that they have to drag the membrane with them in their movements, the narrow differentiation of length and the small number of vibrators to cover such a wide range of pitch (10 or 11 octaves), their being surrounded by liquid, were all objected against Helmholtz, but he and his followers have been able to overcome most of these difficulties. The most sedulous exponent of this theory in recent years is Wilkinson.[2] In order to explain how the resonant response of the membrane can cover

[1] *Sensations of Tone*, p. 145. *See also* Fischer, *Ann. d. Physik*, **25**, 118, 1908; Waetzmann, *Phys. Zeits.*, **9**, 307, 1908; Ranyard, *Comptes Rendus*, **150**, 724, 1910; Schulze, *Ann. d. Physik*, **45**, 283, 1914, and **49**, 683, 1915; Stefanini, *N. Cimento*, **9**, 149, 1915; Budde, *Phys. Zeits.*, **18**, 225, and 249 and 317, 1917; Roaf, *Phil. Mag.*, **43**, 349, 1922; Wegel, *Nature*, **116**, 393, 1925; Bekesy, *Phys. Zeits.*, **29**, 793, 1928; and **30**, 115 and 721, 1929; and **34**, 577, 1933; Tröger, *ibid.*, **31**, 260, 1930; Fletcher, *Acoust. Soc. J.*, **1**, 311, 1930.

[2] *J. Laryng. Otol.*, 1921 and 365, 1927.

the total audible pitch range he points out that these fibres vibrate under quite special conditions, the analogy with a pianoforte being quite out of the question. Taking the ordinary formula for vibrating strings, $n = \frac{1}{2l}\sqrt{\frac{F}{m}}$, if Gray's conclusion is correct, there are considerable variations in F, beside a small variation in l, and also in m, since the strings are loaded with various amounts of liquid (cf. p. 121). By somewhat arbitrary assumptions concerning the magnitude of these fluid columns, Wilkinson is able to make the variations in l, F, and m produce a sufficient range of frequencies in the resonance system. To explain the extremely high pitch sensitivity of the ear, having in view the small number of fibres, Wilkinson employs the principle of maximum stimulation of the nerves. To illustrate this, he suggests the experiment of pressing a point hard into the finger; although there is considerable pressure all around the point, so that a large area surrounding the point is anæmic, yet the sensation is of pressure at a single point. "The point at which we feel the nerve ending to be stimulated is the point at which the maximum degree of stimulation is taking place." So in the cochlea, a pure tone will excite a *region* of the membrane, with maximum response at the level where a cross section of the membrane has a natural frequency of the same period. The pitch sensitivity of the ear is limited by the degree with which the brain can distinguish *two* points of maximum stimulation together. This faculty may well vary in different parts of the scale, just as the application of two pointed objects close together can be felt as two points on the hand, say, whereas they are felt as one point if applied to a less sensitive region like the middle of the back. The resonance theory requires the assumption of very heavy damping on the basilar membrane, otherwise a rapid performance on a musical instrument would be audible as a jangle of notes.

Indirect Evidence for the Resonance Theory. On examination, we find that most of the facts about the sense of hearing fall into line with the resonance theory, in its developed form. The salient criteria are:

(1) Subjective combination tones: The ear-drum and trumpet form a system whose response is non-linear. In fact, owing to the one-sided load of the ear drum, combination tones $p - q$, $p + q$, $p - 2q$, etc. are added to the vibrations due to two simple

tones led to it, if Waetzmann's analogy of the asymmetrically loaded membrane be correct (cf. p. 63). The difficulty of the recognition of König's beat tones as simple tones then disappears, as the difference or summation tone is already present, to an extent dependent on the intensity, in the vibrations impressed on the cochlea, even if absent in the surrounding air.

(2) The variations in sensitivity and damping at different frequencies are what would be expected of an instrument consisting of a set of resonators.

(3) The possibility of fatiguing and even permanently injuring a definite level of the basilar membrane by continued sounding of tones of the corresponding frequency.

(4) The definite pitch and intensity limits.

(5) Deafness to certain definite ranges of pitch, and therefore to certain vowels, due to degradation or destruction of a certain length of the resonating strip; continual "sounds in the head," due to derangements also have a regional tendency in pitch, so that the patient complains "I hear cocks crowing," or "I always hear a bell ringing."

Other Localized Theories. The resonance theory is an essentially localized theory which regards the sound-perception as the recognition by the brain of the stimulation of certain nerves as a pure tone of definite pitch. Other theories have been advanced; some merely put forward other organs in the cochlea as resonators, others differ in the mode by which they imagine the particular nerves to be excited. Some involve rather dubious physics. Hurst [1] suggested that the inward movement of the stapes started a wave down the basilar membrane which was suddenly stopped by the return movement of the stapes. The distance to which the bulge had travelled was recorded by the brain as a note of certain frequency. A better idea from the physical point of view was that of Ewald [2] who pictured stationary undulations in the basilar membrane; all the nerve ends opposite the "antinodes" would be stimulated. But the theory is open to objection on the physiological side, since a number of *discrete* stimuli of different nerves are to combine to give the impression of a simple tone. Both Wilkinson and Ewald have respectively constructed me-

[1] *Liverpool Biol. Soc. Trans.*, **9**, 321, 1895.

[2] *Pflüger's Archiv.*, **93**, 485, 1903. *See also* Ter Kuile, *Pflüger's Archiv.*, **79**, 484, 1900.

chanical models to illustrate the resonance and the pressure pattern theories.

Central Analysis Theories of Hearing. In 1865, Rinne[1] put forward the theory that the ear-drum, and organs connected to it in the inner ear merely copied the vibrations which fell upon them, exciting *all* the auditory nerves to a greater or less extent, and that the analysis into component tones was performed by the brain itself. Such a theory is rather unsatisfactory to physicists as it begs the question of the purpose of all the intricate structure of the inner ear, for why is not then the nervous system applied directly to the ear-drum? Apart from this, the theory requires that a nerve shall be capable of transmitting impulses up to 20,000 per second, and that the brain shall recognize the numbers of these impulses, even if overlaid with impulses at other frequencies. Whereas it is known that, for at least one-thousandth of a second after each stimulation, a nerve fibre is impotent to transmit a further stimulus.

It has been impossible to give more than the main pros and cons of the several theories of hearing. On the whole, the resonance theory holds the field, although modifications may be necessary as research, both physical and physiological, brings further evidence. One of the present troubles is to explain binaural phase perception, on the resonance, or indeed on any other, theory.[2]

The Musical Scale. In spite of the extreme sensitivity of the ear in discriminating pitch, only a limited number of notes is used in music. The ear recognizes that the whole range of notes divides itself up into successive groups, and that each group repeats the musical effect of the others. A group is limited between two notes, the higher being the octave of the lower. The construction of a scale is therefore a question as to how many notes there shall be in the group. The "interval" between two notes is defined by the ratio of their respective frequencies. The smallest interval employed is the semitone ($\frac{16}{15}$), although quarter tones have occasionally been employed. This limit is set by the requirements of musical instruments having a separate key or hole for each note.

[1] *Zeits. f. rath. Med.*, **24**, 12, 1865. *See also* Rutherford, *Brit. Ass. Rep.*, 1886; Wrightson and Keith, *The Mechanism of the Inner Ear*, 1922; Morton, *Phys. Soc. Proc.*, **31**, 101, 1918.

[2] The Physical Society's "Discussion on Audition," 1931, should be consulted for further papers on Hearing.

On a violin or trombone, on the other hand, a player can himself subdivide the semitone. The octave has been divided in a number of ways. Each of these is known as a musical scale or mode, but of them only three survive in current musical practice: (1) the major scale, (2) the minor ascending scale, and (3) the minor descending scale. Each of these three scales starts from its "key note," and by means of a series of intervals which includes two semitones, arrives at the octave, which is the key note of the group above. The scales differ from each other in the relative positions of the two semitone intervals in the group.

The major scale in the table on this page is shown as the interval from one note to the next, then as the interval from the first or keynote, and then as a relative number of vibrations.

While music remains within the compass of such a series of notes all based on one key note—all in one key—there is no difficulty in constructing a musical instrument having octaves of notes, all in the series of intervals of the above table. But composers employ different keys, with frequent changes from one to another, so that, for instance, after a musical composition has been confined to a series of notes based on a key note of frequency 240, the key may change to one based on the second note of the old key, i.e.,

	—	Major-tone.	Minor-tone.	Semi-tone.	Major-tone.	Minor-tone.	Major-tone.	Semi-tone.
Interval from note below	—	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$
Interval from key note	—	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Possible frequencies	240	270	300	320	360	400	450	480

on 270. By applying the same interval ratios to this note, we get the series:—270, 304, 338 360, 405, 450, 506, 540 (major scale), for the notes comprising one octave of the new key, of which only four coincide exactly with any one of the old notes in the table. At this rate a large number of additional notes would be required for every key in which an instrument had to play. To obviate this complication, many schemes for adjusting the intervals in the octave have been proposed, but that which holds the field is the equi-tempered scale. In this scale the octave is divided into

twelve equal semitones, so that the value of the equi-tempered semitone becomes equal to $\sqrt[12]{2} = 1{\cdot}0594$ and a whole tone is $\sqrt[6]{2} = 1{\cdot}1325$, the distinction between major and minor tones being sunk. Since every interval in this scale is a multiple of $\sqrt[12]{2}$, it is now feasible to employ 12 notes to the octave, and yet be able to use keys founded on any one of them as key note.[1]

Harmony. The basis of classical harmony was the common chord, formed of the first, third and fifth of the notes of the scale, the interval between the first and third being $\frac{5}{4}$ (4 semitones) or $\frac{6}{5}$ (3 semitones), and between the first and fifth $\frac{3}{2}$ (7 semitones). These intervals are known as the major third, minor third, and the perfect fifth respectively. Those intervals which the æsthetic taste judges to be concords are all expressible by the ratio of small numbers. Thus the fifth is the most perfect concord after the octave. The equi-tempered scale retains the perfect octave (interval 2), but spoils the perfect fifth, which now becomes $2^{\frac{7}{12}} = 1{\cdot}498$ instead of $1{\cdot}500$. On the other hand, it makes all fifths (i.e., 7 semi-tones) equal, wherever they are chosen on the scale, while all major thirds become $2^{\frac{4}{12}} = 1{\cdot}260$, and minor thirds $2^{\frac{3}{12}} = 1{\cdot}189$. The common chord in the major scale is then made of a major third followed by a minor third, and in the minor scale is made of a minor third followed by a major third. All concords are made up of these three notes in various positions. Any note which is alien to this combination is made to lead in certain stipulated ways to a note which forms part of a concord. Chords having these intruding notes count as discords, and the process by which the return is made to a concord is known as "resolving the discord."

Modern composers have built up scales in which other notes, such as the seventh or the ninth in the scale are regarded as regular constituents of the chord characterizing a key. They do not regard such chords as discords, and consequently leave them unresolved in their compositions. Another departure has been made by Scriabin, who builds up chords based on the upper constituents of the harmonic series of tones—the 8th to the 14th partials, in fact. These he arranges in ascending fourths.

[1] *See* Sizes, *Comptes Rendus*, **164**, 861, 1917; **165**, 264 and 465, 1917; **167**, 229 and 455, 1918; Würschmidt, *Zeits. f. Phys.*, **3**, 80, 1920; **5**, 111 and 198, 1921; Wallot, *Zeits. f. Phys.*, **4**, 151, 1921 and **6**, 73, 1921; Redfield, *Acoust. Soc. J.*, **1**, 249, 1930.

His "Promethean chord" based on 240 (fundamental 40), for example, would contain the following notes (using the frequencies of the natural scale), 240, 338, 427, 600, 800, 1,280. This "synthetic harmony" embraces both forms of common chord. Such innovations leave the key a rather undefined quantity, and the natural development in modern music is to ignore the question of key altogether. For this reason it appears to be beside the point to detail the elaborate theory of harmony developed by Helmholtz on the common chord.

CHAPTER THIRTEEN

TECHNOLOGY

The technical aspects of sound have received a great deal of attention during the last decade, particularly in relation to the development of the acoustics of buildings, of wireless telegraphy and telephony, and the needs of the late European War. The sound problems in the second category mainly concern the interconversion of electrical and sound energy, and so are to be found under the discussion of the telephone and of the analysis of sound. To go beyond this would take us into the domain of electromagnetic radiation, which cannot find a place here. There are other branches of telegraphy involving transmission of sound itself, such as sound-ranging, sound-signalling by land or sea, and depth-sounding. These deserve separate treatment.

Acoustics of Buildings

Reverberation in an Auditorium. Suppose a "pulse" to be produced at some point in a room. A wave of compression spreads out in all directions. In the absence of any reflecting bodies, an auditor would receive a single sharp impression, but the walls of the room reflect the greater portion of the sound, so that a series of waves, generally diminishing in amplitude, and formed by subsequent reflections, pass the observer's ears, until all the energy of the original wave has been dissipated by friction. In place of the single compression, the observer hears a roll of sound, and the time taken for this to die away, i.e., fall below the threshold of audibility, is known as the "time of reverberation" of the sound in this particular room, reckoned from the time that the original pulse was produced, or, in the case of a continuous note, the time that the source stopped sounding. Now suppose such a continuous source, e.g., an organ pipe, to be started and to continue at constant output at some point in the room. The waves spread out and the auditor is conscious of the commencement of the sound when the first of these reaches his ears, and of an

addition to the energy received for every reflected wave which subsequently adds itself to the incident waves still emitted from the source. During this time part of the energy is being absorbed by the walls and other materials in the room, and part is escaping through these walls and via open windows or ventilators. After a few seconds a balance is reached between the energy emitted per second and that lost outside or dissipated; then the intensity at the listener's ear, represented in energy incident on unit area per second, attains a steady average value. If now the source is cut off, this intensity is rapidly diminished, first by loss of the incident radiation and then by successive removals of the reflections of the last waves, until it falls below minimum audibility. The rise and fall of the sound is shown graphically in Fig. 107(*a*). When a succession of different vowels or notes are sounded the effect is

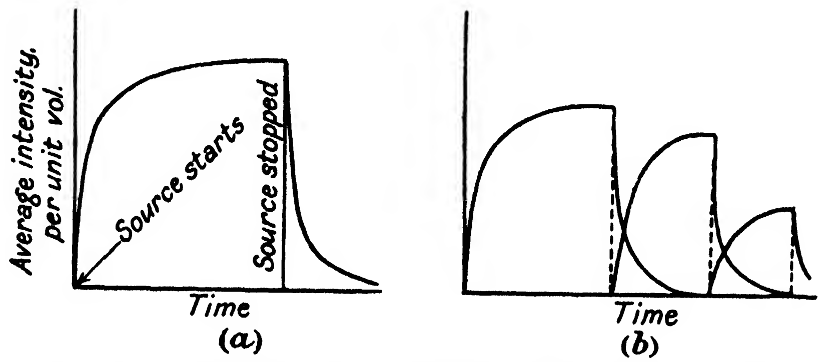


FIG. 107.—Rise and Fall of Average Intensity in a Room.

as in Fig. 107(*b*). Now it is important for distinct rendering of music or speech that each separate sound should give rise to a sufficient intensity in every part of the auditorium and then rapidly decay to give place to the next sound. This is particularly necessary with speech; for music more reverberation is permissible.

Reverberation Formula. A formula expressing the rise and fall of sound in a room was developed by Wallace Sabine [1] on the following assumptions:—

(1) That an average energy per unit volume σ could be conceived.

(2) That dissipative influences were negligible, energy being lost only by "absorption," using this word to include transmission to the outside.

Following Jäger,[2] we can calculate, on these assumptions, the

[1] *Amer. Architect*, 68, 1,900, 1911.

[2] *Akad. Wiss. Wien. Ber.*, 120, 613, 1911. *See also* Buckingham, *Bureau of Standards Sci. Paper*, No. 506, 1925; Eckhardt, *Frank. Inst. J.*, 195, 799, 1925.

energy falling on unit area of the walls per sec., for the energy lying in a cylinder of unit length and unit cross-section, taken anywhere in the room, and moving parallel to its own axis, will fall on an area of the wall $\delta S \cos \delta$, at a rate c cm. per sec., where $\delta S \cos \delta$ is the projection of the wall on the boundary of the cylinder. Thus there falls on this piece of wall an amount of energy $c\sigma\delta S \cos \delta$ due to the energy σ in the unit volume of the cylinder. Now the angle δ may have any value between 0° and 180°, since the sound is travelling in random directions through the room. To find the total energy falling on unit area of the wall, per second, we must take the average value of $\cos \delta$, when δ lies between 0° and 180°. This value is $\frac{1}{4}$, so that the energy falling on unit area per second is $\frac{c\sigma}{4}$.

If α is the average "absorption coefficient" of the walls, defined as the fraction of incident energy which is not reflected, the rate at which sound energy is removed from the room is $\frac{\alpha S c \sigma}{4} = \Omega\sigma$ ergs per second, where S is the total area of the walls and other absorbing materials. The rate at which the total energy $v\sigma$ in the room increases is the difference between the rate of supply from the source ($= Q$ ergs per second, a constant) and the rate of removal by absorption, so that

$$v\frac{d\sigma}{dt} = Q - \Omega\sigma.$$

Substituting $\sigma = B + be^{-\beta t}$ as a solution, we find :—

$$\beta = \frac{\Omega}{v}, \quad B = \frac{Q}{\Omega}.$$

Also, when $t = 0$, $\sigma = 0$, therefore $b = -B$, so that the rise of average sound energy per unit volume from the time the source commences to sound, is given by :—

$$\sigma = \frac{4Q}{\alpha S c}\left(1 - e^{-\frac{\alpha S c}{4v}t}\right)$$

and attains a steady value $\sigma_{max} = \frac{4Q}{\alpha S c}$. Similarly, after the sound ceases, we find :—

$$\sigma = \frac{4Q}{\alpha S c}e^{-\frac{\alpha S c}{4v}t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

to represent the decay of σ.

For a source of standard frequency and standard output measured by the energy produced per second (Q), the average steady intensity of sound (σ_{max}) produced in the room will be inversely as the absorption + transmission αS by the surfaces. On the other hand, if the material of the exposed surfaces be kept the same, the intensity is weaker in a room of larger surface area.

The factor $\frac{\alpha Sc}{4v}$ determines the time of reverberation in the room. Thus if σ_0 denotes the minimum audible intensity, equation (81) can be written :—

$$\sigma_0 = \sigma_{max} e^{-\frac{\alpha Sc}{4v}t_1}$$

where t_1 is the time from the cutting-off of the sound to the time at which the sound becomes inaudible. This gives :—

$$\log_e \frac{\sigma_{max}}{\sigma_0} = \frac{\alpha Sc}{4v} t_1.$$

Both α and σ_0 vary with pitch (cf. p. 264). The standard frequency chosen by Sabine was 256 (middle C).

For reckoning the time of reverberation, a standard steady intensity is required. For this Sabine took "a million times minimum audibility." His method of determining the maximum intensity in a room in terms of the standard will be explained presently. The quantity on the left-hand side then becomes $\log_e 10^6 = 2{\cdot}3 \times 6$, and putting $c = 1{,}120$ ft. per second, we obtain approximately :—

$$t_1 = {\cdot}05 \frac{v}{\alpha S}. \qquad \qquad (92)$$

the foot now being taken as the linear unit.

For a source of constant output, therefore, the time of reverberation should vary directly as the volume of the room, and inversely as the total absorption + transmission of the exposed surfaces. This formula has been verified by Sabine and others experimentally; t_1 was found by a revolving drum chronograph, the turning-off of the source, an organ pipe, made a mark on the drum, and a second mark was made by the observer himself when the reverberating sound became inaudible. As standard material Sabine took an open window as having zero reflecting power. The coefficients of other materials were found in comparison with this; e.g., Sabine found what area of surface of cushions with shut windows would produce the same effect as the maximum possible area of open window and no cushions, and obtained an

absorption coefficient of ·80 for the material of the cushions. To refer all values to his standard steady intensity, Sabine used two organ pipes of equal output and frequency as far as could be judged, first each singly, then two together. Calling the respective reverberations t_1 and t_2,

$$\sigma_0 = \sigma_1 e^{-\beta t_1}$$

$$\sigma_0 = 2\sigma_1 e^{-\beta t_2}.$$

From the first equation $\beta = \frac{1}{t_1} \log_e \frac{\sigma_1}{\sigma_0}$,

from the second $\beta = \frac{1}{t_2} \log \frac{2\sigma_1}{\sigma_0}$

whence
$$\frac{1}{t_1} \log_e \frac{\sigma_1}{\sigma_0} = \frac{1}{t_2}\left(\log_e 2 + \log_e \frac{\sigma_1}{\sigma_0}\right),$$

or
$$\log_e \frac{\sigma_1}{\sigma_0} = \frac{t_1}{t_2 - t_1} \log_e 2.$$

By measuring the "times of reverberation" t_1 and t_2, in the same unchanged room, he was able to determine the value of the intensity, measured as energy σ_1 per unit volume, produced by a single source, in terms of the minimum audible intensity σ_0, and hence of the standard intensity $\sigma_0 \times 10^6$. The standard source chosen was perhaps unreliable, but was sufficiently accurate for this branch of applied acoustics, in which approximate values of the quantities concerned suffice, in view of the large scale of the experiments.

The standard intensity is, by the way, very great, and nothing short of a steam-siren or diaphone could produce it in the average concert-room or theatre. This explains why experimenters generally obtain shorter times under the same conditions, than those of Sabine, which are calculated for the standard intensity.. Sabine concluded that the reverberation was practically independent of the position of the source and the observer, but later work has not confirmed this conclusion, when the walls are of peculiar shape.

Optimum Reverberation. The acceptable time of reverberation in a room is a matter of taste. Statistics have been compiled by Watson [1] and by Lifschitz [2] of the times of reverberation (always

[1] *Frank. Inst. Journ.*, **198**, 73, 1924. *See also* Barus, *Nat. Acad. Sci. Proc.*, **8**, 96 and 118, 1922.

[2] *Phys. Rev.*, **25**, 391, 1925, and **27**, 620, 1926. *See also* Sutherland, *Phys. Soc. Proc.*, **36**, 142, 1924; Backhaus and Trendelenburg, *Wiss. Veröfft. d. Siemens-Konzern*, **4**, 205, 1925; Trendelenburg, *Zeits. tech. Phys.*, **8**, 502, 1927; Meyer, *Elekt. Nach. Tech.*, **4**, 135, 1927.

reckoned on Sabine's standard) and volumes of halls pronounced by public opinion to be acoustically good. It is found that this optimum time of reverberation increases with the volume. From (91) and (92) the actual time of reverberation varies as $\frac{v}{\alpha S}$; if the absorption + transmission per unit area α is kept constant for all halls, then this time will be proportional to the volume divided by the surface area, or, since the latter varies as $\sqrt[3]{v^2}$, if the hall is changed in scale, but not in shape, t_1 will vary as $\sqrt[3]{v}$. Plotting the optimum reverberation against the cube root of the volume of the halls in the collected data, Watson finds that the points lie roughly on a straight line (volumes between 10^5 and 10^6 cu. ft.), which does not, however, pass through the origin. It is therefore proposed to make α the same in all halls; referring to (91), it appears that when this is done, as the volume of the hall increases the output of the source must increase as $v^{\frac{1}{3}}$, if the same average intensity is to be kept· For speech the optimum reverberation, when the hall is full, is given by the statistics in the form:—

$$t' = 0{\cdot}75 + 0{\cdot}175\sqrt[3]{v} \quad . \quad . \quad . \quad . \quad . \quad (93)$$

Lifschitz [1] finds that expert criticism does not think this law to be satisfied when the room is of small volume. A limiting reverberation of 1·03 seconds seems best for all rooms having less than 10,000 cu. ft. of volume.

Correction of Reverberation. The work of correcting an auditorium for too much reverberation on the one hand or for deadness on the other consists in practice of assimilating the times in formulæ (92) and (93); a mere matter of calculation. The volume of the hall is calculated, and substituted in (93) to get the optimum reverberation; going back to (92) it is necessary to adjust αS so that the value of t_1 so obtained shall equal the calculated t'. In all these calculations a latitude of 5 per cent. is permissible. As the total surface of the room cannot generally be altered, the walls, floor or ceiling must be covered or replaced by material of greater or less absorbing power.

Combining the results of Watson for optimum reverberation with Sabine's formula (92), we can draw a graph (Fig. 108) which gives the number of "absorption units," reckoned by the value of αS, which a room should contain, as a function of its volume. Under αS must be included the absorption produced by the average

[1] *Phys. Rev.*, **25**, 391, 1925, and **27**, 620, 1926.

audience; it is usual to reckon this at 4·7 units per head, to tally with corresponding values of αS for *materials*, measured in square feet. Upholstered seats are useful as they provide absorption to take the place of the absent audience when the hall is

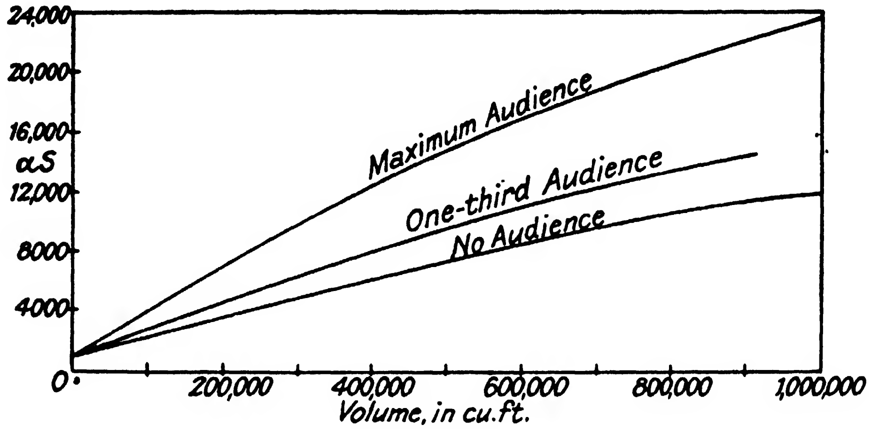


FIG. 108.—Variation of Absorption required in Halls with Volume (*Watson*).

sparsely filled. A table of absorption coefficients will be found on p. 307.

Effect of Frequency on Time of Reverberation. The calculations of Sabine were based on a pitch of 256 (" middle C ") taken as the average musical pitch. Such a compromise has to be adopted, in view of the fact that the sounds in the auditorium cover the complete musical range. When Sabine experimented with organ-pipes of other frequencies he found a change in the time of reverberation. This variation of reverberation with pitch results from two influences: (1) the variation of reflecting power of materials for sounds of different frequency; (2) the variation of the minimum audibility with frequency. The latter influence is indicated on Fig. 102 in the last chapter; the other influence will be discussed in a later section.

Concentration of Sound. We have spoken at present as though the intensity reached after the sound had become established were the same all over the room. This is by no means the case if there are large concave surfaces which tend to bring the sound to a focus, concentrating the sound in some parts to the detriment of others. Where such is the case, an auditor moving about in the room during the production of a steady sound, will hear marked oases and deserts in the sound intensity. " Dead spaces " caused by concentration of the sound into other parts are to be avoided; and large smooth surfaces are detrimental, except behind the

source of sound, where they may serve to send out plane waves to distant parts of the auditorium. When defects due to these are detected, it is advantageous to cover the surface with poorly reflecting material, or to break it up by heavily embossed decoration, the size of the protuberances being of the order of the long wave-lengths, a foot or more across.

The acoustical effect of a proposed design can be studied in advance of construction by making sectional models, both vertical and horizontal, through the proposed position of the source. The paths of sound waves through the room can be studied in the model sections by the spark-pulse method (p. 17), or by making the model a shallow tank of water, and studying the progress of ripples over the surface (p. 19). The first method is more tedious, but gives more definite results and was extensively employed by Sabine in the elucidation of acoustical defects in existing buildings. If the walls of the model are made of some plastic material, the second method allows rapid adjustments of the section, until focussing of the waves is reduced or eliminated.

The paths by which the sound can reach an auditor, e.g., the direct wave and the once-reflected wave, should be made as equal as possible, otherwise, in place of the desirable reverberation the sound may rise again as an echo after it has once died away. The time of elapse between the direct and once-reflected waves to give the impression of a distinct echo is a personal quantity, but it is of the order of a fifth of a second (cf. p. 13). Echoes may be present in a building having a very high ceiling in relation to its horizontal dimensions, and may be remedied by the same means as those adopted for curved surfaces, i.e., by spoiling the reflection.[1]

Other Influences in Buildings. Resonance and interference play a minor part in auditorium acoustics. A certain amount of distortion is produced by the resonance of sections of partition or wall acting as soundboards to an emitted tone of the right frequency, or of volumes of air contained in small rooms, alcoves, etc. The effect is not very noticeable, as the resonant bodies in question are highly damped.

In the reverberation in small rooms, however, natural frequencies of the air space occupying the room are noticeable. When an oscillograph record of the reverberation is taken, Knudsen[2] finds

[1] Aigner, *Akad. Wiss. Wien. Ber.*, **123**, 1,489, 1914.

[2] *Acoust. Soc. J.*, **5**, 127, 1933.

these resonances present no matter what the frequency of the exciting source. These natural oscillations are particularly noticeable when a " reverberation meter "[1] is used in place of the more primitive but effective stop-watch. These mechanical recorders usually employ a warble tone (p. 280) which is turned on for a while, then switched off at the same time as a chronograph is started. The chronograph is stopped again when the intensity received by a microphone falls below a level corresponding to minimum audibility.

When a steady tone of constant frequency is sounding, an interference pattern is produced in the room. The variation in intensity throughout the room due to this cause may be shown by moving a tuned resonator, e.g., a hot-wire microphone, about the room, and noting the difference in response at different points even when there are no focussing surfaces present. During the rendering of speech or music, this pattern is constantly shifting with the wavelength, and moreover is close-woven in the sense that the distances between maxima and minima due to diffraction are small, so that the effect generally passes unnoticed by the listener.

The acceptable time of reverberation varies by about 10 per cent. between music and speaking. Discrimination is now being made between different classes of musical sound, e.g., musical taste requires for the staccato notes of the pianoforte a reverberation differing from that of the sustained notes of the organ.[2]

Measurement of Reflecting Power. In order to find, for building or other purposes, the fraction of incident sound energy reflected by a surface, we have at our disposal both small scale and full scale experiments. Both types are open to criticism, but the results obtained by them show concordance, and are probably accurate enough for technical use. The small scale methods are based on observations of Tuma[3] that when stationary waves are formed between a source of sound and an imperfectly reflecting wall, pseudo nodes and antinodes are formed, having respectively less and greater pressure amplitudes than those indicated by the simple theory of stationary waves, which supposes

[1] Meyer and Just, *E.N.T.*, **5**, 293, 1928; *Zeits. f. tech. Phys.*, **10**, 309, 1929.

[2] For an account of the present position in this subject *see* Sabine, *Frank. Inst. J.*, **217**, 443, 1934.

[3] *Akad. Wiss. Wien.*, **111**, 402, 1902.

the waves to be reflected without loss of amplitude. A similar remark applies to the displacement amplitudes. The idea was developed by Taylor [1] using a pipe resonator having the thick wooden stop at the end covered with the material in question. This resonator was excited by a suitably tuned organ pipe. He determined the relative displacement amplitudes at the pseudo nodes and antinodes by means of a Rayleigh disc resonator to which the point of observation was connected by a long search tube. This method is open to criticism by virtue of the stationary vibrations which would tend to form in the search tube, causing deflections of the disc dependent on its position relative to these (cf. also p. 243)

Weisbach [2] repeated the method, using a microphone in the pipe resonator itself in place of the Rayleigh disc. The tube was made wide compared with the microphone and the amplitude of response measured on an Einthoven string galvanometer. The writer [3] has used the hot-wire grid (p. 185) in such a resonator to determine the displacement amplitude at a given point, (1) with a stop of thick teak to give as nearly as possible perfect reflection, and (2) with this stop covered by a layer of the material to be investigated. The reflection coefficient can be calculated from the ratio of the amplitudes in the two cases at any point except at the nodes where the ratio is indeterminate. Referring to p. 43 we see that the ratio of the amplitudes in the incident and reflected waves which make up the stationary vibration is calculable, assuming that the phase change on reflection is unaffected by the substitution. The ratio b^2/a^2 is the reflection coefficient.

Of methods using large sheets of the material, the substitution method used by Sabine in his reverberation experiments has been already mentioned. Other investigators rely, in principle, on the measurement of the response of a suitable detector to a sound produced nearby with and without a sheet of the material in the vicinity. The method is not so simple as it sounds. If the experiment is performed indoors, as is usual, or even if it is performed out of doors near the ground, reflections are produced by all neighbouring objects, forming, when the source is steady, interference patterns on both sides of the sheet. This makes possible

[1] *Phys. Rev.*, **2**, 270, 1913. *See also* Eckhardt and Chrisler, *Bureau of Standards*, No. 526, 1926; Paris, *Phil. Mag.*, **4**, 907, 1927; *Roy. Soc. Proc.*, **115**, 407, 1927; *Phys. Soc. Proc.*, **39**, 269, 1927.

[2] *Ann. d. Physik*, **33**, 763, 1910. [3] *Roy. Soc. Proc.*, **112**, 538, 1926.

only a comparative method, in which nothing is moved between two measurements except the sheet of material. Watson[1] placed this over the doorway between two rooms, focussed upon it the sound of an organ pipe blown at constant pressure, and measured by Rayleigh discs the intensity reflected to A in the first room, and transmitted to B in the second (Fig. 109). The observer was enclosed in a box in order that his movements should not alter the diffraction pattern. As a working assumption, the deflection of the disc had to be taken as proportional to reflection or transmission produced by each sheet.

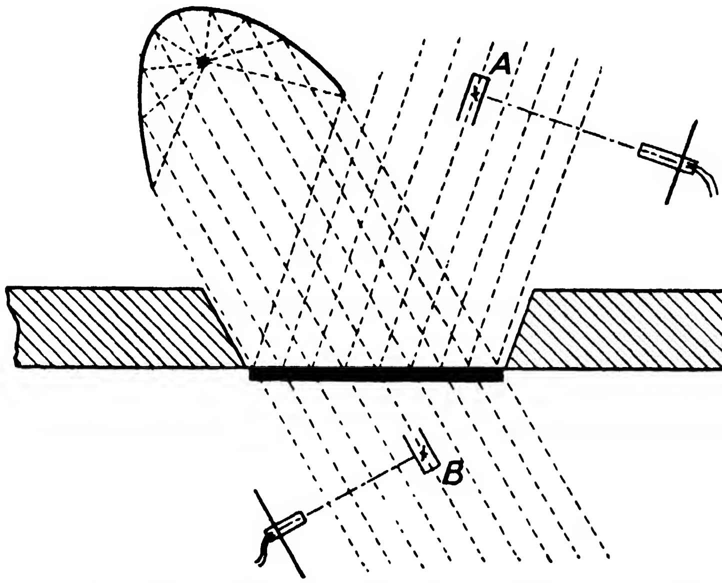


FIG. 109.—Measurement of Absorption Coefficients (*Watson*).

Other workers have followed similar lines. An organ pipe is a rather unreliable source to use with a resonator, seeing that the latter is so sensitive to change of frequency; others have employed tuning forks as sources, and telephone receivers. It is necessary to ensure constant amplitude of the source as the varying conditions tend to react on it.

Absorption Coefficients and their Use. Absorption coefficients (given by the fraction of incident energy not returned) for some common materials are shown in a table for $n = 512$. The numbers represent average values obtained by the above methods, and do not distinguish between absorption and trans-

[1] *Phys. Rev.*, 7, 125, 1916. *See also* Weisbach, *loc. cit.*; Kurokawa, *Inst. Elec. Engin., Japan Proc.*, 415, 113, 1923; Davis and Littler, *Phil. Mag.*, 3, 273, 1927.

mission, that is, they are actually reflection coefficients subtracted from unity.

Asbestos cloth ($\frac{3}{4}$ in.)	·26
Carpet ($\frac{1}{2}$ in.)	·30
Concrete	·17
Cork (2 in.)	·23
Glass	·027
Hair felt ($1\frac{1}{2}$ in.)	·58
Marble	·01
Common plaster	·03
"Acoustic" plaster	·30
Wood sheathing	·06

To find the number of absorption units in a room it is then necessary to take each area and multiply by the appropriate coefficient. The total gives the quantity αS. An example will make this clear :—

Acoustic Data of Great Hall, University College, London.
Volume : 170,000 cu. ft.

		Units
Wood, ceiling, floor, wall panels, etc.	16,000 sq. ft. @ ·06 =	1,000
Plaster ; upper part of walls	9,250 sq. ft. @ ·03 =	280
Glass ; windows	2,820 sq. ft. @ ·027 =	70
	Total =	1,350

With a full audience of a thousand persons, and an extra 4·7 units for each, the total absorption equals 6,000 units. With one-third audience, 3,000. Reference to Fig. 109 shows that more absorption must be introduced (1,000 more units) to reduce the reverberation at a sparsely attended meeting. It was therefore recommended to cover the upper part of the walls with "acoustic plaster," which is marketed for the purpose.

Acoustical Properties of Materials. As regards their action on incident sound waves, materials may be divided into two classes, porous and hard. We saw in Chapter VII that the propagation of sound through narrow channels is attended by rapid reduction of amplitude, by reason of the energy lost in friction. This is the accepted explanation of the true absorbent qualities of porous materials. The absorption is a function of the size of the pores, and a geometric function of the length of the pores, which is the

thickness of the material, i.e., if one inch of material absorbs a quarter of the incident energy, two inches will absorb also a quarter of the remainder, making a loss of seven-sixteenths in all. The intensity thus falls in passing through a material according to an exponential law :—

$$I_x = I_0 e^{-ax},$$

representing the intensity at a distance x from the datum where it is I_0. Some of the sound reflected from porous materials is not turned back at the surface but penetrates the material first. This is shown in Fig. 110 (after Watson [1]), wherein we see that the fraction reflected increases first and tends to a limit.

A partition impervious to air cannot reduce the sound intensity

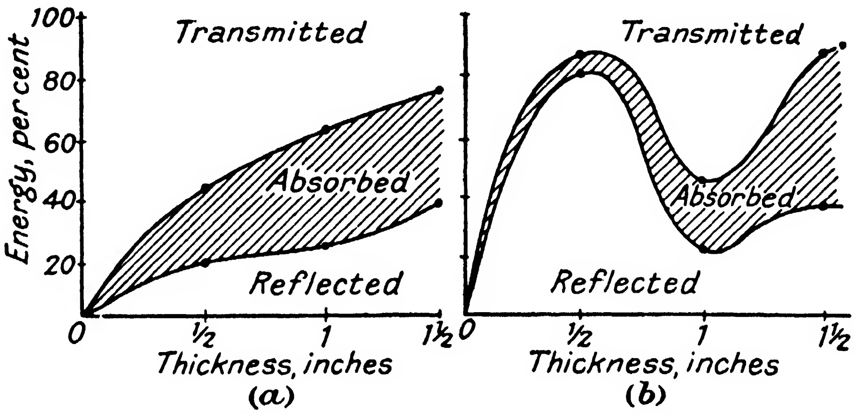


FIG. 110.—Action of Materials on Incident Sound, (*a*) Porous, (*b*) Non-Porous (*Watson*).

by damping air waves set up in it. A rigid partition, prevented from vibrating to and fro, can only transmit the sound by longitudinal waves such that the displacements are at right angles to the surface of the material, and can absorb the energy only by the friction induced by such vibrations. Both these effects are very small since the forces on the surface due to the air waves are generally quite small, so that such a body forms a nearly perfect reflector. In practice, such walls, especially wooden partitions and floors, are only held tightly by their edges and are free to execute lateral vibrations like a drum, under the forcing of the incident air waves, and so to send out corresponding air waves both in front and behind, but mainly in the latter direction. The extent of this response for a given partition will naturally depend

[1] *Phys. Rev.*, **7**, 125, 1916. *See also* Kreüger, *Ing. Vetensk. Akad. Handl.*, **38**, 1924; Paul Sabine, *Phys. Rev.*, **16**, 514, 1920; Davis and Littler, *Phil. Mag.*, **2**, 177, 1927.

on the frequency, and at one pitch there will be resonance, or if the frequency of the source be kept constant, and successive layers be added to the partition, the response will depend on the thickness—at one particular thickness there will be resonance. These considerations explain the anomalous reverberation in buildings at certain frequencies, and why a partition of double thickness may transmit more sound than a single partition. A case of the latter is shown in Fig. 110. Some experiments by Watson on plaster partitions also gave support to this theory of transmission by forced vibration. He arranged a series of wooden beams to press on the partition with a force which could be gradually increased. As the partition was "tuned up" in this way, its power of transmitting notes was changed, and moreover the tone which it transmitted best, its own natural frequency, changed. When the pressure caused cracks to develop in the partition, there were sudden drops in the transmission, until the cleavage was sufficient to open vents in the partition, when of course a considerable proportion of sound got through.

Silencing.[1] Before passing to the insulation of buildings from noise a few words may be said on the prevention of noise in machinery, etc. This problem is becoming an acute one in modern urban life, but the actual remedies lie mainly outside the domain of sound. The main considerations which affect the problems are as follows:

(1) Prevention of moving contact between hard unyielding bodies, e.g., of wheels on road or rail in traction.

(2) Prevention of sudden discontinuities or accelerations in the motion, the "chattering" of valves, the sudden exhaust of high-pressure gases into the air, to be obviated by gradual lowering of the pressure through a silencer.

(3) Oiling of moving parts to prevent squeaks induced by friction.

(4) Balancing of the machinery so that as far as possible moving parts have equal inertia.

Insulation of Sound. Having reduced the noise of machinery to the greatest possible extent, the next problem, if the machinery is located near or in a building, is to insulate it so that as little of the vibrations as possible is transmitted through the floor and walls. The remedy is to introduce discontinuities in the solid path of the waves between the two systems. A motor or stationary

[1] *See also* pp. 233 and 265; Kaye, Smithsonian Rep., 159, 1933.

engine has its bedplate bolted to a concrete floor with layers of wood and felt between. Occasionally it is necessary to construct sound-proof rooms for experiments. In building the walls of such a room the insulation principles already laid down are followed, viz., rigidity of casing, discontinuities in the material, inner materials containing air spaces or pores.[1]

Bedell describes[2] the construction of a room at the Bell Telephone Laboratories designed to simulate an unlimited acoustic field. The walls of the room are covered all over with a number of layers of flannel and muslin separated by air spaces, varying from $\frac{1}{2}$ to 3 in. in thickness, the floor having a grating supported free of the absorbent for the experimenters to walk on. Measurements of the absorption coefficient of the material both by tube methods and in the complete room showed this to average 97 per cent. above 150 c./sec. Such rooms are, of course, of great interest in measurements of audition.

Meyer[3] has tested the frequency characteristics of this type of absorbent, more particularly in the case where the membrane enclosing the air space is not porous. In such a case the natural damped vibrations of the air cell play a part in its sound-absorbing qualities. The electrical analogue of this system is a resistance in series with an inductance and a condenser. The mass m of the membrane fulfils the function of the inductance; the capacitance is that of the cavity and $= l/\rho c^2$, where l is the depth, supposed small compared to the wave-length of the sound. The stiffness of the membrane contributes to the resistance. The natural (fundamental) frequency is then $\frac{c}{2\pi} \cdot \sqrt{\frac{\rho}{ml}}$ when the resistance is neglected. The absorption is naturally very selective. Damping of the air cavity by introduction of cotton-wool increases the absorption but makes the resonance peak sharper; this effect the author ascribes to coupling between membrane and air-cell. He suggests that such a system—e.g., a paper membrane 5 cm. from a wall, with the interspace stuffed with cotton-wool—would be useful in technical acoustics when it is desired to absorb the lower frequencies at the expense of the higher.

Constable[4] has also considered the effect of an absorbent lining

[1] Zwaardemaker, *Konink. Akad. Amsterdam Proc.*, **12**, 706, 1910.
[2] *Acoust. Soc. J.*, **8**, 118, 1936. [3] *Elekt. Nach. Tech.*, **13**, 95, 1936.
[4] *Phys. Soc. Proc.*, **48**, 690, and 914, 1936.

between double partitions, both of which may drum. His theoretical treatment is more general than Meyer's, embracing as it does the ricochetting of the sound energy within the cavity between the two panels, some being absorbed by the lining and another fraction being transmitted through the panel at each rebound. If the absorption coefficient of the lining is not too great, a simple formula for the insulation is obtained, viz.:

$$R = \frac{r_1 r_2}{A} \times \text{(the total absorbing power of cavity)},$$

where R is the net sound reduction, r_1 and r_2 are those due to the panels and A is the area of the panels. He confirmed this formula by tests of double aluminium partitions lined with felt. In conjunction with Aston,[1] this author has also obtained vibration patterns of glass windows and brick walls. In the former case, the vibrations were excited by a near-by loud-speaker, and the amplitude measured by search coils stuck on the glass within the field of an electromagnet. For the brick wall a brass rod attached to a moving-coil loud-speaker was pressed against the wall. The amplitude contours fall into patterns which have a rather remote resemblance to Chladni figures.

For ordinary building an elaborate construction is unnecessary, sufficient insulation being obtainable by using double partitions, floors and doors, with hairfelt or some other insulator between, and making the outer walls massive and rigid. Pipes and metal supports running from one room to another are efficient conductors of sound in the direction of their length, especially if merely suspended instead of being encased in plaster, and should not run directly from one room to the next.

Particular attention has to be paid to the ventilating system, as the pipes composing this debouch directly into the rooms. Transmission of sound by the material of the pipe may be prevented by discontinuities and plaster casing, but the enclosed air acts as a speaking tube between different rooms unless the vibrations of the air column are broken up. This may be accomplished by introducing baffles either in the form of hairfelt, covered with cloth that hair may not be blown along the duct, or of balls of metal gauze. A spreader in the form of an absorbent sheet placed in front of ventilator openings has been found effective. Such devices divert or damp the rapid vibrations which transmit sound,

[1] *Phys. Soc. Proc.*, 48, 919, 1936.

while impeding very little the direct air-flow induced by a powerful fan.

Sound Ranging and Signalling

Sound Ranging. This problem is that of determining the position of an explosive sound from the instants of reception of the sound at three or more listening stations.[1] The recording is done by some sort of microphone, but the currents induced in the microphone circuit are brought to a central station, where they each actuate a separate style writing on a common revolving drum (cf. Regnault's apparatus, p. 7) or deflect a beam of light acting on a moving film in a camera.[2] The three traces are drawn side by side, and from the distance separating the three "kicks" due to the arrival of the sound at the detectors, the position of the source relative to these can be found.

Thus let t_1, t_2, t_3, be the times for the sound to reach the detectors from the source. If t_1 is the shortest time, a circle of radius t_1c with the source as centre represents the distance reached by the wave at this instant (Fig. 111). Circles drawn with the other two stations as centre and having radii $(t_2 - t_1)c$, $(t_3 - t_1)c$ respectively will therefore touch the first circle since $t_2 - t_1$, $t_3 - t_1$, represent the remaining time required for the spherical wave to reach these two stations. Then having the positions of the three stations shown on a map, we can draw a circle of radius $(t_2 - t_1)c$ round the second, and $(t_3 - t_1)c$ round the third; the problem is then to find the centre of a circle which will touch these two and pass through the first station. Practically this is done by having a number of concentric circles drawn on tracing paper, and pushing the paper over the map until the circle which fits best is found.

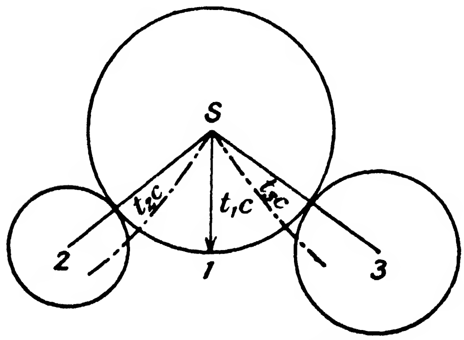


Fig. 111.—Sound-Ranging.

An alternative construction is derived from the consideration that as far as stations 1 and 2 are concerned the locus of S is a curve such that the differences of distance from any point on the curve to the two stations is always $(t_1 - t_2)c$; the curve is therefore a hyperbola. Considera-

[1] *See* Nature, **104**, 278, 1919; Trowbridge, *Frank. Inst. J.*, **189**, 193, 1920.

[2] Blondel, *Comptes Rendus*, **175**, 1,371, 1922.

tion of the 1st and 3rd stations shows that the locus of the source with regard to these is another hyperbola. The two hyperbolas intersect at S. Various instruments have been employed to construct rapidly these hyperbolas.

This sound-ranging device was employed by both sides to locate large guns during the European War, but first apparently by the French under the direction of Esclangon.[1] In reality, sound-ranging, so simple in theory, was not accomplished without a great deal of preliminary experiment. The greatest difficulty was that, owing to the powerful guns employed, the projectile left the muzzle with a speed exceeding sound. Accordingly three sounds were in general registered by each observation post: (1) the *onde de choc* forming the envelope of waves sent out by the projectile as it progressed at a speed exceeding that of sound (cf. p. 26 and Fig. 10*a*); (2) the " gun-wave " which left the muzzle with the projectile

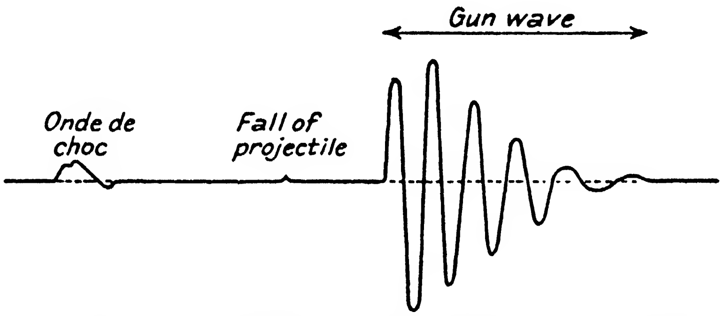


FIG. 112.—Response of Manometer to Firing a Gun (*Esclangon*).

and travelled entirely as a sound wave to the post; (3) the fall or explosion of the projectile which travelled from another spot at a speed c and started some seconds later than (2). Obviously only (2) was of value in determining the *emplacement* of the gun, and unless the gun waves could be picked out from the other sounds, the sound-ranging of such guns would be impossible.

Both *onde de choc* and gunwave give rise to the sensation of detonation (cf. p. 276) being in general followed by a roll of sound due to successive reflections, but the evolution of pressure being slow in either case, definite tones are not heard. Esclangon noticed, however, that the effect of the gun-wave on a resisting surface was much more pronounced than that of the *onde de choc*, the former being able in fact to break windows on occasion. The gun wave involves much slower evolution and much greater amplitude of

[1] *Revue Scientifique*, 115, and 369, 1921.

air motion, and can be detected by a manometer of large volume or instrument of large capacitance, whereas the *onde de choc* is too transient and alternates too rapidly to produce appreciable movements of the large mass though still not fast enough to give the sensation of pitch. As detectors a number of manometers consisting of large reservoirs of air whose motion was detected by membranes, manometric flames, or hot wires were constructed. The typical response in one of these is shown in Fig. 112. This shows how the sound required is easily sorted from the others by the large response of the manometer. Sound ranging was then accomplished from the traces of several of these manometers to the incidence of the gun wave.

From the relative time elapsing between the arrival of these three waves at the station, valuable information as to the muzzle velocity and range of the gun was obtainable. Of course the value of c under the conditions prevailing at the time should be employed, and as this is but imperfectly known the method is open to error. To some extent this can be overcome by testing the results against gun-fire from positions known at the time. Under water this objection does not arise as the velocity of sound therein is enormous as compared with any currents that may exist, and the method was so used for locating U-boats by the British. On sighting a submarine a vessel would explode a small depth charge, whose position was found relative to several submerged detectors near the coast, and so gave the position of the menaced vessel. The explosion of land mines was similarly located, by waves passing through the earth's surface, but in this instance again the value of the velocity is uncertain.

Sound Signals. For signals required to transmit over short distances, there is employed one or other of the devices described in Chapter V for producing a large amplitude of vibration in a diaphragm coupled with a horn. When, however, it is necessary to transmit a fog-signal audible for several miles round, from a lighthouse for example, greater power must be employed. The instrument most in favour is known as the "diaphone," and uses compressed air.[1] It is in principle an oscillating engine, employ-

[1] King, *Roy. Soc. Phil. Trans.*, **218**, 211, 1919. *See also* Mallock, *Roy. Soc. Proc.*, **91**, 71, 1914; Wilson, *Phys. Rev.*, **15**, 178, 1920; Mallett, *Phys. Soc. Proc.*, **39**, 251, 1927.

ing a cylinder, piston, and valves, but fed with compressed air instead of steam, and governed so as to run at half the frequency of the tone to be emitted. This is the auxiliary apparatus and serves to open a valve between the reservoir and a horn, at each end of the piston's stroke. By this means a series of powerful puffs of air, with a very sharp "admission" and "cut-off," is delivered into the horn at its resonant frequency, producing a sound which can be heard for several miles and whose pitch is very nearly independent of the pressure of air in the reservoir. On hearing such a sound in conjunction with a simultaneous wireless signal, or light signal in clear weather, a vessel at sea can estimate its distance from the lighthouse by the time which elapses between the receipt of the practically instantaneous light signal and the sound signal.[1] As the propagation of sound in the atmosphere is attended with all the uncertainties discussed in the first chapter the diaphone is being superseded for such purposes by the submarine signals whose transmission through the sea is more certain.

Dahl and Denk[2] have made a study of the propagation of sound in a gaseous medium whose motion is of complex character. In some instances of the propagation of sound from fog-horns over the sea they noted marked variations in time of the intensity of sound recorded by a condenser microphone at a given point in the sound field. Over a period of three seconds, during which the output of the transmitter remained quite constant, variations of intensity at the source amounted to 10 db. This is ascribed to the turbulence in the air. Rolling and whirling air bodies distributed irregularly through the structure act as scattering centres, particularly important for a sound having a wave-length of the same order as the diameter of the air pockets. They suggest that turbulence in the atmosphere might be studied by such acoustical means.

For signalling through the sea, the submarine bell has long been employed. Normally only the bell and its clapper are submerged, the apparatus floating so that the striking mechanism is above water. The latter is worked by compressed air and actuates the clapper at fixed intervals corresponding to the flashes of light given by a lighthouse. Other bells of less importance to navigation have no mechanism, the clapper hanging free and

[1] Joly, *Roy. Soc. Proc.*, **92**, 101, 1914, and *Phil. Mag.*, **36**, 1, 1918.

[2] *Nature*, **139**, 550, 1937.

the bell striking irregularly under the action of the roll of the buoy to which it is attached on the sea. The sound from such a bell penetrates the sea to a considerable distance, and the submarine bell has stood the test of many years' experience, but when an unusually large range is required, more elaborate and expensive apparatus is needed. The density of water being so much greater than that of air, a considerable expenditure of energy is necessary to produce compressional waves of the needed amplitude, and instruments like the ordinary telephone transmitter are useless.

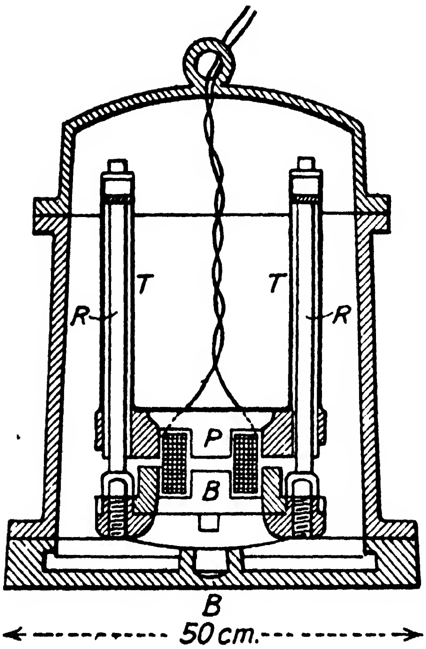


Fig. 113. — Submarine Oscillator (*Du Bois Raymond, Hahnemann, and Hecht*).

The commonest signal, using diaphragm vibrations, is the Fessenden oscillator.[1] Two types are in use, based on the electro-dynamic (cf. p. 130) and electro-magnetic principles respectively. The latter type, as manufactured by the Signal-Ges., Kiel, is shown in section in Fig. 113. The diaphragm is formed of the base *B* of the iron casing of the instrument, which also supports two copper rods *R*, *R* fixed at their upper ends to the ends of two copper tubes *T*, *T* which support another cast-iron plate *P*, over and just clearing *B*. Alternating current led in from the top excites two coils with the requisite frequency, causing violent periodic attraction between *B* and *P*, and exciting longitudinal vibrations in *R*, *R* and *T*, *T*. The net effect is that the base is thrown into powerful transverse vibration, which sends out sound waves into the water.

Both types serve as transmitters and receivers. The submarine signal requires some form of submarine microphone for detection, instead of unaided listening—unless indeed a man should be sus-

[1] *U.S. Pat.*, 1217, 1885. *See also* Du Bois Raymond, Hahnemann and Hecht, *Zeits. tech. Phys.*, **2**, 1 and 337, 1921; Lichte, *Phys. Zeits.*, **20**, 385, 1919, and *Zeits. tech. Phys.*, **2**, 12 and 33, 1921; Du Bois Raymond, *Zeits. tech. Phys.*, **1**, 166, 1920; Ludewig, *Phys. Zeits.*, **21**, 305, 1920.

pended from the ship head downwards in the water. The receiver consists essentially of an elastic metal plate fitted into the side of the ship at some depth below the water surface, which vibrates in sympathy with the sound waves impinging on it. Behind the plate is an air cavity and a tube leading to the listener's ears, so that the sound is ultimately converted into air waves, and detected as such. There is usually no difficulty in recognizing the comparatively high-pitched note of the bell or oscillator from the noise of the machinery which the plate also picks up.

Finding the Direction of a Source of Sound. We have seen in the preceding chapter that the binaural faculty enables us to get a very good idea of the direction in which a source of sound lies. This faculty was used during the war of 1914-18 for locating hostile sounds in all three of the elements, with the aid of intensifying apparatus. The sounds whose direction was sought were usually those of hostile air-craft, and artificial ears consisting of two long conical trumpets were mounted with parallel axes at the ends of a strut several feet in length, which could be turned so as to point towards the source. Connecting tubes ran from the narrow ends of the trumpets to the listener's ears, and their position was adjusted until the intensity at the two ears was the same. In order to get the elevation as well as the bearing of the aeroplane, two pairs of trumpets, each pair having two listening tubes, and two listeners one for each azimuth, are employed, each having control of the appropriate movements. Experience was necessary before the listeners could accurately follow the motion of the aeroplane, and avoid impeding each other's adjustments. Fig. 114 shows the apparatus devised by the French engineers, in which the four trumpets are arranged in a cross. The British instrument had the trumpets arranged in a T pattern on two struts at right angles. For night observation it was arranged to co-ordinate the movements of the listening apparatus with those of the searchlights.

The sensitivity of search on apparatus having trumpets 15 ft. long and 12 ft. apart was reckoned to be 0·1 deg. under favourable conditions. To get greater magnification and consequent sensitiveness of the apparatus, large spherical mirrors of concrete or similar material were constructed by the participating armies, and the listening and direction finding carried out in the neighbourhood of their foci, with apparatus, of course, on a smaller

scale. Generally the mirrors could be turned about an axis to assist in the adjustment.[1]

The listening apparatus led incidentally to a study of the sounds produced by an aeroplane.[2] Among periodic sounds which were discernible that of the exhaust and a number of tones from the air-screw due to lateral vibration of the blades stood out prominently, ranging from 100 to 400 vibrations per second. When the plane is double engined, beats may sometimes be heard between the two engines; there are also the Æolian tones of the struts, but these are probably too weak to be detected at a dis-

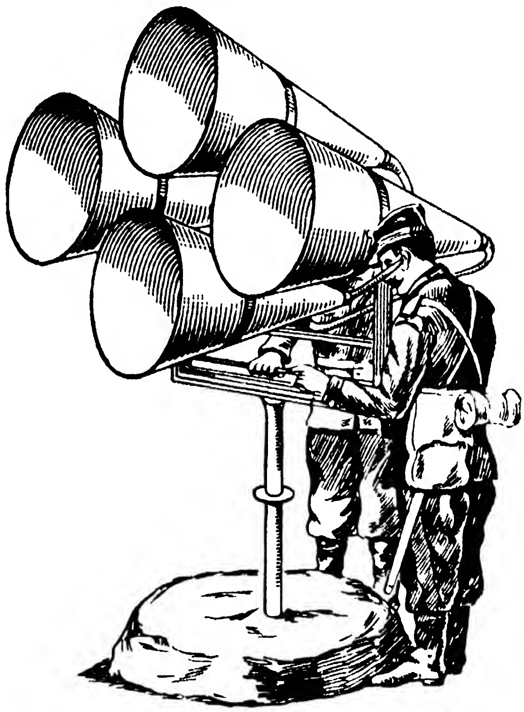


FIG. 114.—Direction Finder.

tance. Beside these there are the "reflection tones" (cf. p. 59).

Similar apparatus is employed for direction finding at sea, one microphone being carried on the port and one on the starboard. The first use of two submarine microphones for direction finding was made by Gray and Murdy, but more extensive tests were made by Blake.[3] Submarine signals sent out at regular intervals

[1] Tucker, *Roy. Aero. Soc. Journ.*, **28**, 504, 1924. *See also* Fage, *Roy. Soc. Proc.*, **107**, 451, 1925; Paris, *Phil. Mag.*, **13**, 99, 1932 and **16**, 50, 1933.

[2] Waetzmann, *Zeits. f. tech. Physik*, **4**, 193, 1923.

[3] *Elect. World*, **49**, 754, 1907; Amerio, *Acad. Linc. Rend.*, **9**, 262, 1929.

by a lightship or buoy are picked up by these microphones, and if adjusted to give equality of response when equally excited they will produce equal sounds when the ship is steering directly to the source. Otherwise the signal will appear louder on that side of the course to which it lies, and the pilot on the bridge holding the stethoscopic connections can steer the vessel accordingly.

Fig. 115 shows a submarine sound-ranging apparatus used by the Germans in the war.[1] The two microphones M laid at a fixed distance d apart in the water actuate loud-speaking telephones T in the air tubes applied to the observer's ears at E. When these have equal sensitivity and are similarly situated in the tube they will reproduce to the observer the impressions on the microphones, and tell him on which side the sound lies. Now instead of turning the system of microphones to face the source, as in the military apparatus, he pushes back the loud-speaker in its tube on the side of the source. This alters, through the increased sound path, the phase and intensity of the sound received at this ear, until ultimately the phases and intensities at the ear pieces are equal, the sound then seems to be straight ahead. If l is the distance of this shift, then l is the extra path *in air* required by the sound on this side, to compensate the additional path $d \sin \phi$ *in water* of the sound on the other side. Hence $\sin \phi = 4{\cdot}3\frac{l}{d}$, taking 4·3 as the ratio of the velocities in water and air respectively, and so ϕ can be determined. Care must be taken against resonance in one or other of the air tubes. This, of course, would render indications of the apparatus useless; but the apparatus

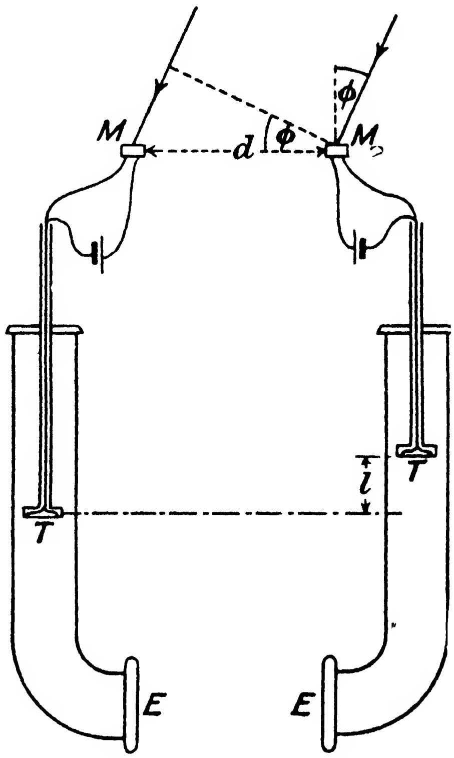


FIG. 115.—Submarine Direction Finder.

[1] Aigner, *Unterwasserschalltechnik*, 310, 1922. *See also* Troller, *La Nature*, **49**, 4, 1921; Hayes, *Am. Phil. Soc. Proc.*, **59**, 371, 1920.

was actually used for pulses such as those from guns. Direction finding by means of a single microphone was found possible by the British, using a "light-body" hydrophone suggested by Bragg[1] (Fig. 116). This instrument consists of a large diaphragm in a lens-shaped case, open to the water on both sides. The carbon microphone itself was placed in a central boss of the diaphragm, and responded to its movements. The principle on which it works is that when a body is in forced vibration under the action of another body the relative amplitudes are inversely as the respective masses. The diaphragm, conversely to that of the Fessenden oscillator, is a light body, less dense than water (a volume of air

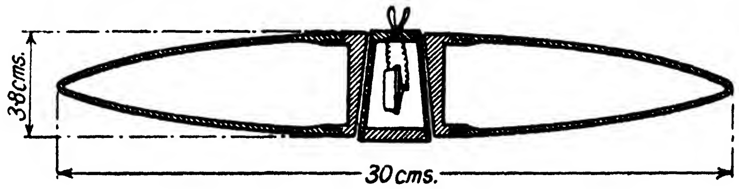


FIG. 116.—Light Body Hydrophone ([*Bragg*] *Wood & Young*).

in a glass case, in the example figured), and so the amplitude of the diaphragm exceeds that of the water, securing enhanced response of the microphone without the bugbear of resonance. It was found that, with the case open at both sides, the response was least when the case and diaphragm were turned edge-on to the sound. With a diaphragm shielded on one side by a baffle, direction is found by turning the hydrophone to face the incident sound, when the response is a maximum.

Echo Sounding. The depth of the sea, or, more strictly speaking, the distance from the surface to the nearest large solid mass, can be determined by timing a sound to the bottom and back. Two methods can be employed. By Behm's method, a small charge is exploded in the water near one side of a ship. The initial and reflected pulse, received on the other side, are recorded by microphones stylographically. On the same graph, and side by side with the microphone traces, appears a sine wave due to

[1] *See* Wood and Young, *Roy. Soc. Proc.*, **100**, 252 and 261, 1921. *See also* Hopwood, *Nature*, **103**, 467, 1919; W. H. Bragg, *Engineering*, **107**, 776, 1919; Wood and Browne, *Phys. Soc. Proc.*, **25**, 183, 1923.

a fork of 1,440 frequency. Now since sound travels 1,440 m. per second in the sea, the number of waves between the initial "kicks" of the two microphones represents the depth sounding in half-metres (the sound traverses the distance twice). Behm claims an accuracy of $\frac{1}{4}$ m. in the estimation. It is necessary to screen the microphone from the direct sound propagated under the keel. Behm [1] has also arranged that the depth can be directly recorded on a dial for the use of pilots. In the dial recorder a disc is set in motion by the firing of the cartridge, and stopped by the arrival of the echo; the angle through which the disc has revolved is calibrated to read heights directly. The apparatus has also been recently employed in measuring the height of an airship (Zeppelin ZR3) above the land.

In the pattern used by the British and United States Navies,[2] the sound is produced by an oscillator excited at certain definite instants. The current is supplied by a contact spindle rotating at constant speed. Also on the spindle is another contact set in an ebonite disc which periodically connects the telephones to the microphone receiving the echo. No sound is heard in the telephones unless the receiving circuit is closed at the instant the echo is received. The contact brush can be displaced round the spindle until the echo is caught, by an amount depending on the depth of the water. The two types differ in detail, but this description illustrates in broad outlines the mechanism of both.

The energy reflected, when a pulse or a sound consisting of a few long-period waves is returned from a surface, will be small, unless the surface is of considerable extent, for only a small part of the emitted energy will fall upon it. For this reason the attempt to use the above apparatus, for detecting the presence of an iceberg or of a large ship in the neighbourhood of a vessel carrying an echo-sounding device, fails. If, however, waves of very short length are emitted through an aperture, interference will destroy the energy diffracted out of the direct path, and a beam of sound will be produced, just as the exceedingly short waves constituting light are concentrated in a beam on passing through an orifice. To render this device effective, waves of frequency above the

[1] *Jahrbuch d. Wiss. f. Luftfahrt*, 56, 1925.

[2] Hayes, *Frank. Inst. J.*, **197**, 323, 1924. *See also Phys. Soc. Exhib. Rep.*, 1926; *Nature*, **116**, 689, 1925; Brillié, *Le Génie Civil.*, **80**, 378 and 387, and 427, 1922; Eisner and Krüger, *Hochfreq.*, **42**, 64, 1933; Delsasso, *Acoust. Soc. J.*, **6**, 1, 1934.

audible limit—supersonic waves—are necessary. It was for this purpose, i.e., sounding, that Langevin[1] first employed the piezo-electric quartz resonator (p. 251). An *ad hoc* aperture is not necessary, the two metal plates forming the electrodes of the crystal resonator (*EE* Fig. 94) being of sufficient length to ensure that the longitudinal waves shall be directed in a beam limited by the planes of the two electrodes. The technique of sounding by means of these ultra-sonic waves is being developed by Boyle and collaborators in Canada.[2]

Langevin obtained a great improvement in the efficiency of the sender by making the electrodes of thick 3 cm. steel, rigidly hold-

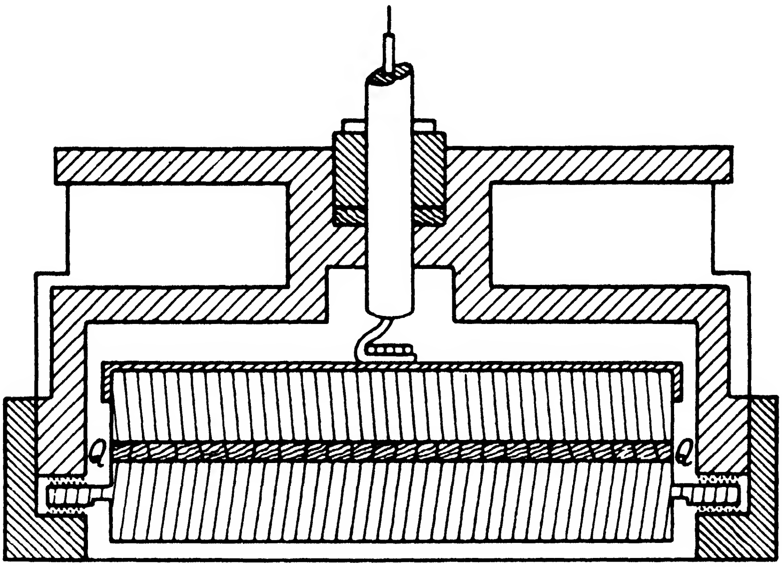


FIG. 117.—Piezo-electric source for Echo-sounding.

ing a number of quartz slabs (*QQ*, Fig. 117) in a sort of sandwich, so that the mass of crystal and steel was forced to vibrate as a whole. This mass possessed much greater elasticity than the original arrangement, and with one steel face in contact with the water it was estimated that, using a peak potential of 2,500 volts between the metal plates, one kilowatt of ultra-sonic power was obtained. Magneto-striction has also been used.

The Langevin-Florisson system for ultra-sonic echo-sounding consists of such a piezo-electric sandwich in contact with the

[1] *See La Nature*, **49**, 4 and 125, 1921.

[2] In *Roy. Soc. Canada Trans.*, Boyle, Lehmann and Reid, **19**, **194**, **1925**; Boyle and Reid, **20**, 233, 1926; Boyle and Taylor, **20**, 245, 1926. *Also* Boyle and Lehmann, *Phys. Rev.*, **27**, 518, 1926; Boyle and Taylor, *Roy. Soc. Canada Trans.*, **21**, 79, 1927; Boyle and Lehmann, **21**, 115, 1927.

water, with its controlling electric circuit in which a short train of rapidly damped electric oscillations is excited by means of an electric spark. This wave-train is converted into a corresponding wave-train of about $\frac{1}{1000}$ second duration, i.e., short compared with the shortest echo-time encountered. The crystal acts as its own detector both of the emitted and returned ultra-sonic waves, as the electric circuit also contains an amplifier which passes on the fluctuations in the circuit, due to departure and arrival of ultra-sonic waves, to a sensitive string galvanometer contained in an "analyser." This analyser is an instrument like Behm's recorder which sets in motion at constant speed a spot of light moving along a scale; and exhibiting "kinks" in its motion at the passage and return of the waves in the manner previously described. For greater accuracy the constant speed of the beam of light is controlled by a phonic motor, worked off a tuning fork. A relative error of 1 per cent. may be introduced within the specified range (4 to 360 metres) on account of variations in velocity with depth; some of this error could, if required, be reduced by correction from tables.

Sound Waves through the Earth's Surface. The Geophone. Sensitive sound detectors for waves passing through the earth have been designed and used in connection with mining, both military and civil. The Geophone, invented by the French during the late war, is actually a small seismograph, consisting of a very heavy lead weight surrounded by a light case to which it is connected circumferentially. On receipt of an impulse the lead weight remains comparatively motionless because it is suspended between the two discs forming the case, and because its mass is so great; so that the motion is taken up by the case. The relative motion of the lead and the case causes compressions and rarefactions in the listening tube at the back. In operation the front of the case is pressed against the earth, preferably rock, and the two free ends of the listening tube to the ear. If two are used relative direction can be determined by the binaural principle. The geophone is now being used in mine rescue work.[1]

Echo-prospecting. Under this name we may class the methods now being developed to determine the depth of mineral strata below the surface of the earth, by reflection of sound waves in the earth from the surface of discontinuity between the softer

[1] *See* Leighton, King and Shaw, *Roy. Soc. Canada Proc.*, **11**, 73, 1917; Waetzmann, *Naturwiss.*, **15**, 401, 1927.

earth and the harder strata. These methods have been exploited by Ambronn,[1] and the details are still secret, but the general principle is illustrated in Fig. 118.

This represents a layer of ore in which the speed of sound is 600 m. per second, beneath a soft layer in which it is only 300 m. per second, and which extends to the ground level. At O compressional waves are excited in the earth. On reaching the surface of separation AB they are partly reflected and partly refracted, but if they are incident on AB at the critical angle (p. 14), they are entirely reflected. The value of the critical angle in this case is 30 degrees for $\sin i = \frac{300}{600} = \frac{1}{2}$. Of the rays which strike AB, some (incident at i less than 30°) will enter the ore,

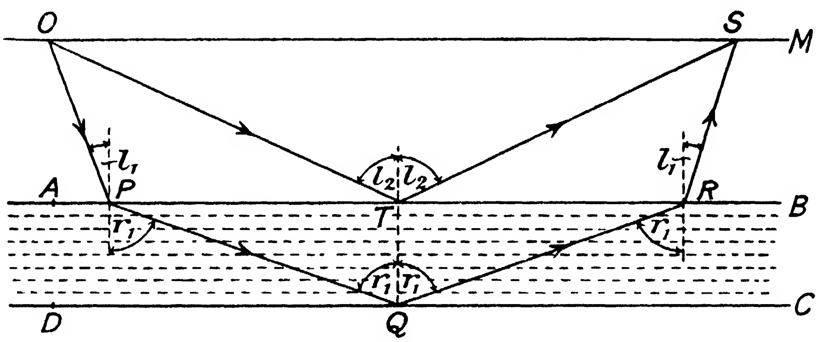


Fig. 118.—Echo-Prospecting.

be partially or entirely reflected at CD (depending on the stratum below CD), refracted again at AB and reach the surface again, following a path such as $OPQRS$; the rest of the energy which they represent returning to the surface direct from P. The rays which strike AB at an angle greater than 30° will be entirely reflected. A certain one of these rays, OTS, incident at an angle i_2, will strike the surface at S, the same point as the ray OP which traversed the lower medium. If $OA = l_1$ and $AD = l_2$, the respective depths of the strata; the time taken for the path $OTS = \dfrac{2l_1}{V_1 \cos i_2}$; for the path $OPQRS$ it is $\dfrac{2l_1}{V_1 \cos i_1} + \dfrac{2l_2}{V_2 \cos r_1}$, where i_1 and r_1 are related by $\dfrac{\sin i_1}{\sin r_1} = \dfrac{V_1}{V_2}$.

It is fairly obvious, under the conditions assumed, $V_2 = 2V_1$, that if S is near O the wave will traverse the path OTS more quickly than the path $OPQRS$, but if we take for S a point dis-

[1] *Die anwendung phys. Auszschlussmethoden in Berg-, Tief- u. Wasserbau,* 1921. *See also* Berger, *Schalltechnik,* 87, 1926.

tant from O, the latter wave will have accomplished a greater portion of its journey in the ore, where its velocity is twice that of the wave following the shorter distance OTS in the soft earth. The refracted wave may therefore arrive before the totally reflected one.

If now, we set up a number of observation stations along the surface OM, and ignore the direct surface wave, the return wave will arrive in regularly increasing times as we recede from O, until suddenly there is a kink in the time curve along OM, representing the refracted wave which has penetrated the ore and is now gaining upon the totally reflected wave. The existence of this discontinuity in the times of transit is a sure sign of a harder stratum below. From these times and from the directions i_1 and i_2 from which the waves return to the surface—found by binaural geophones—the quantities l_1, l_2, V_1, V_2 can be calculated from the preceding equations, giving valuable information as to the nature of the stratum, $ABCD$.

The method should also be applicable when the upper medium is the sea or a lake, and the lower one its bed.

Reproduction of Sound

Mechanical Recording. The gramophone has developed out of the sound analysers of Scott and Edison (cf. p. 207). All the early instruments employed a needle moving along a sinuous groove and directly attached to the centre of a diaphragm clamped to the throat of a small horn. This horn at first increased in flare and length, to the advantage of the reproduction, until æsthetic considerations stepped in and demanded that it should be concealed in some form of cabinet. In spite of the poor reproduction in the bass of such a short horn as—in spite of ingenious foldings—is necessary to fit the confines of the cabinet, this type has persisted.[1]

It is sometimes thought that, for perfect reproduction free from distortion, the trace of the groove should exactly correspond to the wave-pattern in the air outside the horn, that, e.g., a sine wave should be reproduced as a sine curve on the record. A little thought, however, will show that this is not essential. All that we are concerned with in reproduction is the correspondence between the final sound as given out by the instrument when

[1] *See* Williams, *Frank. Inst. J.*, **202**, 413, 1926; Thomas, *J. Sci. Inst.*, **8**, 363, 1931.

the record is played, and the original sound. It does not matter what form the trace takes as long as any change or distortion is compensated in the rest of the reproducing system, diaphragm, horn, etc. As a matter of fact both parts of the system are designed to have a response proportional to the intensity, so that for a range over the whole gamut at the same (physical) loudness, both record and reproduction should give constant energy, n^2a^2, which means that the amplitude, a, is to be inversely as the frequency n. Since $2\pi na$ is the mean velocity in S.H.M., this is sometimes known as the "constant velocity" system of recording. The natural frequency of the diaphragm, usually of mica, is made high in the musical scale by making it thin and clamping it tightly at the edges. Maxfield and Harrison[1] have used the principle of mechanical impedance in order to design a sound box—this being the name given to the diaphragm and needle assemblage—which shall transform the vibrations of the needle most efficiently into sound waves. The diaphragm and the stylus bar which acts as a transformer between it and the pick-up, possess both inertance (in virtue of their respective mass) and capacitance or compliance (due to stiffness). The compliance due to the arm of the stylus near its junction with the diaphragm and that of the air chamber facing it must be in shunt upon that at the edge of the diaphragm where it is clamped, and the stylus where it is pivoted, for if the former were made zero, i.e., were perfectly rigid, they would act as a short circuit, in the sense that no oscillation would be transmitted into the horn. Although it is not possible to calculate these quantities exactly, it is evident that the mechanical impedance of the pick-up must be matched to that of the diaphragm, and this again to that of the horn. The latter point has already been dealt with. In order that the diaphragm may execute piston-like vibrations it must have a high natural frequency, so that it does not develop nodal lines, with consequent out-of-phase sections when responding to notes of high pitch, and must further be loaded by an air cavity; this is the purpose of the air chamber in the throat of the horn. The impedance of this cavity (calculated as for the Helmholtz resonator (p. 229), is to equal that of the horn (p. 240) and that of the diaphragm treated as a piston (= its mass merely). As the diaphragm has, in fact, a certain amount of compliance the upper arm of the stylus bar must be made somewhat resilient to cope with

[1] *Bell Syst. Tech. J.*, **5**, 493, 1926.

this. This was noted as an empirical fact long before these theoretical ideas were worked out. As with any transmitting system the ideal aimed at is to have the impedance of the system a pure resistance over the range of frequencies to be transmitted. At those frequencies, for which the system behaves as a whole as a pure reactance, it acts as a filter. Apart from this extreme case, any reactance in the impedance, i.e., any component of pressure in phase with the velocity, will do no useful work but may do considerable damage by its reaction upon the needle, causing the latter to resist being guided by the groove, and so producing wear. With a hard needle the record will suffer; if a soft fibre needle is used the damage is less serious as it is the needle which is worn.

Electrical Pick-ups. The new vogue which the gramophone received about 1927 was due rather to the introduction of electrical methods of recording than to the improvement in the mechanical reproduction outlined above. In this system, which is like a telephone system with an indefinite time lag, the oscillations of the microphone diaphragm are converted into the corresponding electrical oscillations, amplified if necessary and returned as mechanical oscillations of the recording needle, usually by electro-magnets.[1] A similar circuit terminating in a loud speaker enables the sound to be picked up from the record and reconverted into sound waves in the air at will. This arrangement holds many advantages over the old method. The electrical amplifier enables the instruments and soloists to be placed at reasonable distances from the apparatus which is recording their efforts and to be grouped so that the balance of tone is not upset, whereas formerly the soloist had his head almost inside the horn and was literally hemmed in by a very small orchestra whose accompaniment he effectively swamped in the resulting record. Acoustic adjustment of the room becomes possible instead of the former excessive reverberation which was necessary to add to the depth of the recording groove. Finally electric filters, always more compact and easier to design than acoustic filters, can be incorporated in the circuit, to relieve distortion, tone down the high frequencies of the scratch, etc. Unfortunately, a filter which removes the scratch noise also takes out all the upper tones in the reproduction. To get over this difficulty, Pierce and Hunt[2] suggest

[1] *See* Whitaker, *Sci. Inst. J.*, 5, 35, 1928; Kellogg, *J. Amer. I.E.E.*, 46, 1,041, 1927.

[2] *Acoust. Soc. J.*, 10, 14, 1938; *see also* Braunmühl, *Akust. Zeits.*, 3, 350, 1938.

that the electric record after being picked up should first be passed through an amplifier which exaggerates the extreme treble at the expense of the bass and, *ipso facto*, masks the scratch noise. When the wave-form is then passed on to the loud-speaker, it will have the proper proportions re-established by another amplifier which over-emphasizes the bass. In the resulting reproduction the high frequencies are retained, but the scratch noise—it is said—is mislaid.

Loud Speakers. We have already referred in a number of places to the principle which should underlie a loud speaker. As far as the acoustical part of the apparatus is concerned theory would indicate the exponential horn (p. 242) as the most effective type. The piston type is, however, more often met with. This consists of a large waxed paper diaphragm, one or two feet across, which is set back at its centre to form an obtuse angled cone. Sometimes two cones one behind the other may be found. The cone is actuated at its centre by the electrical mechanism. As the diaphragm is comparatively loose it has no marked resonance though such resonance as there is lies in the bass. Owing to the asymmetric loading the membrane is liable to produce distortion, particularly at higher frequencies. Formerly the centre of the diaphragm was vibrated by a steel stylus in an alternating magnetic field but, *ceteris paribus*, the moving coil type of reproducer is found to produce less distortion.[1] This bears the same relation to the older type as the D'Arsonval to the tangent galvanometer. It consists of a light coil to which the current is fed, and which is fixed to the diaphragm and oscillates in the magnetic field of a permanent magnet. Other distortion-less loud speakers have been operated on the electrostatic system, i.e., they are overgrown condenser microphones ; while for frequencies above 10,000 cycles/sec. at which the response of these falls off, one operating on the piezo-electric principle (cf. Chap. XI) is now available.

In the apparatus used by Davis[2] for obtaining rapidly the response curve of a loud speaker, the voltage from the microphone is taken after amplification to one pair of electrodes of a cathode ray oscillograph. To the other pair of plates at right angles to the first is led from the valve oscillator a D.C. voltage whose value is proportional to the log of the frequency. This is accom-

[1] *See* McLachlan, *Roy. Soc. Proc.*, **122**, **604**, **1929**.

[2] *Phil. Mag.*, **15**, **309**, **1933**.

plished by passing the output through a frequency-weighting network and amplifier.

Thus the electron spot in the viewing screen receives a horizontal displacement proportional to the frequency and a vertical one proportional to the output of the microphone. As then the oscillator is taken through the gamut, the spot on the screen traces out the response curve (which can be photographed) of the loud speaker as recorded by the distortion-free microphone. The frequency-weighting network, to which reference has been made, consists essentially of a resistance-capacity stage of amplification in which the coupling condenser is small. Consequently the amplification factor increases nearly as the frequency. In the rectifier which follows, the D.C. output is made nearly proportional to the log of the frequency.

Sound Film. Two methods of recording sound on film are in commercial use. In one method light is reflected from a mirror which oscillates with the sound on the same principle as the phonodeik (p. 208), is passed through a slit and produces a record of *variable width* having a serrated edge. The other recorder is virtually a string oscillograph in which there are two strings or ribbons of duralumin $0{\cdot}0005 \times 0{\cdot}006$ in. forming a " light valve " through which the light passes to affect the film to a greater or less extent. The record on the developed film is of fixed width but *variable density*. The ribbons are tensioned to a natural frequency of 9,500 cycles per second, which is above the band at present recorded. There is a certain amount of " background noise " due to faults in the recording apparatus. It is obvious that this background becomes more obnoxious as the level of recorded sound is lowered. A steady current is accordingly passed through the ribbons so as to close the gap completely when no sound is being recorded. As the intensity level of the latter increases the biassing current is automatically reduced releasing the ribbons, for the background noise will then become more completely masked by the wanted sound.[1]

In reproduction the film runs *continuously* through a gate in the projector, a short distance ahead of the picture gate itself. The light from an incandescent filament after crossing the sound track falls on a photoelectric cell, the current from which is amplified and fed to the loud speakers.

[1] *See* Bristol, *Frank. Inst. J.*, **205**, 179, 1928; Sandvick, *ibid.*, **207**, 839, 1929; Frederick, *Rev. Sci. Inst.*, **5**, 117, 1934.

We must also mention here the process sometimes known as the Blattnerphone, whereby speech or music may be recorded by magnetic means on a steel strip and reproduced later. This is now frequently used by the B.B.C. when a temporary record of a speech is required during broadcasting, to be radiated later in the day in another programme. Although this official adoption is recent, the process dates back to the early days of wireless telegraphy and was a device used for tape-recording by Poulson. A continuous steel band is carried round on pulleys in front of a coil through which an alternating current engendered by the speech or music passes. This magnetizes the steel in the direction of its motion in a corresponding fashion. When reproduction is desired, the band is passed in front of another coil with Permalloy core and the induced current is fed into an amplifier. The record can be "washed off" for subsequent use by passing it near a sufficiently strong direct current or alternating current in this wise, that the first saturates the iron, the latter demagnetizes it by taking it through hysteresis cycles of successively smaller amplitudes as the band retreats from its influence.

Electric Musical Instruments.—These are of three main types: (1) Instruments which retain, in their construction, considerable portions of conventional instruments. Such are the pianofortes in which the vibrations of the wires, struck in the usual fashion, are picked up by electromagnets near them and the resulting induced currents after amplification are passed to a loud speaker. Such pianofortes have no soundboards.

(2) Instruments which claim to produce "music from the æther" and are actually self-excited valve oscillators giving notes whose quality and frequency are determined by the circuit details and, in particular, the capacity in the tuned portion. The capacity is varied by moving a sliding condenser or, more simply, by approaching the hand to give an infinite variation in frequency. Others have keyboards giving a step-by-step variation of the capacity of the effective condenser.

(3) Keyboard instruments in which the frequency of each note depends on periodic variations in capacity of condensers with revolving plates or on the number of studs on a revolving axle which pass an electromagnetic pick-up in one second. In some, for example, the Hammond organ, the designer has so shaped the stud, that, after a little judicious filtering of the electric current,

a S.H.M. of known frequency is obtained from every pick-up. The quality, which is capable of being pre-set by the player, is made up of mixtures of the fundamental with harmonics, derived from S.H.M.'s picked up further along the rotor, where there is a greater number of studs per revolution. The resulting synthesized current is amplified and fed to the loud-speakers. Others purposely pick up a distorted wave, by using saw-edge teeth on the axle, and derive their desired smoothed current by suitable filters. In the last class, the rotating axle and stationary electromagnets is sometimes replaced by a stroboscopic disc interrupting periodically the light which falls on a photo-electric cell, the current from which is the source of the sound.

The quality of these instruments depends ultimately on the faithfulness of the reproduction in the loud-speaker, and it is this limitation which at the moment impedes their more general adoption.

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